INTRODUCTION TO HYDROLOGY

WARREN VIESSMAN, JR.

TERENCE E. HARBAUGH

JOHN W. KNAPP

INTRODUCTION TO HYDROLOGY

The Intext Series in Civil Engineering

Consulting Editor

RUSSELL C. BRINKER
Visiting Professor of Civil Engineering
New Mexico State University

BOUCHARD and MOFFITT—Surveying, 5th ed.

BRINKER—Elementary Surveying, 5th ed.

CLARK, VIESSMAN, and HAMMER—Water Supply and Pollution Control, 2d ed.

GHALI and NEVILLE—Structural Analysis: A Unified Classical and Matrix Approach

JUMIKIS—Foundation Engineering

McCORMAC—Structural Analysis, 2d ed.

McCORMAC—Structural Steel Design, 2d ed.

MEYER—Route Surveying and Design, 4th ed.

MOFFITT—Photogrammetry, 2d ed.

SALMON and JOHNSON—Steel Structures: Design and Behavior

URAL—Matrix Operations and Use of Computers in Structural Engineering

VIESSMAN, HARBAUGH, and KNAPP—Introduction to Hydrology

WANG—Matrix Methods of Structural Analysis, 2d. ed.

WANG and SALMON—Reinforced Concrete Design

WINFREY—Economic Analysis for Highways

INTRODUCTION TO HYDROLOGY

WARREN VIESSMAN, JR.
Director, Water Resources Research Institute
and Professor of Civil Engineering
University of Nebraska

TERENCE E. HARBAUGH
Professor of Civil Engineering
University of Missouri—Rolla

JOHN W. KNAPP
Professor of Civil Engineering
Virginia Military Institute

Intext Educational Publishers
New York/London

In memory of our friend and colleague
Terence E. Harbaugh

U-I-RC

Copyright © 1972 by Intext, Inc.
All rights reserved.

Library of Congress Catalog Card Number
72-84131
ISBN 0-7002-2403-3

CONTENTS

Preface . . . . . *vii*

1	Introduction	1
2	Precipitation and Hydrologic Abstractions	13
3	Stream Flow	58
4	Point-Frequency Analysis	106
5	Regional Analysis: Joint Distributions and Time Series	147
6	Hydrologic and Hydraulic Routing	173
7	Groundwater Hydrology	223
8	Minor Structure Design	269
9	Major Structure Design	316
10	Hydrologic Simulation	330
11	Snow Hydrology	351
	Appendix A	397
	Appendix B	401
	Appendix C	402

Index 405

PREFACE

All aspects of the environment have received increased emphasis in recent years, and a desire to minimize the ecologic impact on development and use of our natural resources is firmly established. As a result, it is more important than ever for young engineers and scientists to understand the nature of the environment before attempting to manipulate it or govern its uses. This book deals with one important aspect—water.

Material presented is suitable for either a one- or a two-semester beginning course in hydrology. Although primarily intended as an undergraduate book, the contents are sufficiently rigorous for a first graduate course as well. Advanced students will find it necessary to consult other references and the literature. The book emphasizes hydrologic principles and their practical application to realistic problems. In this respect, practitioners should find the material useful as a reference.

Frequent citations are made to computer applications and several programs included. Many solved examples illustrate the principles developed. Statistical and numerical methods are widely employed.

Many sources supply subject matter for the book. The authors hope that suitable acknowledgment has been given to all of them, and wish to thank their colleagues and students for helpful comments and review.

Warren Viessman, Jr.
Terence E. Harbaugh
John W. Knapp

1

INTRODUCTION

1-1 Hydrology Defined

Hydrology is one of the earth sciences. It encompasses the occurrence, distribution, movement, and properties of the waters of the earth and their environmental relationships. Because of its scope, it is closely related to such fields as geology, climatology, meteorology, and oceanography, to name but a few. The emphasis of this book is on engineering hydrology, "the practicing art."

1-2 A Brief History

Ancient philosophers focused their attention on the nature of processes involved in production of surface flows and other phenomena related to the origin and occurrence of water in various stages of the perpetual cycle of water being conveyed from the sea to the atmosphere to the land and back again to the sea. Unfortunately, early speculation was often faulty.[1,4–6]* For example, Homer believed in the existence of large subterranean reservoirs which furnished rivers, seas, springs, and deep wells. It is interesting to note, however, that Homer did understand the dependence of flow in the Greek aqueducts on both conveyance cross section and velocity. This knowledge was lost to the Romans and the proper relationship between area, velocity, and rate of flow remained unknown until Leonardo da Vinci discovered it during the Italian Renaissance.

During the first century B.C. Marcus Vitruvius, in Volume 8 of his treatise *De Architectura Libri Decem* (the engineer's chief handbook during the Middle Ages) set forth a theory generally considered to be the predecessor of modern notions of the hydrologic cycle. He hypothesized that rain and snow falling in mountainous areas infiltrated the earth's surface and later appeared in the lowlands as streams and springs.

*Superior numbers indicate references at end of chapter.

In spite of the inaccurate theories proposed in ancient times, it is only fair to state that practical application of various hydrologic principles was often carried out with considerable success. For example, about 4000 B.C. a dam was constructed across the Nile to permit reclamation of previously barren lands for agricultural production. Several thousand years later a canal to convey fresh water from Cairo to Suez was built. Mesopotamian towns were protected against floods by high earthen walls. The Greek and Roman aqueducts and early Chinese irrigation and flood-control works are also significant projects.

Near the end of the fifteenth century the trend toward a more scientific approach to hydrology based on observation of hydrologic phenomena became evident. Leonardo da Vinci and Bernard Palissy independently reached an accurate understanding of the water cycle. They apparently based their theories more on observation than on purely philosophical reasoning. Nevertheless, until the seventeenth century it seems evident that little if any effort was directed toward obtaining quantitative measurements of hydrologic variables.

The advent of what might be called the "modern" science of hydrology is usually considered to begin with the studies of such pioneers as Perrault, Mariotte, and Halley in the seventeenth century.[1,4] Perrault obtained measurements of rainfall in the Seine River drainage basin over a period of three years. Using these and measurements of runoff, and knowing the drainage-area size, he showed that rainfall was adequate in quantity to account for river flows. He also made measurements of evaporation and capillarity. Mariotte gauged the velocity of flow of the River Seine. Recorded velocities were translated into terms of discharge by introducing measurements of the river cross section. The English astronomer Halley measured the rate of evaporation of the Mediterranean Sea and concluded that the amount of water evaporated was sufficient to account for the outflow of rivers tributary to the sea. Measurements such as these, although crude, permitted reliable conclusions to be drawn regarding the hydrologic phenomena being studied.

The eighteenth century brought forth numerous advances in hydraulic theory and instrumentation. The Bernoulli piezometer, the Pitot tube, Bernoulli's theorem, and the Chézy formula are some examples.[8]

During the nineteenth century experimental hydrology flourished. Significant advances were made in groundwater hydrology and in the measurement of surface water. Such significant contributions as Hagen-Poiseuille's capillary flow equation, Darcy's law of flow in porous media, and the Dupuit-Thiem well formula were evolved.[9–11] The beginning of systematic stream gauging can also be traced to this period. Although the basis for modern hydrology was well established in the nineteenth century,

much of the effort was empirical in nature. The fundamentals of physical hydrology had not yet been well established or widely recognized. In the early years of the twentieth century the inadequacies of many earlier empirical formulations became well known. As a result, interested governmental agencies began to develop their own programs of hydrologic research. From about 1930 to 1950, rational analyses began to replace empiricism.[3] Sherman's unit hydrograph, Horton's infiltration theory, and Theis's nonequilibrium approach to well hydraulics are outstanding examples of the great progress made.[12–14]

Since 1950 a theoretical approach to hydrologic problems has largely replaced less sophisticated methods of the past. Advances in scientific knowledge permit a better understanding of the physical basis of hydrologic relationships, and the advent and continued development of high-speed digital computers have made possible, in both a practical and an economic sense, extensive mathematical manipulations that would have been overwhelming in the past. Effective utilization of "men, models, methods, and machines" appears to be at hand for the science of hydrology.[7]

For a more comprehensive historical treatment, the reader is referred to the works of Meinzer, Jones, Biswas, et al.[1,2,4,5,15]

1-3 The Hydrologic Cycle

The hydrologic cycle is a continuous process by which water is transported from the oceans to the atmosphere to the land and back to the sea. Many subcycles exist. Evaporation of inland water and its subsequent precipitation over land before returning to the ocean is one example. The driving force for the global water-transport system is provided by the sun which furnishes the energy required for evaporation. Note that the water quality also changes during passage through the cycle; for example, sea water is converted to fresh water through evaporation.

The complete water cycle is global in nature. World water problems require studies on regional, national, international, continental and global scales.[17] Practical significance of the fact that the total supply of fresh water available to the earth is limited and very small compared with the salt-water content of the oceans has received little attention. Thus waters flowing in one country cannot be available at the same time for use in other regions of the world. Raymond L. Nace of the U.S. Geological Survey has aptly stated that "water resources are a global problem with local roots."[17] Modern hydrologists are obligated to cope with problems requiring definition in varying scales of significant order of magnitude difference. In addition, developing techniques to control weather must

receive careful attention, since climatological changes in one area can profoundly affect the hydrology and therefore the water resources of other regions.

1-4 The Hydrologic Budget

Because the total quantity of water available to the earth is finite and indestructible, the global hydrologic system may be looked upon as closed. Open hydrologic subsystems are abundant, however, and these are usually the type analyzed by the engineering hydrologist. Regardless of the nature of the system, a water budget can be developed to account for the hydrologic components.

To illustrate, assume the simple and highly restricted system of Fig. 1-1.

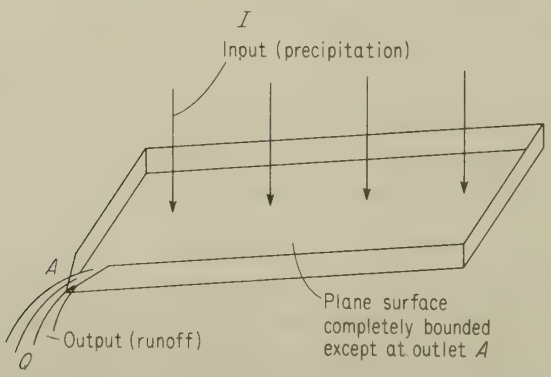


FIG. 1-1. Simple hydrologic system model.

Consider a completely impervious inclined plane surface (water cannot be transmitted through the surface), confined on all four sides with an outlet at corner A. Since this surface is assumed to be a perfect plane, there are no depressions in which water can be stored. If a rainstorm input is applied, an output or outflow, designated as surface runoff, will be developed at A. The hydrologic water budget for this system can be represented by the following differential equation:

$$I - Q = \frac{dS}{dt} \tag{1-1}$$

where I is inflow per unit time, Q is outflow per unit time and dS/dt is the change in storage within the system per unit time. Until a minimal depth is accumulated on the surface, outflow cannot occur, but, as a storm intensifies, the depth retained on the surface (surface detention) increases. At the cessation of input, water held on the surface becomes the outflow from the system. For the example cited, all input eventually becomes out-

flow, neglecting the small quantity electrically bound to the surface and any evaporation during the period of input (a reasonable assumption for the system depicted). This elementary illustration should suggest that any hydrologic system can be described by a hydrologic budget which accounts for the disposition of inputs to the system and changes in storage. The simplicity of the budget equation is often misleading, however, since, as will be seen later, the terms in the equation cannot always be adequately or easily quantified.

A more generalized version of the hydrologic budget will explain the various components of a hydrologic cycle and provide an insight to problem-solving techniques for composite hydrologic regions. Such regions may be topographically defined, politically limited, or arbitrarily specified. A watershed or drainage basin is a topographically defined area drained by a river/stream or system of connecting rivers/streams such that all outflow is discharged through a single outlet. Past water-resources studies have often been conducted on watersheds or drainage basins because these areas simplify application of the water budget. Theoretically, an accounting is possible for any kind of region, although data availability and degree of refinement of analytical methods will determine the feasibility of this in a practical sense.

To demonstrate the nature of a generalized hydrologic budget, use is made of Figs. 1-2, 1-3, and 1-4. Figure 1-2 is a conceptual model of the

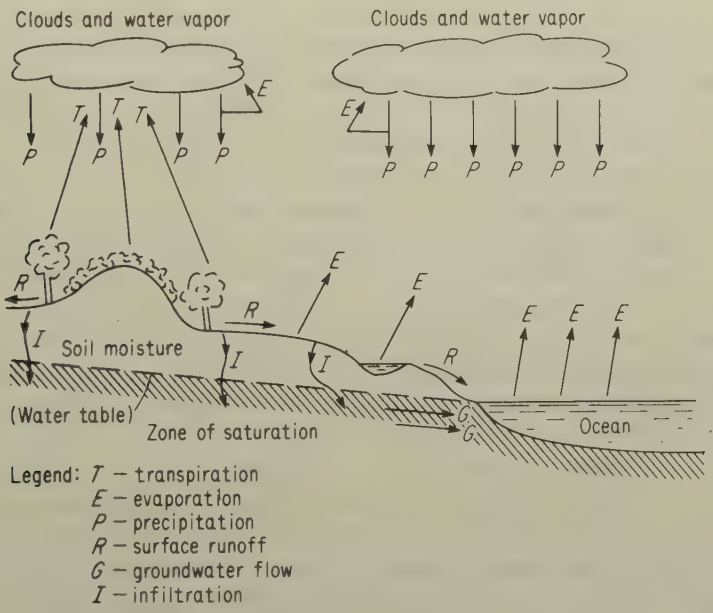


FIG. 1-2. The hydrologic cycle.

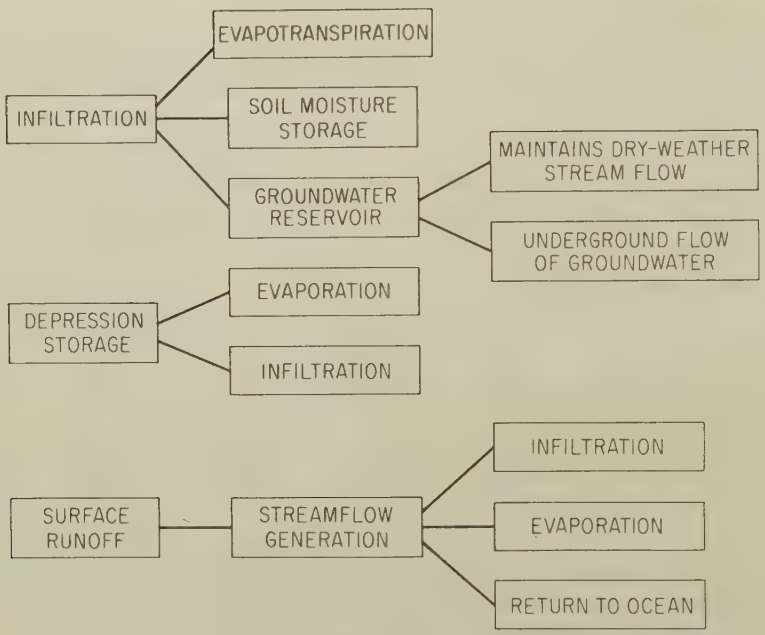


FIG. 1-3. Flow diagrams indicating disposition of infiltration, depression storage, and surface runoff.

hydrologic cycle. Precipitation in the form of rain, snow, hail, etc., comes from atmospheric water vapor and constitutes the primary input. Some rainfall may be intercepted by trees, grass, other vegetation, and structural objects, and will eventually return to the atmosphere by evaporation. Once precipitation reaches the ground, some of it may fill depressions (become depression storage), part may penetrate the ground (infiltrate) to replenish soil moisture and groundwater reservoirs, and some may become surface runoff—that is, flow over the earth's surface to a defined channel such as a stream. The flow diagrams of Fig. 1-3 show the disposition of infiltration, depression storage, and surface runoff.

Water entering the ground may take several paths. Some may be directly evaporated if adequate transfer from the soil to the surface is maintained. This can easily occur where a high groundwater table (free water surface) is within the limits of capillary transport to the ground surface. Vegetation using soil moisture or ground water directly can also transmit infiltrated water to the atmosphere by a process known as *transpiration*. Infiltrated water may likewise replenish soil-moisture deficiencies and enter storage provided in groundwater reservoirs which in turn maintain dry-weather stream flow. Important bodies of groundwater are usually flowing so that infiltrated water reaching the saturated zone

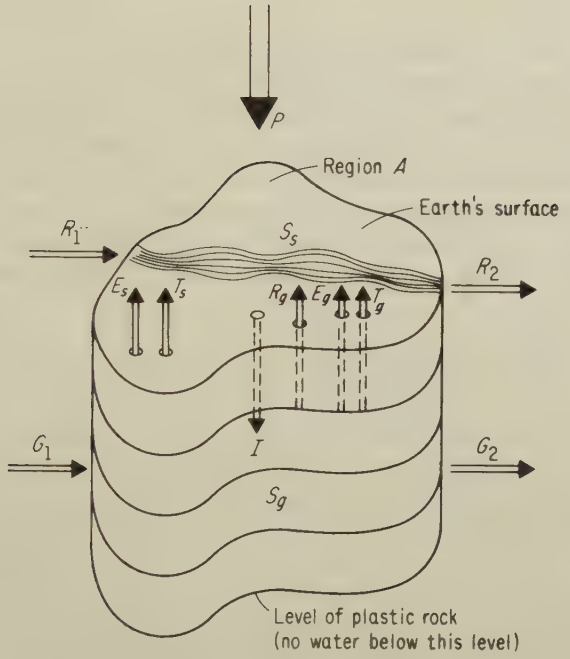


FIG. 1-4. Schematic diagram of hydrologic cycle for a region.

may be transported for considerable distances before it is discharged. Groundwater movement is subject, of course, to physical and geological constraints.

Water stored in depressions will eventually evaporate or infiltrate the ground surface. Surface runoff ultimately reaches minor channels (gullies, rivulets, and the like), flows to major streams and rivers, and finally reaches an ocean. Along the course of a stream, evaporation and infiltration can also occur.

The foregoing discussion suggests that the hydrologic cycle, while simple in concept, is actually exceedingly complex. Paths taken by particles of water precipitated in any area are numerous and varied before the sea is reached. The time scale is on the order of seconds, minutes, days, or years.

A general hydrologic equation can be developed based on the concepts illustrated in Figs. 1-2 and 1-3. Figure 1-4, a more abstract version of Fig. 1-2, represents schematically the hydrologic cycle of a region and it serves a useful purpose, since it is readily translated into mathematical terms. Hydrologic variables P, E, T, R, G, and I are as defined in Fig. 1-2. Subscripts g and s are introduced to denote vectors originating above and

below the earth's surface respectively. For example, R_g signifies groundwater flow which is effluent to a surface stream, and E_s represents evaporation from surface water bodies or other surface storage areas. Letter S stands for storage. The region under consideration specified as A has a lower boundary below which water will not be found. The upper boundary is the earth's surface. Vertical bounds are arbitrarily set as projections of the periphery of the region. Remember from Eq. 1-1 that the water budget is a balance between inflows, outflows, and changes in storage. Then Fig. 1-4 can be translated into the following mathematical statements:

(1) hydrologic budget above the surface

$$P + R_1 - R_2 + R_g - E_s - T_s - I = \Delta S_s \tag{1-2}$$

(2) hydrologic budget below the surface

$$I + G_1 - G_2 - R_g - E_g - T_g = \Delta S_g \tag{1-3}$$

(3) hydrologic budget for the earth (sum of Eqs. 1-2 and 1-3)

$$P - (R_2 - R_1) - (E_s + E_g) - (T_s + T_g) - (G_2 - G_1) = \Delta(S_s + S_g) \tag{1-4}$$

where all values are given in units of volume per unit time.

If the subscripts are dropped from Eq. 1-4 so that letters without subscripts refer to total precipitation and net values of surface flow, underground flow, evaporation, transpiration, and storage, the hydrologic budget for a region can be written simply as

$$P - R - G - E - T = \Delta S \tag{1-5}$$

This is the basic equation of hydrology. For the simplified hydrologic system of Fig. 1-1, terms G, E, and T do not apply and Eq. 1-5 reduces to

$$P - R = \Delta S \tag{1-6}$$

which is basically the same as the differential Eq. 1-1. The general equation is applicable to exercises of any degree of complexity and therefore basic to the solution of all hydrologic problems.

1-5 Application of the Hydrologic Equation

The difficulty in solving practical problems lies mainly in the inability to properly measure or estimate the various hydrologic equation terms. For local studies, reliable estimates often are made, but on a global scale quantification is usually crude. Precipitation is measured by rain or snow gauges located throughout an area. Surface flows can be measured using various devices such as weirs, flumes, velocity meters, and depth gauges located in the rivers and streams of the area. Under good conditions these measurements are frequently 95 percent or more accurate, but large

floods cannot be directly measured by current methods and data on such events are sorely needed. Soil moisture can be measured using neutron probes and gravimetric methods; infiltration determined locally by infiltrometers or estimated through the use of precipitation-runoff data. Areal estimates of soil moisture and infiltration are generally very crude, however. The extent and rate of movement of groundwater are usually exceedingly difficult to determine and adequate data on quantities of groundwater normally do not exist. Knowledge of the geology of a region is essential for groundwater estimates if they are to be more than just rough guides. Determination of the quantities of water evaporated and transpired is also extremely difficult under the present state of development of the science. Most estimates of evapotranspiration are obtained by using evaporation pans, energy budgets, mass-transfer methods, or empirical relationships. A predicament inherent in the analysis of large drainage basins is the fact that rates of evaporation, transpiration, and groundwater movement are often highly heterogeneous.

The hydrologic equation is an exceedingly useful tool and the reader should understand that it can be employed in various ways to estimate the magnitude and time distribution of hydrologic variables. An introductory example is given here, and others will be found throughout the book.

Example 1-1 In a given year, a 10,000-sq-mi watershed received 20 in. of precipitation. The annual rate of flow measured in the river draining the area was found to be 6,000 cfs (cubic feet per second). Make a rough estimate of the combined amounts of water evaporated and transpired from the region during the year of record.

Beginning with the basic hydrologic equation

$$P - R - G - E - T = \Delta S \qquad \textbf{(1-5)}$$

and since evaporation and transpiration can be combined,

$$ET = P - R - G - \Delta S \qquad (1\text{-}7)$$

The term ET is the unknown to be evaluated and P and R are specified. The equation thus has 5 variables and 3 unknowns and cannot be solved without additional information.

In order to get a solution, two assumptions will be made. First, since the drainage area is quite large (measured in hundreds of square miles), a presumption that the groundwater divide (boundary) follows the surface divide will probably be reasonable. In this case the G component may be considered zero. The vector R_g exists but is included in R. The foregoing assumption is usually not valid for small areas and must therefore be used carefully. It will also be presupposed that $\Delta S = 0$ thus implying that the groundwater reservoir volume has not changed during the year. For such short periods this fiction can be very inaccurate, even for well-watered regions with balanced withdrawals and good recharge potentials. In arid areas where groundwater is being mined (ΔS consistently negative) it would be an unreasonable supposition in many cases.

Nevertheless, the assumption will be made here for illustrative purposes and qualified by saying that past records of water levels in the area have revealed an approximate constancy in groundwater storage. Hydrology is not an exact science, and reasonable well-founded assumptions are required if practical problems are to be solved.

Using the simplifications just outlined, the working relationship reduces to

$$ET = P - R$$

which can be solved directly. First change R into inches per year so that the units are compatible:

$$\frac{\text{ft}^3}{\text{sec}} \times \frac{1}{\text{area} - (\text{ft}^2)} \times \frac{\text{sec}}{\text{yr}} \times \frac{\text{in.}}{\text{ft}} = R, \text{ in.}$$

$$R = \frac{6000 \times 86{,}400 \times 365 \times 12}{10^4 \times (5280)^2} = 8.1 \text{ in.}$$

Therefore, $ET = 20 - 8.1 = 11.9$ in./yr

The amount of evapotranspiration for the year in question is estimated to be 11.9 in. This is admittedly a crude approximation but could serve as a useful guide for water-resources planning.

1-6 Hydrologic Data

Hydrologic data are needed to describe precipitation; streamflows; evaporation; soil moisture; snow fields; sedimentation; transpiration; infiltration; water quality; air, soil and water temperatures; and other variables or components of hydrologic systems. Sources of data are numerous with the U.S. Geological Survey the primary one for streamflow and groundwater facts. The U.S. Weather Bureau (NOAA or National Oceanic and Atmospheric Administration) is the major collector of precipitation data. Many other federal, state, and local agencies and other organizations also compile hydrologic data. For a complete listing of these, see Refs. 3 and 16.

1-7 Application of Hydrology to Environmental Problems

The principle that man cannot exist without water is fundamental. It is also true that at times water becomes a threat to the health and welfare of individuals, communities, and even regions. Because water is such an indispensable and yet potentially damaging resource, its effective management has become an important goal of modern society.

Everyone uses water in a variety of ways; for drinking, bathing, in industry and agriculture, and in many other important applications. In 1967 the U.S. Geological Survey estimated that the average per capita rate of consumption of water in American cities was 155 gallons per day, triple the amount used a hundred years ago. In future years it appears

that the average rate will continue to increase although at a pace likely to be tempered by technological changes and improved management policies. A little reflection on the nature of the trend in population shift from rural to urban centers underlines the increasing burden being placed on the water resources of heavily populated regions.

Although the use of water for domestic activities is vital, its use in industry, commerce, agriculture, recreation, and esthetics is also basic to the national welfare. Water for recreation has become the most important single use in some multiple-purpose water-resource development projects, and this drift is expected to accelerate.

Our national concern over environmental quality is tied in part to the country's water resource. It is clear that improved techniques for planning, managing, and developing the environment are high on the priority list. A greater emphasis will have to be placed on humanistic and social values. At the same time, effective planning and management will result only if we are able clearly to understand the physical systems with which we must deal. This is the challenge to the hydrologist. Reliable mathematical models are needed to evaluate and predict the performance of hydrologic systems under expected or known conditions of stress. The effects of physical works such as dams, drainage systems, channelization, and the like must be understood before construction begins. The hydrologist has to play a fundamental role in our quest for a better managed and understood environment. His competence will have a tremendous impact on the success of all water-resources activities, regardless of scale.

It is the intent of this book to provide students with a basic understanding of the physical aspects of hydrologic systems, and the manner in which hydrologic components can be synthesized for effective planning and preparation of design models.

References

1. Jones, P. B., G. D. Walker, R. W. Harden, and L. L. McDaniels, "The Development of the Science of Hydrology," Circular No. 63-03, Texas Water Commission, April 1963.

2. Mead, W. D., *Notes on Hydrology.* Chicago: D. W. Mead, 1904.

3. Chow, Ven Te (ed.), *Handbook of Applied Hydrology.* New York: McGraw-Hill Book Company, 1964.

4. Meinzer, O. E., *Hydrology,* Vol. IX of *Physics of the Earth.* New York: McGraw-Hill Book Company, 1942. Reprinted by Dover Publications, Inc., New York, 1949.

5. Krynine, P. D., "On the Antiquity of Sedimentation and Hydrology," *Bull. Geol. Soc. Am.,* Vol. 70 (1960).

6. Kazmann, Raphael G., *Modern Hydrology.* New York: Harper & Row Publishers, 1965.

7. Snyder, Willard M., and John B. Stall, "Men, Models, Methods, and Machines in Hydrologic Analysis," *Proc. A.S.C.E., J. Hy. Div.,* Vol. 91, No. HY2 (March 1965).

8. Rouse, Hunter, and Simon Ince, *History of Hydraulics*, Iowa Institute of Hydraulic Research, State University of Iowa, 1957.

9. Hagen, G. H. L., "Ueber die Bewegung des Wassers in engen cylindrischen Rohren," *Poggendorfs Ann. Physik. Chem.*, Vol. 16 (1839).

10. Darcy, Henri, *Les fontaines publiques de la ville de Dijon.* Paris: V. Dalmont, 1856.

11. Dupuit, A. J., *Études théoriques et practiques sur le mouvement des eaux dans les canauxs découverts et à travers les terrains perméables.* 2nd ed., Paris: Dunod, 1863.

12. Sherman, L. K., "Stream Flow from Rainfall by the Unit-Graph Method," *Eng. News-Rec.,* Vol. 108 (1932).

13. Horton, R. E., "The Role of Infiltration in the Hydrologic Cycle," *Trans. Am. Geophys. Union*, Vol. 14, 1933.

14. Theis, C. V., "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of a Well Using Ground Water Recharge," *Trans. Am. Geophys. Union*, Vol. 16 (1935).

15. Biswas, Asit K., "Hydrologic Engineering Prior to 600 B.C.," *Proc. A.S.C.E., J. Hyd. Div.*, Proc. Paper 5431, Vol. 93 No. HY5 (September 1967).

16. Todd, D. K. (ed.), *The Water Encyclopedia.* New York: Water Information Center, Inc., 1970.

17. Nace, Raymond L., "Water Resources: A Global Problem with Local Roots," *Environmental Science and Technology*, Vol. 1, No. 7 (July 1967).

2

PRECIPITATION AND HYDROLOGIC ABSTRACTIONS

Streamflow in a region is a function of hydrologic input and physical, vegetative, and climatic characteristics. As indicated by the hydrologic equation, all water derived from precipitation does not appear as streamflow. Various fractions of the gross precipitation input are diverted into paths which do not terminate in the regional surface transport system. The geologic, topographic, vegetative, and man-made features of the earth all are important modifiers of any area's streamflow characteristics.

Once precipitation strikes the ground it may go into storage on the surface, or in the soil and groundwater reservoirs beneath the surface. Character of soil and rocks largely determines which storage system the precipitated water will enter. Opportunity for evaporation and transpiration is also affected by geologic and topographic features.

2-1 Water Vapor

The fraction of water vapor in the atmosphere is very small compared with quantities of other gases present, but it is exceedingly important and is largely responsible for prevailing weather conditions. Precipitation is derived from this atmospheric water. Moisture content of the air is also a significant factor in local evaporation processes. Thus it is necessary for a hydrologist to be acquainted with ways for evaluating the atmospheric water-vapor content and to understand the thermodynamic effects of atmospheric moisture.

Under most conditions of practical interest (modest ranges of pressure and temperature, providing the condensation point is excluded) water

vapor essentially obeys the gas laws. Atmospheric moisture is derived from evaporation and transpiration, the principal source being evaporation from the oceans. Precipitation over the United States comes largely from oceanic evaporation, the water vapor being transported over the continent by the primary atmospheric circulation system.

Measures of water vapor or atmospheric humidity are related basically to conditions of evaporation and condensation occurring over a level surface of pure water. Consider a closed system containing approximately equal volumes of water and air maintained at the same temperature. If the initial condition of the air is dry, evaporation takes place and the quantity of water vapor in the air increases. A measurement of pressure in the air space will reveal that as evaporation proceeds, pressure in the air space increases because of an increase in partial pressure of the water vapor (vapor pressure). Evaporation continues until vapor pressure of the overlying air equals the surface vapor tension [a measure of the excess of water molecules leaving (evaporating from) the water surface over those returning]. At this point evaporation ceases, and if the temperatures of the air space and water are equal, the air space is said to be *saturated*. If the container had been open instead of closed, the equilibrium would not have been reached, and all of the water would eventually have evaporated.

Measures of Atmospheric Moisture

Some commonly used measures of atmospheric moisture or humidity are vapor pressure, absolute humidity, specific humidity, mixing ratio, relative humidity, and dew-point temperature.

Partial pressure is the pressure exerted on the surface of a container by an individual component of a gaseous mixture. In accordance with Dalton's law for gaseous mixtures, each gas exerts a pressure approximately proportional to the product of the total pressure of the mixture and the volume percentage of the gas. The partial pressure exerted by water vapor is called *vapor pressure*. Saturation vapor pressure is the partial pressure of water vapor at a specific temperature in saturated air.

Absolute humidity is the mass of water vapor contained in a unit volume of space. Units of measurement are normally grams per cubic meter (abbreviated g/m^3). It can be determined by using the equation of state for an ideal gas. In meteorological applications, the equation of state is generally written in a form slightly different from that most commonly used. If we consider M is the mass of some volume V which has a gram-molecular weight m, then the number of gram molecules n contained in volume V will be determined by the relation

$$n = M/m \tag{2-1}$$

The usual form of the equation of state is

$$pV = nRT \tag{2-2}$$

where R is the universal gas constant and T the absolute temperature. When the value for n from Eq. 2-1 is inserted into Eq. 2-2 the following expression results:

$$pV = (M/m)RT \tag{2-3}$$

By definition, $V/M = 1/\rho = v$ where v is the specific volume (cm^3/g). Introducing this in Eq. 2-3 yields

$$pv = (R/m)T \tag{2-4}$$

or

$$p = \rho(R/m)T \tag{2-5}$$

Equations 2-4 and 2-5 are commonly used in meteorologic work. Occasionally the term R/m is called R' or B and is termed an *individual gas constant* which varies with the gas.

It follows that the equation of state for water vapor may be written as

$$e = \rho_w(R/m_w)T \tag{2-6}$$

where ρ_w is the vapor density or absolute humidity and e is the partial pressure of the water vapor.

Specific humidity is the weight of water vapor contained within a unit weight of moist air. It is normally expressed as grams per gram or grams per kilogram and may be calculated using

$$q = 0.622\,e/(p - 0.378e) \tag{2-7}$$

where q is the value of specific humidity in grams per gram (abbreviated g/g), e the vapor pressure in millibars, and p the total pressure of the moist air in millibars.

The definition of mixing ratio is similar to that for specific humidity with the exception that it is the ratio of the mass of water vapor to the mass of dry air rather than to the mass of moist air. For most practical purposes the specific humidity and mixing ratio can be considered equal.

The dewpoint temperature is that temperature at which an air mass just becomes saturated when cooled at constant pressure and when moisture is neither added nor removed.

Relative humidity is the ratio of the water vapor pressure existing to that which would prevail under saturated conditions. It may be stated in percent in the following form:

$$H = 100e/e_s \tag{2-8}$$

where e_s is the saturated vapor pressure. It should be noted that the ratio of vapor densities could also be used. The relative humidity is a term widely used in weather analyses, since most of the atmosphere is un-

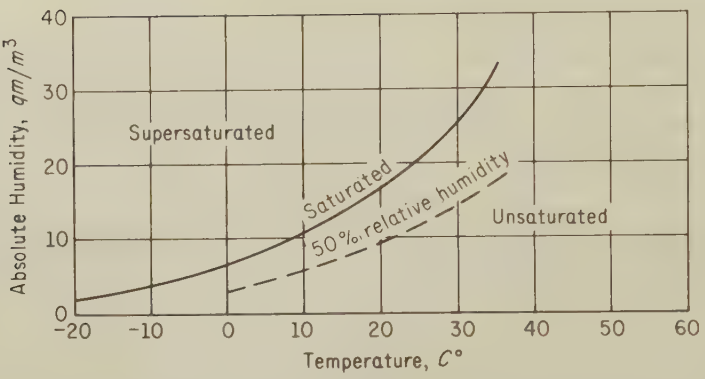


FIG. 2-1. Saturation and 50 percent relative humidity curves.

saturated and it is useful to determine the degree of saturation that exists. Figure 2-1 gives saturation and 50 percent relative humidity curves.

Amount of Precipitable Water

Estimates of the amount of precipitation that might occur over a given region with favorable conditions are often useful. These may be obtained by calculating the amount of water contained in a column of atmosphere extending up from the earth's surface. This quantity is known as the *precipitable water W*, although it cannot all be removed from the atmosphere by natural processes. Precipitable water is usually expressed in centimeters or inches.

An equation for computing the amount of precipitable water in the atmosphere can be derived as follows. Consider a column of air having a square base one centimeter on a side. The total water mass contained in this column between elevation zero and some height z would be

$$W = \int_0^z \rho_w \, dz \tag{2-9}$$

where ρ_w is the absolute humidity. The hydrostatic equation may be written as

$$dp = -\rho g \, dz \tag{2-10}$$

where ρ is the total air density and is thus the sum of the vapor density and the total density of the dry gases ρ_d, or $\rho = (\rho_w + \rho_d)$. The hydrostatic equation may be solved for dz, yielding

$$dz = -\frac{dp}{\rho g} \tag{2-11}$$

Substituting the above expressions for ρ and dz in Eq. 2-9, we obtain

$$W = \frac{1}{g} \int_{p}^{p_0} \frac{\rho_w}{(\rho_w + \rho_d)} \, dp \tag{2-12}$$

The ratio of densities shown above is actually the specific humidity q, and since q is approximately equal to $0.622e/p$, Eq. 2-12 may be written as

$$W = \frac{0.622}{g} \int_{p}^{p_0} e \frac{dp}{p} \tag{2-13}$$

with little loss of accuracy, where W is given in (g/cm^2). Considering the density of the precipitable water equal to 1, and since integration is performed over a unit area, the value of W is also the depth of precipitable water in centimeters. The integral can be evaluated graphically or by dividing the atmosphere into layers of approximately uniform specific humidities and solving for these individually and then summing. Figure 2-2 illustrates the average amount of precipitable water for the continental United States up to an elevation of 8 km.[32] Example 2-1 illustrates a computational procedure.

Example 2-1 Given the following data on elevation, temperature, and vapor pressure, compute the depth of precipitable water in inches for a column of atmosphere 18,000 ft in height.

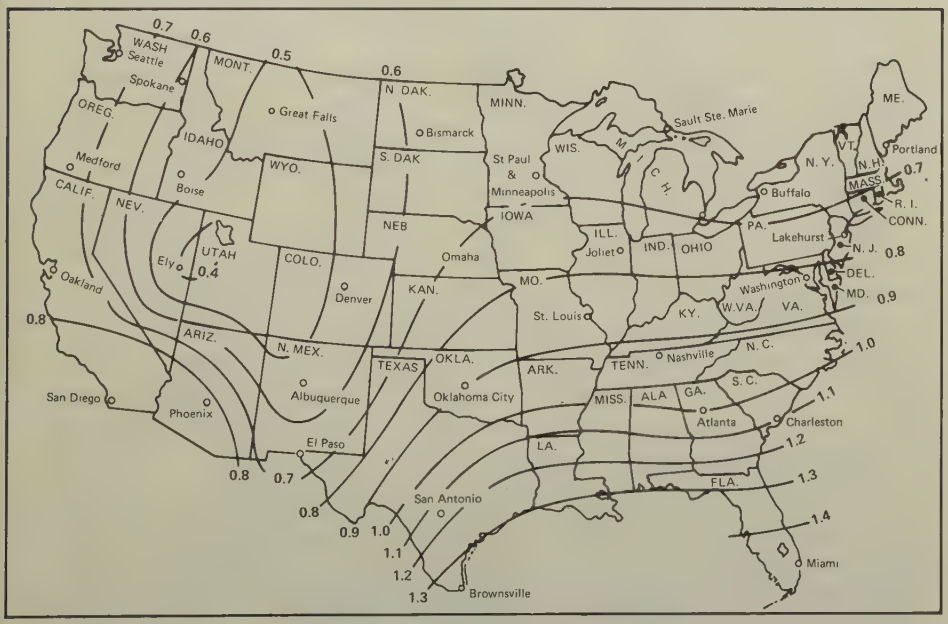


FIG. 2-2. Mean precipitable water in inches, to an elevation of 8 km. (U.S. Weather Bureau.)

Elevation, KF	0	2	4	6	8	10	12	14	16	18
Temp., °F	58	52	44	38	30	23	16	8	2	−6
Pressure, millibars	1013	942	875	812	753	697	644	595	550	500
Vapor pressure, millibars	7.0	5.0	3.8	3.2	2.0	1.6	1.1	0.8	0.6	0.4

Equation 2-13 can be converted to yield W in inches by multiplying by the appropriate conversion factor. This yields

$$W = 0.0004 \int_{p}^{p_0} q \, dp \tag{2-14}$$

where W is the precipitable water in inches, q the specific humidity in grams per kilogram (g/kg), and p is pressure in millibars (mb). To solve this we will divide the column into 2,000-ft zones and use mean values for each zone. Thus Eq. 2-14 can be written as

$$W = 0.0004 \Sigma \bar{q} \Delta p \tag{2-15}$$

where $\bar{q}$ is the mean value of q for each zone. The computations are shown in the following table. The final result is

$$W = 0.0004 \times 1013 = 0.41 \text{ in.}$$

Calculations for Example 2-1

Elevation, 1000's ft	Specific humidity, g/kg = $622e/p$	Mean specific humidity $\bar{q}$	Δp	$\bar{q}\Delta p$
0	4.32	3.81	71	271
2	3.30	2.98	67	199
4	2.66	2.55	63	161
6	2.45	2.05	59	121
8	1.65	1.53	56	86
10	1.41	1.23	53	65
12	1.06	.95	49	47
14	.84	.76	45	34
16	.68	.59	50	29
18	.50			
				$\Sigma = 1{,}013$

Geographic and Temporal Variations

The quantity of atmospheric water vapor varies with location and time. These variations may be attributed mainly to temperature and source of supply considerations. The greatest concentrations can be found near the ocean surface in the tropics, the concentrations generally decreasing with latitude, altitude, and distance inland from coastal areas.

About half the atmospheric moisture can be found within the first mile above the earth's surface. This is because vertical transport of vapor is

mainly through convective action, which is slight at higher altitudes. It is also of interest that there is not necessarily any relation between the amount of atmospheric water vapor over a region and the resulting precipitation. The amount of water vapor contained over dry areas of the Southwest, for example, at times exceeds that over considerably more humid northern regions, even though the latter areas experience precipitation while the former do not.

2-2 Precipitation

Precipitation is the primary input vector of the hydrologic cycle. Its primary forms are rain, snow, and hail and several variations of these forms such as drizzle and sleet. Precipitation is derived from atmospheric water, its form and quantity thus being influenced by the action of other climatic factors such as wind, temperature, and atmospheric pressure. Atmospheric moisture is a necessary but not sufficient condition for precipitation. Continental air masses are usually very dry, so that most precipitation is derived from moist maritime air which originates over the oceans. In North America about 50 percent of the evaporated water is taken up by continental air and moves back again to the sea. This is generally contradictory to some proposed theories which indicate a relationship between the amount of water evaporated in a region and its precipitation.

Because considerable water may accumulate in clouds without any precipitation resulting, condensation can occur independently of precipitation. It is therefore useful to consider each of these processes individually so that a better understanding of the occurrence of precipitation can be had.

Condensation

Condensation may be attributed to one or more of the following causes: (1) dynamic or adiabatic cooling, (2) mixing of air masses having different temperatures, (3) contact cooling, and (4) cooling by radiation. However, the most important cause is dynamic cooling, and it is this process which produces nearly all precipitation. Condensation rates associated with other causes are usually small and these rarely produce appreciable precipitation. Dew, frost, and fog are precipitation forms commonly associated with radiational and contact cooling.

The condensation of water vapor into cloud droplets (average size $^1/_{2500}$ in. diameter) occurs on small hygroscopic particles which are termed *condensation nuclei.* Particles of ocean salt and combustion products containing sulfurous and nitrous acids constitute the most active nuclei. Such nuclei are usually less than 1 micron (μ) in diameter, but very large nuclei

on the order of 5 μ are known to be present occasionally. The number of salt nuclei present range from about 10 to 1,000 per cc,[29] while the number of combustion nuclei is directly related to the extent and nature of regional industrial operations. Under conditions usually encountered, it takes anywhere from several seconds to produce a droplet of about 10 μ to approximately a day for a small raindrop (about 3 mm) to grow on a condensation nuclei. This natural growth process is extremely slow and at best produces only very small cloud droplets. As a result, the condensation mechanism cannot alone be expected to produce any significant form of precipitation.

Formation of Precipitation

Two processes are considered to be capable of supporting the growth of droplets of sufficient mass (droplets of from about 500 to 4,000 μ in diameter) to overcome air resistance and consequently fall to the earth as precipitation. These are known as the *ice crystal process* and the *coalescence process.*

The coalescence process is one by which the small cloud droplets increase their size due to contact with other droplets through collision. Water droplets may be considered as falling bodies which are subjected to both gravitational and air resistance effects. Fall velocities at equilibrium (terminal velocities) are proportional to the square of the radius of the droplet, and thus the larger droplets will descend more quickly than the smaller ones. As a result, smaller droplets are often overtaken by larger droplets, and the resulting collisions tend to unite the drops, producing increasingly larger particles. Very large drops (order of 7 mm in diameter) break up into small droplets which repeat the coalescence process and produce somewhat of a chain effect. In this manner, sufficient large raindrops may be produced to generate significant precipitation. This process is considered to be particularly important in tropical regions or in warm clouds.

An important type of coalescence is known to occur when ice crystals and water droplets are found to exist together at subfreezing temperatures down to about $-40°$ C. Under these conditions, certain particles of clay minerals and organic and ordinary ocean salts serve as freezing nuclei so that ice crystals are formed. The vapor pressure under these conditions is higher over the water droplets than over the ice crystals, and thus condensations occurs on the surface of the crystals. The ice crystals grow in size, and uneven particle-size distributions develop which further favor growth through contact with other particles. This is considered to be a very important precipitation-producing mechanism.

The artificial inducement of precipitation has been studied extensively in recent years, and these studies are continuing. It has been demon-

strated that condensation nuclei supplied to clouds can induce precipitation. The ability of man to insure the production of precipitation or to control its geographic location or timing has not yet been attained. Nevertheless, the future of this phase of weather control is promising.

Many legal as well as technological problems are associated with the prospects of "rainmaking" processes. Of interest here is the impact on hydrologic estimates that uncontrolled or only partially controlled artificial precipitation might have. Many naturally occurring hydologic variables are considered as statistical variates which are either randomly distributed or which are distributed with a random component. If the distribution or time series of the variable can be modeled, an inference as to frequency of occurrence of significant hydrologic events of a given magnitude (such as precipitation) can be made. If, however, artificial controls are used and if the effects of these cannot be reliably predicted, frequency analyses may prove to be totally unreliable tools.

Precipitation Types

Dynamic or adiabatic cooling is the primary cause of condensation and is responsible for most rainfall. Thus it can be seen that vertical transport of air masses is a requirement for precipitation. Precipitation may be classified according to the conditions which generate vertical air motion. In this respect, the three major categories of precipitation type are *convectional*, *orographic*, and *cyclonic*.

Convective Precipitation Convectional precipitation is typical of the tropics and is brought about by heating of the air at the interface with the ground. This heated air expands with a resultant reduction in weight. During this period, increasing quantities of water vapor are taken up, the warm moisture-laden air becomes unstable, and pronounced vertical currents are developed. Dynamic cooling takes place, causing condensation and precipitation. Convective precipitation may be in the form of light showers or storms of extremely high intensity (thunderstorms are a typical example).

Orographic Precipitation Orographic precipitation results from the mechanical lifting of moist horizontal air currents over natural barriers such as mountain ranges. The rainfalls of the Pacific Northwest are typical examples.

Cyclonic Precipitation Cyclonic precipitation is associated with the movement of air masses from high-pressure regions to low-pressure regions. These pressure differences are created by the unequal heating of the earth's surface.

Cyclonic precipitation may be classified as frontal or nonfrontal. Any barometric low can produce nonfrontal precipitation as air is lifted through horizontal convergence of the inflow into a low pressure area.

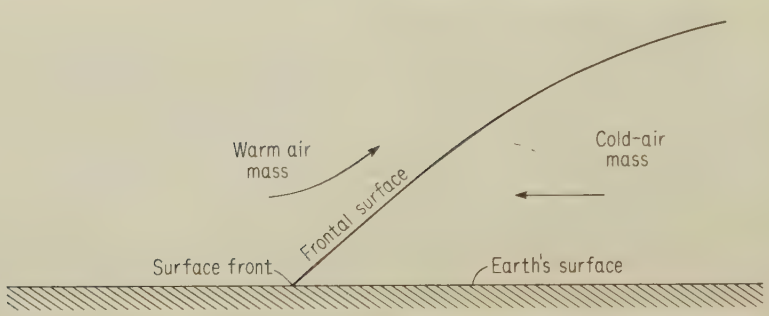


FIG. 2-3. Vertical cross section through a frontal surface.

Frontal precipitation results from the lifting of warm air over cold air at the contact zone between air masses having different characteristics. If the air masses are moving so that warm air replaces colder air, the front is known as a *warm front*; if, on the other hand, cold air displaces warm air, the front is said to be *cold*. If the front is not in motion, it is said to be a *stationary front*. Figure 2-3 illustrates a vertical section through a frontal surface.

Thunderstorms

Many areas of the United States are subjected to severe convective storms which are generally identified as thunderstorms due to their electrical nature. These storms, although usually very local in nature, are often productive of very intense rainfalls which are highly significant when local and urban drainage works are considered.

Thunderstorm cells develop from vertical air movements associated with intense surface heating or orographic effects. There are three primary stages in the life history of a thunderstorm. These are the *cumulus stage*, the *mature stage*, and the *dissipating stage*. Figure 2-4 illustrates each of these.

All thunderstorms begin as cumulous clouds, although few such clouds ever reach the stage of development needed to produce such a storm. The cumulus stage is characterized by strong updrafts which often reach altitudes of over 25,000 ft. Vertical wind speeds at upper levels are often as great as 35 mph. As indicated in Fig. 2-4a, there is considerable horizontal inflow of air (entrainment) during the cumulus stage. This is an important element in the development of the storm, as additional moisture is provided. Air temperatures inside the cell are greater than those outside as indicated by the convexity of the isotherms viewed from above. The number and size of water droplets increase as the stage progresses. The duration of the cumulus stage is approximately 10 to 15 min.

The strong updrafts and entrainment support increased condensation and the development of water droplets and ice crystals. Finally, when the particles increase in size and number so that surface precipitation occurs,

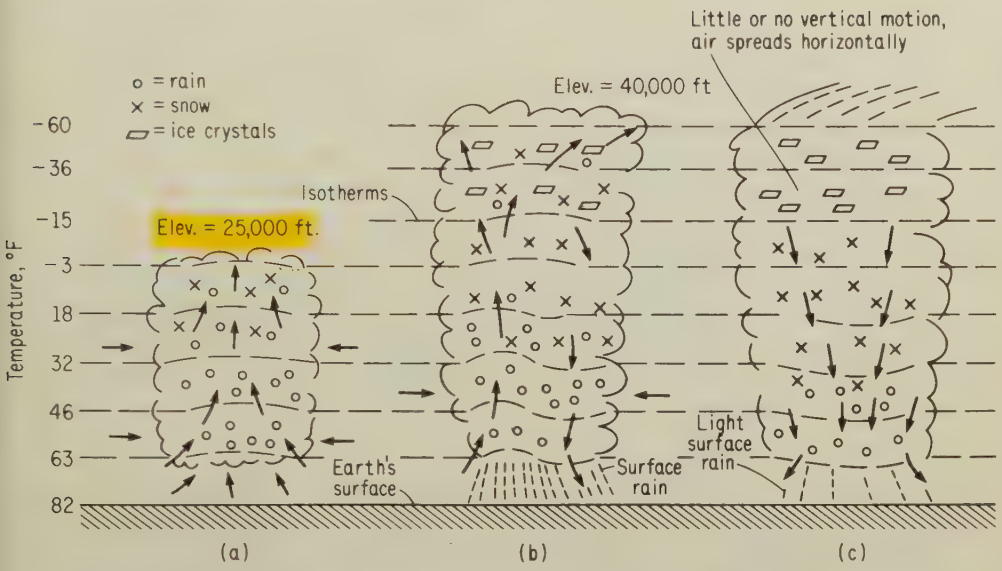


FIG. 2-4. Cumulus, mature, and dissipating stages of a thunderstorm cell. (Department of the Army.)

the storm is said to be in the mature stage. In this stage, strong downdrafts are created as falling rain and ice crystals cool the air below. Updraft velocities at the higher altitudes reach up to 70 mph in the early periods of the mature stage. Downdraft speeds of over 20 mph are usual above about 5,000 ft in elevation. At lower levels, frictional resistance tends to decrease the downdraft velocity. Gusty surface winds move outward from the region of rainfall. Heavy precipitation is often derived during this period which is usually on the order of 15 to 30 min.

In the final or dissipating stage, the downdraft becomes predominant until all of the air within the cell is descending and being dynamically heated. Since the updraft ceases, the mechanism for condensation ends and precipitation tails off and ends.

Precipitation Data

Considerable data on precipitation are available in several publications of the U.S. Weather Bureau.[33,34] Other sources include various state and federal agencies engaged in water-resources work. For critical regional studies, it is recommended that all possible data be compiled and often the establishment of a gauging network will be necessary.

Precipitation Variability

Precipitation varies geographically, temporally and seasonally. Figure 2-5 indicates the mean annual precipitation for the continental United

Average Annual Precipitation (inches)
Period 1899-1938
Ridge line "Applachian Mountain" system
Ridge line "Sierra – Cascade" system
Rocky Mountain divide
Colorado – Columbia divide

FIG. 2-5. Mean annual precipitation. (U.S. Department of Agriculture Soil Conservation Service.)

States, while Fig. 2-6 gives an example of seasonal differences. It should be understood that both regional and temporal variations in precipitation are very important in water-resources planning and hydrologic studies. For example, it may be very important to know that the cycle of minimum precipitation coincides with the peak growing season in a particular area, or that the period of heaviest rainfall should be avoided in scheduling certain construction activities.

Time variations in rainfall intensity are extremely important in the rainfall-runoff process, particularly in urban areas (see Fig. 2-8a). The aereal distribution is also significant and highly correlated with the time history of outflow (see Fig. 2-8b). These considerations are discussed in greater detail in following chapters (see particularly Secs. 4-7, 4-9, 5-5, 8-1, 8-4, 8-5, and 8-8).

2-3 Distribution of the Precipitation Input

Total precipitation is distributed in numerous ways. That intercepted by vegetation and trees may be equivalent to the total precipitation input for relatively small storms. Once interception storage is filled, raindrops begin falling from leaves and grass, where water stored on these surfaces eventually becomes depleted through evaporation. Precipitation which reaches the ground may take several paths. Some water will fill depressions and eventually evaporate; some will infiltrate the soil. Part of the infiltrated water may strike relatively impervious strata near the soil surface and flow approximately parallel to it as interflow until an outlet is reached. Other portions may replenish soil moisture in the upper soil zone, and some infiltrated water may reach the groundwater reservoir which sustains dry-weather stream flow. The component of the precipitation input that exceeds the local infiltration rate will develop a film of water on the surface (surface detention) until overland flow commences. Detention depths varying from $\frac{1}{8}$ in. to $1\frac{1}{2}$ in. for various conditions of slope and surface type have been reported.[2] Overland flow ultimately reaches defined channels and becomes streamflow.

Figure 2-7 illustrates in a general way the disposition of a uniform storm input to a natural drainage basin. While such an input is not to be expected in nature, the indicated relationships are representative of actual conditions. Modifications resulting from nonuniform storms will be discussed as they arise.

In Fig. 2-7a note that the storm input is distributed uniformly over time t_a at a rate equal to i (dimensionally equal to LT^{-1}). This input is dissected into components i_1 through i_4, the sum of which is equal to i at any time t. Figure 2-7b illustrates the manner in which infiltrated water is further subdivided into interflow, groundwater and soil moisture. Fig-

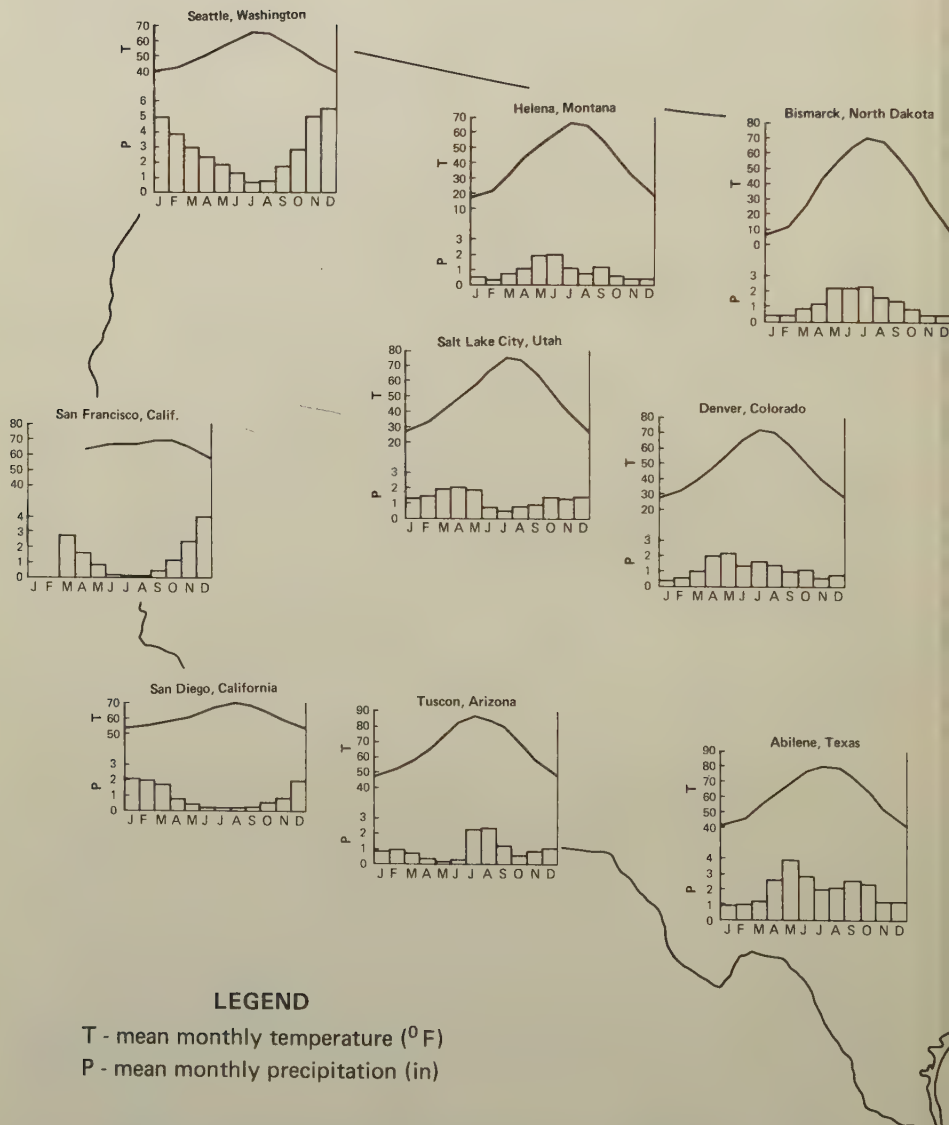


FIG. 2-6. Precipitation and temperature distributions. (U.S. Department of Agriculture Soil Conservation Service.)

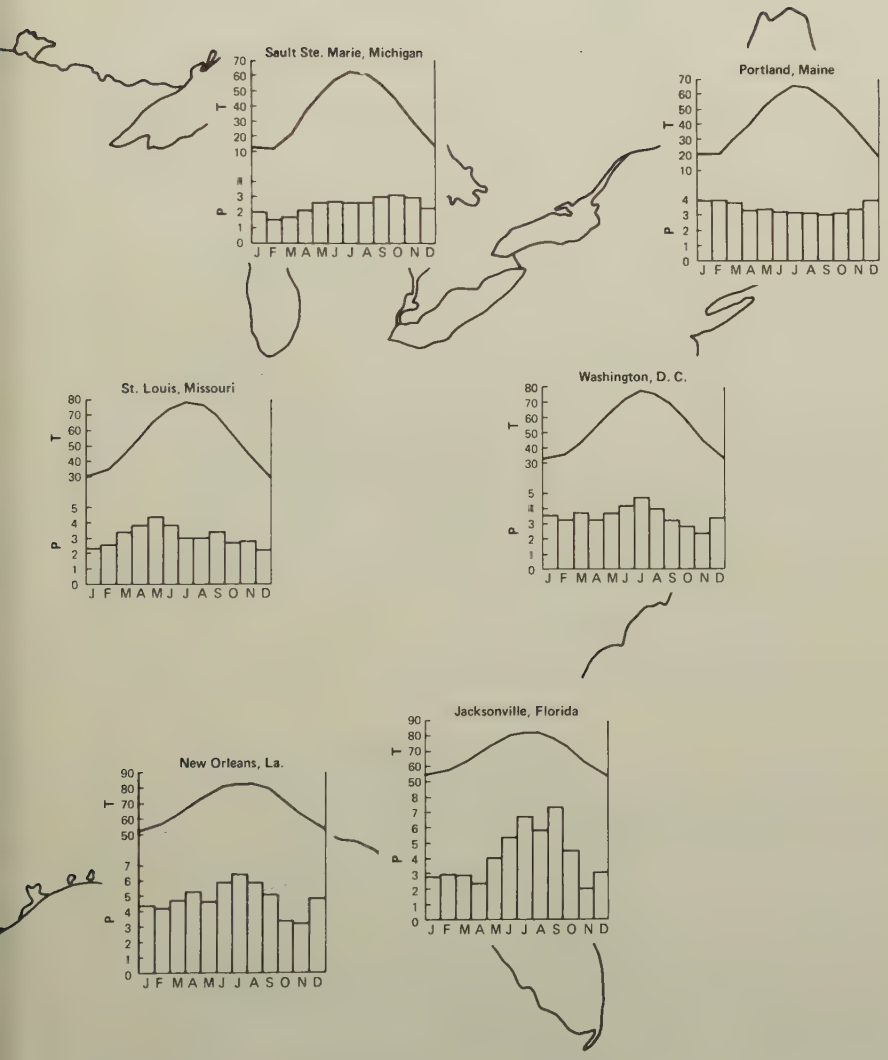


FIG. 2-6. (*Continued*)

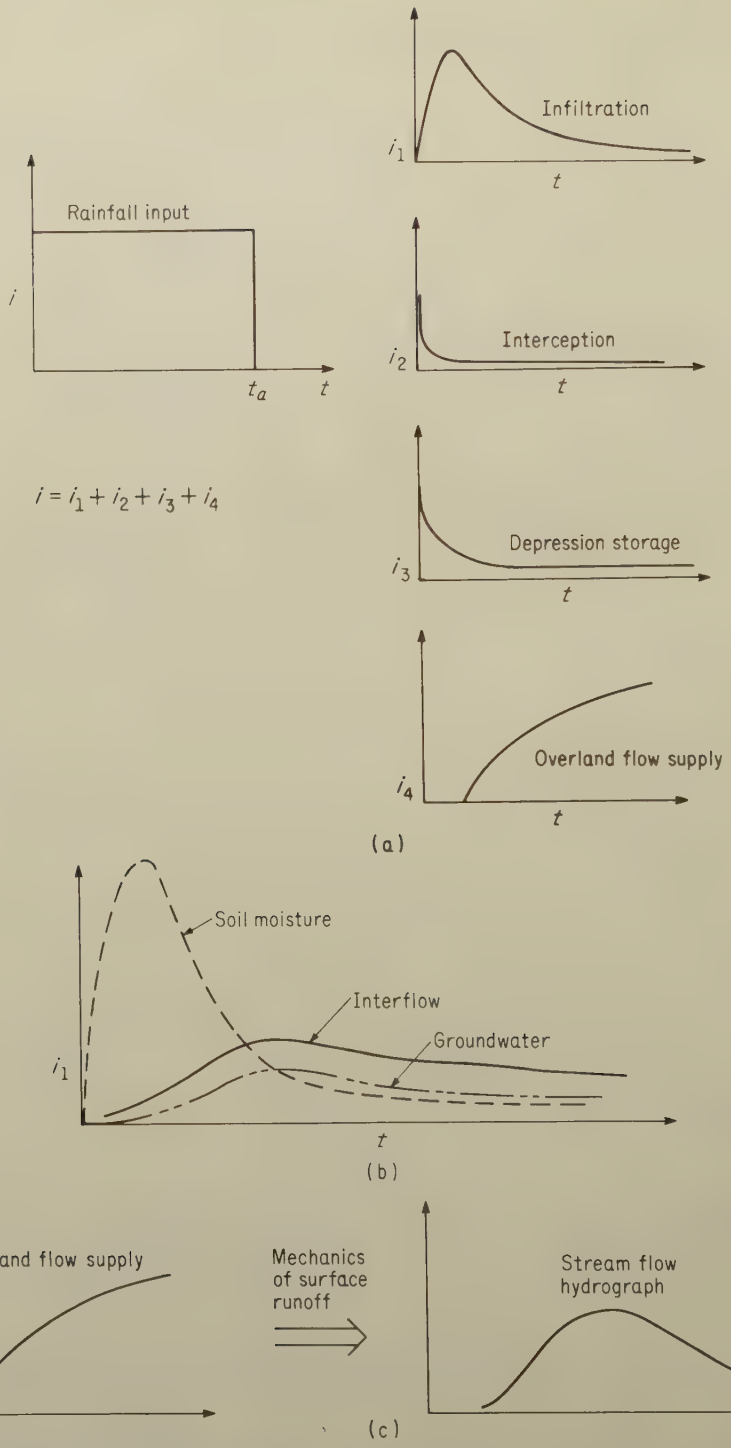


FIG. 2-7. The runoff process: (a) disposition of precipitation; (b) components of infiltration; and (c) disposition of overland flow supply.

ure 2-7c shows the transition from overland flow supply into streamflow. The mechanics of these processes will be treated in detail in later sections. The nature of the curves presented depicts the general runoff process. It should be realized, however, that actual graphs of infiltration and/or other factors versus time might appear quite different in form and relative magnitude when compared with these illustrations because of the effects of nonuniform storm patterns, antecedent conditions, and other basin characteristics.

The rate and areal distribution of runoff from a drainage basin are determined by a combination of physiographic and climatic factors. Important climatic factors include the form of precipitation (rain, snow, hail), the type of precipitation (convective, orogrophic, cyclonic), the quantity and time distribution of the precipitation, the character of the regional vegetative cover, prevailing evapotranspiration characteristics and the status of the soil moisture reservoir. Physiographic factors of significance include geometric properties of the drainage basin, land-use characteristics, soil type, geologic structure, characteristics of drainage channels (geometry, slope, roughness, and storage capacity).

Large drainage basins often react differently from smaller ones when subjected to a precipitation input. This can be explained in part by such factors as geologic age, relative impact of land-use practices, size differential, variations in storage characteristics, and other causes. Chow defines a small watershed as a drainage basin whose characteristics do not filter out (1) fluctuations characteristic of high-intensity, short-duration storms; or (2) the effects of land-management practices.[1] On this basis, small basins may vary from less than an acre up to 100 sq mi. A large basin is one in which channel storage effectively does filter out the high frequencies of imposed precipitation and effects of land-use practices.

2-4 Gross and Net Precipitation Inputs

In this chapter we are mainly concerned with surface runoff but as can be seen from the preceding discussion, the linkage between surface and subsurface flows must be accounted for. In the runoff process, the final outcome of a storm event—the outflow hydrograph—is of vital concern. This involves first a knowledge of the causative storm and secondly a means to relate this storm input to the outflow hydrograph.

A rain-gauge network can provide a measure of the time distribution of gross precipitation. Such data can be displayed graphically as shown on Fig. 2-8, placed on magnetic tape or punched cards for computer use, or stored in some other form. The net (excess) precipitation which contributes directly to the surface runoff supply is equivalent to the gross precipitation minus losses to interception, storm period evaporation, depression storage and infiltration. The relationship between excess pre-

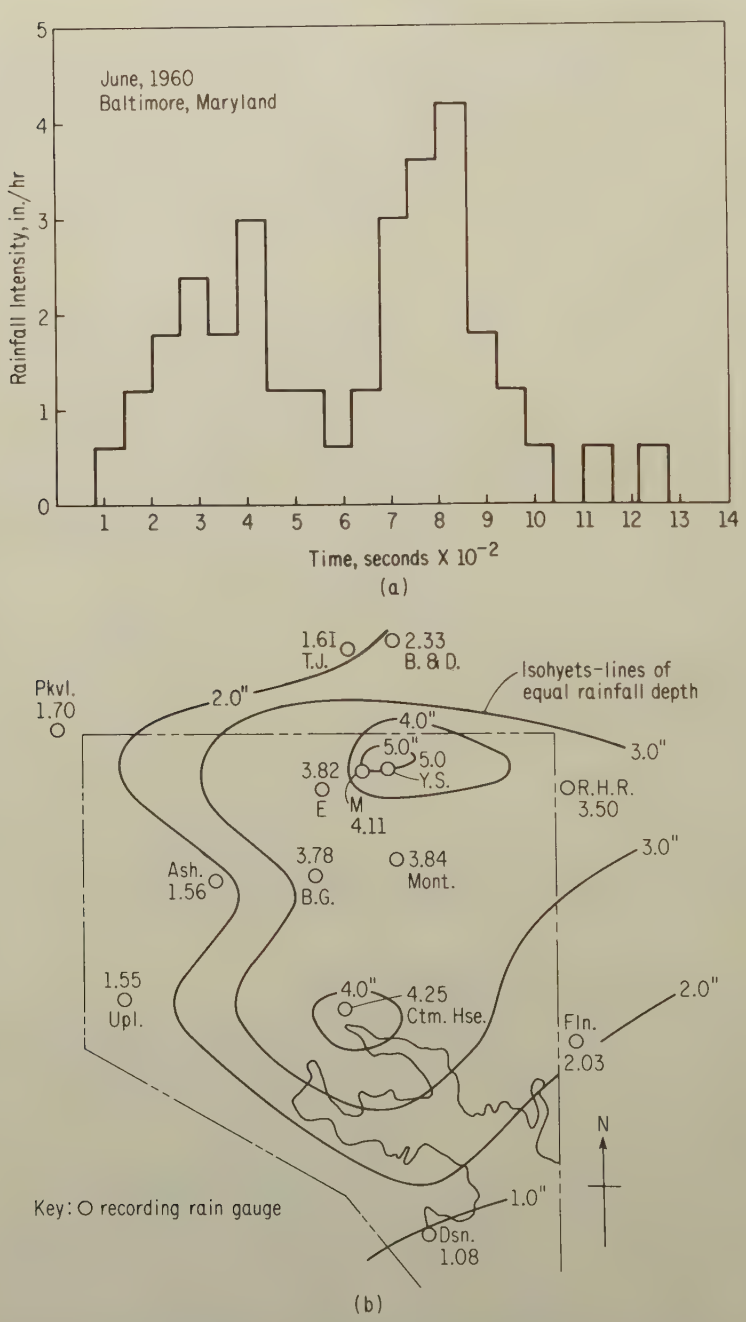


FIG. 2-8. (a) Rainfall distribution in a convective storm. (b) Isohyetal pattern, storm of September 10, 1957, Baltimore, Maryland.

cipitation P_e and gross precipitation P_g is thus,

$$P_e = P_g - \Sigma \text{ losses} \tag{2-16}$$

where the losses include all deductions from the gross storm input.

2-5 Losses From Precipitation

Losses from precipitation include interception, evaporation, depression storage, and infiltration. These are defined as "losses" in the sense that they are not available for direct surface runoff. Hydrologically, no component is ever really lost. To an agriculturalist, infiltration is the gain and surface runoff the loss. Nevertheless, the term *loss* will be used as defined here to avoid confusion and provide consistency in treatment.

2-6 Evaporation

Evaporation is the process by which water is transferred from land and water masses of the earth to the atmosphere. Because there is a continuous exchange of water molecules between an evaporating surface and its overlying atmosphere, it is common in hydrologic practice to define evaporation as the *net rate of vapor transfer.* It is a function of solar radiation, differences in vapor pressure between a water surface and the overlying air, temperature, wind, atmospheric pressure, and quality of evaporating water. Conversion of snow or ice into water vapor is in reality sublimation rather than evaporation, since water molecules do not pass through a liquid phase. Otherwise the effects of these two processes are the same.

The role of evaporation in the hydrologic cycle has been discussed. The practical significance of evaporation losses to a particular region will be illustrated.

Mean annual lake evaporation for the continental United States varies from a low of about 20 in. in the northeast to a high of about 86 in. in southern California. The mean rate of lake evaporation in arid and semiarid regions is often in excess of the local precipitation depth for that area. As a result, significant quantities of water are lost to those areas for beneficial uses.

About 75 in. of water are estimated to be evaporated annually from the pool of Elephant Butte Reservoir in New Mexico, a storage site located along the Rio Grande River which furnishes irrigation water to much of southern New Mexico. The average surface area of the pool is roughly 13,760 acres, so approximately 86,000 acre-ft of water are evaporated annually. For a normal annual irrigation requirement 3 acre-ft per acre, a loss in irrigation potential of about 29,000 acres is indicated. This

same amount of water could meet the water supply needs of New York City (about one billion gpd) for almost one month. Evaporation loss from a larger reservoir such as Lake Mead (mean surface area of 124,200 acres and mean annual evaporation of 849,000 acre-ft) could supply the city of New York for approximately 1 yr.[14]

These examples illustrate the importance of evaporation in water-development planning. About 30 in. of water are received annually as precipitation in the United States, with perhaps 70 percent of this amount returned to the atmosphere through evapotranspiration, and the remaining 30 percent appearing as runoff to the oceans. Figure 2-9 shows the mean annual evaporation from shallow lakes and reservoirs in the United States.

Determination of evaporation is generally based on the water budget, energy budget, mass-transfer techniques, or pan-evaporation data. Direct measurements of evaporation are not easily obtained for large bodies of water because of the extensive surfaces involved. In fact, of all variables included in the general hydrologic equation, surface runoff is the only one that readily permits direct evaluation, since it is confined within well-defined geometric boundaries that permit determination of both rate and cross-sectional area of flow. Choice of method used to determine evaporation depends on required accuracy of results and type of instrumentation available. Accuracy is related to the varying degree of reliability with which the method's parameters can be determined.

Evaporation is a crucial consideration in water-resources planning and management programs, in evaluating the potential for water-resources development, and in water-supply studies. During storm periods, vapor pressure gradients are reduced and evaporation is usually not significant. In very dry regions, for example, the mean hourly evaporation rate during precipitation-free periods would be on the order of 0.008 in./hr. Comparing this with a relatively small storm having an average intensity of 0.5 in./hr, it is easily seen that during important storm periods evaporation can be safely disregarded with respect to flow estimation. As a result, it is common practice to make no deduction from gross storm precipitation for evapotranspiration.

Water Budget The water budget method for determining reservoir evaporation has already been mentioned in discussing the use of the hydrologic equation. Basically it is a very simple procedure but it seldom produces reliable results. In this method, reservoir (lake, pool) evaporation E_s can be computed by rearranging Eq. 1-2

$$E_s = P + R_1 - R_2 + R_g - T_s - I - \Delta S_s \qquad (2\text{-}17)$$

It is useful to deal with the net transfer of seepage through the ground $(R_g - I) = 0_s$ and consider that the transpiration term T_s equals zero.

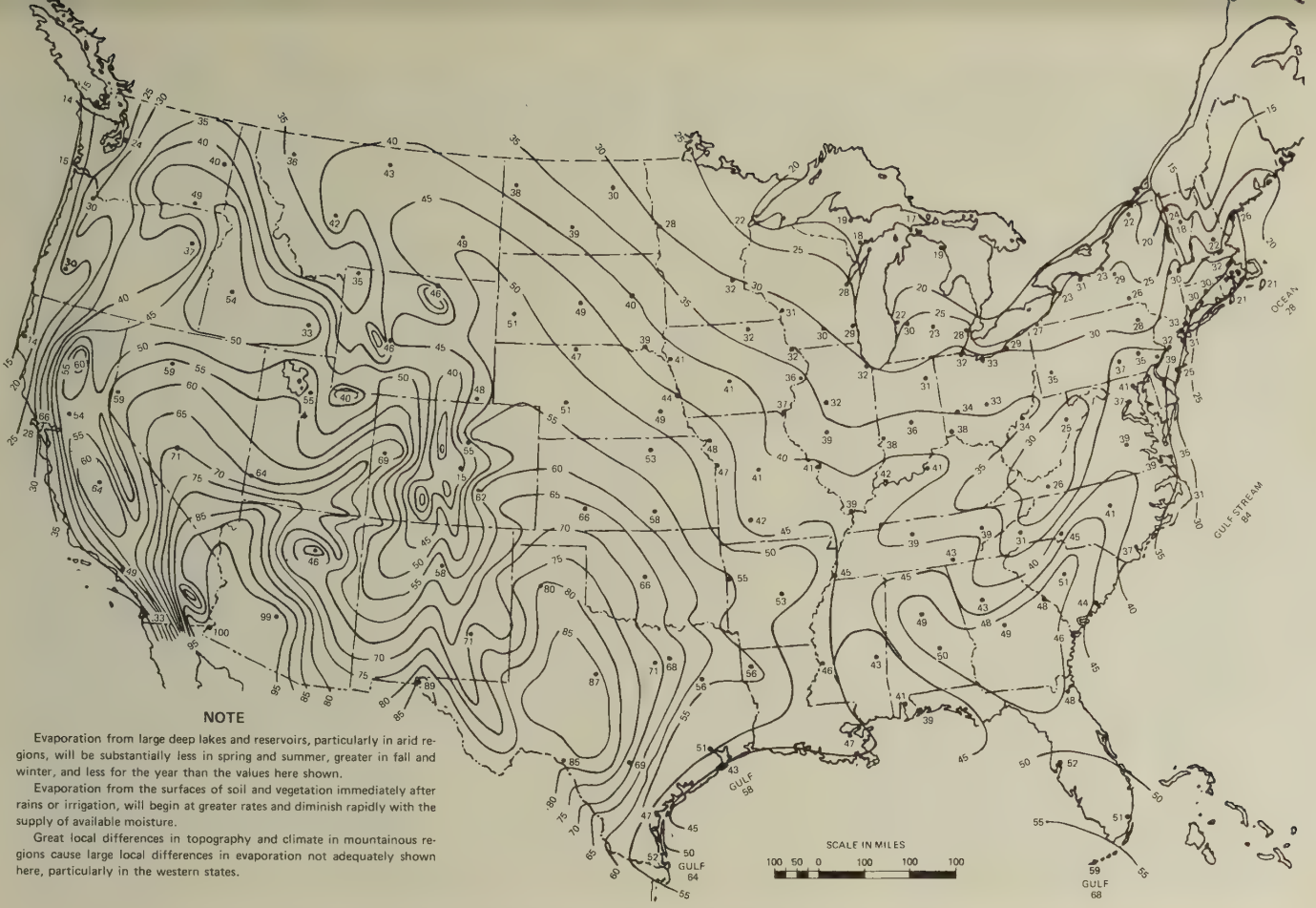

NOTE
Evaporation from large deep lakes and reservoirs, particularly in arid regions, will be substantially less in spring and summer, greater in fall and winter, and less for the year than the values here shown.
Evaporation from the surfaces of soil and vegetation immediately after rains or irrigation, will begin at greater rates and diminish rapidly with the supply of available moisture.
Great local differences in topography and climate in mountainous regions cause large local differences in evaporation not adequately shown here, particularly in the western states.
SCALE IN MILES
100 50 0 100 100 100
GULF 64
GULF 68
GULF 58
GULF STREAM 84
OCEAN 78

FIG. 2-9. Mean annual evaporation from shallow lakes and reservoirs, in inches. (U.S. Department of Agriculture Soil Conservation Service.)

With these few modifications, Eq. 2-17 becomes

$$E_s = P + R_1 - R_2 - O_s - \Delta S_s \tag{2-18}$$

All terms are in volume units for a time period of interest, and Δt should be at least a week. In general, however, the method would more likely be used to estimate monthly or annual evaporation from a particular reservoir. Note that all errors in measuring inflow, outflow, precipitation, net seepage and change in storage are reflected in the final estimate of evaporation. Precipitation, runoff, and changes in storage can often be determined within reasonable limits of accuracy, but evaluation of the net seepage O_s is frequently subject to appreciable errors and if the magnitude of O_s is on the order of E_s, very large errors are possible. Seepage estimates usually come from measurements of groundwater levels and/or soil permeability. The water budget is usable on a continuous basis if a stage-seepage relationship for the lake can be established.

In cases where the water budget for a lake is defined by only two unknowns, net seepage and evaporation, these losses can be separated and evaluated in a relatively simple manner by assuming that evaporation is proportional to the product $u(e_0 - e_a)$. This is the mass-transfer product described later in the section on mass-transfer techniques. The variables are wind velocity u, saturation vapor pressure e_0 (related to water surface temperature), and vapor pressure of the air e_a. When the product $u(e_0 - e_a)$ is zero, evaporation may be neglected.

Periods of no surface inflow or outflow are desirable for net seepage determination since during such intervals the only losses are evaporation and seepage. Under these conditions, whenever the mass-transfer product is equal to zero, the change in water elevation is considered equivalent to the net seepage loss. Normally a daily plot of change in elevation versus $u(e_0 - e_a)$ is obtained and a best fitting line constructed. The intercept of this line on the change in stage axis is the net seepage rate. Values of net seepage estimated in this manner can be used on a long-term basis in the water-budget equation if the net seepage does not change appreciably over extended periods. Unfortunately this condition is rarely representative since net seepage is a function of reservoir stage and season of the year in many cases. Unless these effects can be calculated, net seepage values determined from limited data have little utility.

Good estimates of evaporation using the water budget equation have been obtained as exemplified by research conducted on Lake Hefner, Oklahoma.[12] Under optimal conditions, the order of accuracy of the method is about 10 percent.

Example 1-1 illustrated use of the water budget in resolving basin evapotranspiration. For such estimates, reliable values can be expected if the period of time chosen is 1 yr or longer. Short period values may

also be obtained if observations are adequate. Mean annual evapotranspiration is successfully judged by using long-time averages of precipitation and surface flows, and competent information on the fluctuation of storage. Adequate short-period estimates are also possible if variables in the budget equation can be satisfactorily quantified on a short-term basis.

Energy-Budget Method The energy-budget method illustrates an application of the continuity equation written in terms of energy. It has been employed to compute the evaporation from oceans and lakes, i.e., for Lake Hefner in Oklahoma and at Elephant Butte reservoir in New Mexico.[4,12] The equation accounts for incoming and outgoing energy balanced by the amount of energy stored in the system. Accuracy of estimates of evaporation using the energy budget is highly dependent upon the reliability and preciseness of measurement data. Under good conditions, average errors of perhaps 10 percent for summer periods and 20 percent for winter months can be expected.

The energy budget equation for a lake may be written as

$$Q_0 = Q_s - Q_r + Q_a - Q_{ar} + Q_v - Q_{bs} - Q_e - Q_h - Q_w \quad (2\text{-}19)$$

where Q_0 = increase in stored energy by the water
Q_s = solar radiation incident at the water surface
Q_r = reflected solar radiation
Q_a = incoming long-wave radiation from the atmosphere
Q_v = net energy advected (net energy content of incoming and outgoing water) into the water body
Q_{ar} = reflected long-wave radiation
Q_{bs} = long-wave radiation emitted by the water
Q_e = energy used in evaporation
Q_h = energy conducted from water mass as sensible heat
Q_w = energy advected by evaporated water

and all terms are in calories per square centimeter per day (cal/cm^2-day). Heating brought about by chemical changes and biological processes is neglected as is the energy transfer which occurs at the water-ground interface. Transformation of kinetic energy into thermal energy is also excluded. These factors are usually very small, in a quantitative sense, when compared with other terms in the budget if large reservoirs are considered. As a result, their omission has little affect on reliability of results.

During winter months when ice cover is partial or complete, the energy budget only occasionally yields adequate results, seeing that it is difficult to measure reflected solar radiation, ice-surface temperature and areal extent of ice cover. Daily evaporation estimates based on the energy budget are not feasible in most cases because reliable determination of

changes in stored energy for such short periods is impractical. Periods of a week or longer are more likely to provide satisfactory measurements.

In using the energy-budget approach, it has been demonstrated that the required accuracy of measurement is not the same for all variables.[4] For example, errors in measurement of incoming long-wave radiation as small as 2 percent can introduce errors of 3 to 15 percent in estimates of monthly evaporation, while errors on the order of 10 percent in measurements of reflected solar energy may cause errors of only 1 to 5 percent in calculated monthly evaporation.

To permit determination of evaporation by Eq. 2-19, it is common to use the following relationships:

$$B = Q_h/Q_e \tag{2-20}$$

where B is known as Bowen's ratio,[13] and

$$Q_w = c_p Q_e (T_e - T_b)/L \tag{2-21}$$

where c_p is the specific heat of water (cal/g-°C), T_e is the temperature of evaporated water (°C), T_b is the temperature of an arbitrary datum usually taken as 0°C, and L is the latent heat of vaporization (cal/g). Introducing these expressions in Eq. 2-19 and solving for Q_e,

$$Q_e = \frac{Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} - Q_0 + Q_v}{1 + B + c_p(T_e - T_b)/L} \tag{2-22}$$

To determine depth of water evaporated per unit time, the expression

$$E = Q_e/\rho L \tag{2-23}$$

may be used where E is evaporation (cm^3/cm^2-day), and ρ is the mass density of evaporated water (g/cm^3). The energy budget equation thus becomes,

$$E = \frac{Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} - Q_0 + Q_v}{\rho[L(1 + B) + c_p(T_e - T_b)]} \tag{2-24}$$

The Bowen ratio can be computed using

$$B = 0.61 \frac{p}{1000} \frac{(T_0 - T_a)}{(e_0 - e_a)} \tag{2-25}$$

where p is atmospheric pressure in millibars (mb), T_0 is the water surface temperature (°C), T_a is air temperature (°C), e_0 the saturation vapor pressure at the water surface temperature (mb), and e_a the vapor pressure of the air. This expression circumvents the problem of evaluating the sensible heat term, which does not lend itself to direct measurement.

Mass-Transfer Techniques

Mass-transfer equations are based primarily on the concept of turbulent transfer of water vapor (by eddy motion) from an evaporating surface to the atmosphere. Many equations, both theoretical and empirical,

have been developed. Most are similar in form to a relationship between evaporation and vapor pressure first recognized by Dalton,[14]

$$E = \kappa(e_0 - e_a) \tag{2-26}$$

where E is the direct evaporation, κ a coefficient dependent upon wind velocity, atmospheric pressure and other factors, and e_0 and e_a are the saturation vapor pressure at water surface temperature and the vapor pressure of air respectively.

Theoretical mass-transfer equations are based on the concepts of discontinuous and continuous mixing at the air-liquid interface.

Empirical approaches often require exacting and costly instrumentation and observations, so their general utility is limited. Complexity of the equations varies from simple expressions such as Eq. 2-26 to complex relationships like Sutton's equation for a circular lake of radius r given below.[15]

$$E = \frac{0.623}{p} G' \rho u^{\frac{2-n}{2+n}} r^{\frac{4+n}{2+n}} (e_0 - e_a) \tag{2-27}$$

where E is the evaporation (cm/day), ρ the mass density of the air (g/cm^3), u the average wind velocity (cm/sec), r the radius of circular lake (cm), p the atmospheric pressure (mb), e_0 and e_a are as previously defined, n is an empirical constant, and G' a complex function.

A commonly used empirical equation has been developed by Meyer.[16] This equation takes the form

$$E = C(e_0 - e_a)\left(1 + \frac{W}{10}\right) \tag{2-28}$$

where E is the evaporation in inches depth, e_0 and e_a are as previously defined but in units of inches of Hg, W is the wind velocity in mph messured about 25 ft above the water surface, and C is an empirical coefficient. For daily data on an ordinary lake, C will be about 0.36. For wet soil surfaces, small puddles and shallow pans, the value of C is approximately 0.50.

Example 2-2 Find the daily evaporation E from a lake for a day during which the following mean values were obtained: air temperature 87°F, water temperature 63°F, wind speed 10 mph, and relative humidity 20 percent.

Interpolating from Table 2-1,

$$e_0 = 0.58 \text{ in. Hg}$$
$$e_a = 1.29 \times 0.20 = 0.26 \text{ in. Hg}$$

Using Eq. 2-28,

$$E = 0.36(0.58 - 0.26)(1 + 10/10)$$
$$= 0.36 \times 0.32 \times 2$$
$$= 0.23 \text{ in./day}$$

TABLE 2-1 Water Vapor Pressure at Various Temperatures

Temp, °F	Vapor pressure		
	Millibars	Psi	(in. Hg)
32	6.11	0.09	0.18
40	8.36	0.12	0.25
50	12.19	0.18	0.36
60	17.51	0.26	0.52
70	24.79	0.36	0.74
80	34.61	0.51	1.03
90	47.68	0.70	1.42
100	64.88	0.96	1.94

Investigations of the utility of mass-transfer equations conducted at Lake Hefner indicated that a simple equation using wind speed and vapor pressure differences yielded as good results as any that were tested.[12] The equation evolved was

$$E = Nu(e_0 - e_a) \tag{2-29}$$

in which E is the evaporation (cm/day), N a coefficient, u the wind velocity at 2 meters above the water surface (meters/sec), and e_0 and e_a are as previously defined (mb). The value of N can be determined by comparative studies between mass-transfer and energy-budget methods. This is the preferred approach. If such an evaluation is not available, it can be approximated using,

$$N = 0.0291/A^{0.05} \tag{2-30}$$

where A is the surface area of the water surface in square meters. For values of A less that about 4×10^6 m^2, variations in wind exposure may become important and Eq. 2-30 should be used with caution. When N is based on comparative studies using the energy budget, average errors in evaporation estimates of about 15 percent can be expected, while errors of roughly 30 percent would likely obtain if Eq. 2-30 was employed.

Evaporation Determination Using Pans

The most widely used method of finding reservoir evaporation is by means of evaporating pans. The standard weather bureau Class A pan, built of unpainted galvanized iron, is currently the most popular. It is 4 ft in diameter, 10 in. deep, and is mounted 12 in. above the ground on a wooden frame. Relationships developed between pan and actual evaporation from large bodies of water such as lakes indicate multiplying the former by a factor of 0.70 to 0.75 (pan coefficient) gives the equivalent lake evaporation. Ratios of annual reservoir evaporation to pan evaporation are consistent from year to year and region to region, while monthly ratios often show considerable variation.

Estimates of reservoir evaporation based on short-period pan observations (less than 1 yr) may be seriously in error. Use of a pan coefficient to estimate evaporation from an ungaged location should reflect the geographic variability in heat transfer through the sides of the class A pan. For lakes subjected to significant amounts of advected energy, local pan-lake relationships should be established.

Available data indicate that the annual ratio of lake evaporation to class A pan evaporation is essentially 0.70 provided net advection is balanced by the change in energy stored, conduction through the pan is negligible, and the pan is located so that its exposure conditions are representative of the body of water being considered.

Selection of a Method for Determining Evaporation

Any of the methods described can be employed to determine evaporation. Usually, instrumentation for energy budget and mass-transfer methods is quite expensive and the cost to maintain observations is substantial. For these reasons, the water-budget method and use of evaporation pans are more common. The pan method is the least expensive and will frequently provide good estimates of annual evaporation. Any approach selected is dependent, however, upon the degree of accuracy required. As our ability to evaluate the terms in the water budget and energy budget improves, so will the resulting estimates of evaporation.

Nature of Evaporating Surface

Evaporation from a particular surface is directly related to the opportunity for evaporation (availability of water) provided by that surface. For open bodies of water, evaporation opportunity is 100 percent, while for soils it varies from a high of 100 percent when the soil is highly saturated—for example during storm periods—to essentially zero at different stages. Other types of surfaces provide diverse degrees of evaporation opportunity and except in rare cases, these will almost always vary widely with time.

Evaporation Reduction

Evaporation losses can be greatly significant at any location. Consequently, the concept of evaporation reduction is receiving widespread attention. Evaporation losses from soils can be controlled by employing various types of mulches or by chemical alteration. They may be reduced from open waters by (1) storing water in covered reservoirs, (2) making increased use of underground storage, (3) controlling aquatic growths, (4) building storage reservoirs with minimal surface area, and (5) through use of chemicals. Some of these approaches may be impractical, (covering large reservoirs), or uneconomical (large-scale vegetation control). All have potential, however, under the proper circumstances.

The first four approaches need no explanation of the mechanism expected to control evaporation. The fifth method, chemical means, requires further comment. Research has shown that certain types of organic compounds such as hexadecanol and octadecanol form monomolecular films which are effective as evaporation inhibitors.[4,17] Studies by the Bureau of Reclamation indicate that evaporation may be suppressed by as much as 64 percent with hexadecanol films in 4-ft diameter pans under controlled conditions. Actual reductions on large bodies of water would be significantly less than this, however, due to problems in maintaining films against wind and wave action. Evaporation reduction in the range of 22 to 35 percent has been observed for some studies on small lakes of roughly 100 acres in size with reductions of 9 to 14 percent reported for Lake Hefner in Oklahoma (2,500 acres).[18] Wind was a major problem at Lake Hefner, however.

In Australia, extended tests on medium-sized lakes (less than 2,500 acres) have indicated savings of 30 to 50 percent, although adverse winds were generally not encountered.[18] Although considerable research and development work remains, use of monomolecular films to control evaporation appears promising. Franzini has indicated that the cost per acre-foot of water saved by evaporation suppression is, in fact, competitive with various alternate means of increasing local water supplies, and these costs will likely decrease with advances in research.[17]

2-7 Transpiration

Root systems of plants absorb water in varying quantities. Most of this water is transmitted through the plant and escapes through pores in the leaf system. This is known as *stomatal transpiration.* Plants also lose water by other mechanisms, but usually this is negligible compared with that lost through the microscopic leaf apertures. Transpiration is basically a process by which water is evaporated from the airspaces in plant leaves. Therefore it is controlled essentially by the same factors which dominate evaporation, namely, solar radiation, temperature, wind velocity, and vapor pressure gradients. In addition, transpiration is affected to some degree by the character of the plant and plant density.

Soil moisture content, when reduced to the wilting point (stage at which plants wilt and do not recover in a humid atmosphere) also affects transpiration. The effects of decreased soil moisture above the wilting point are not clearly established and somewhat controversial. Nevertheless, it appears that as long as soil moisture lies between the limits of the wilting point and field capacity (amount of water retained in a soil against gravity after percolation ceases), transpiration is not materially affected. Saturated soils can sometimes adversely affect plant life.

Diffusion of water vapor from plant leaves to the atmosphere is

proportional to the vapor pressure gradient at the leaf-atmosphere interface. Upon absorbing solar radiation, leaves tend to become warmer than the surrounding air (often by as much as 5 to 10°F). The amount of water vapor held by the air at the leaf-air interface thus increases, more rapid water losses are favored, and transpiration follows a diurnal cycle which is approximately that of light intensity. It has also been demonstrated that transpiration and rate of plant growth are related. Below a temperature of about 40°F the amount of water transpired is considered negligible.

Different species and types of plants often display considerably different demands on soil moisture even if the same environmental conditions prevail. For example, an oak tree may transpire as much as 170 quarts of water a day, whereas a corn plant will transpire only about 2 quarts. The area covered by the two root systems is of course significantly different. Various species also indicate different patterns in seasonal demands for water. Agricultural products obviously have their periods of greatest transpiration at the peak of the growing season.

Precise values for quantities of water transpired are difficult to acquire, since many variables are active in the process and these range widely from one region to another. Available estimates should be used with caution and the conditions under which they were obtained should be determined before applying the data. Adequate relationships between climatic factors and transpiration become prerequisites if data derived in one climatic region are to have general utility.

Transpiration may be measured in the laboratory by using tanks wherein evaporation is eliminated and water losses are found by weighing. Coefficients must be derived before such data can be applied to field conditions and, even then the observations usually provide little more than an index to field water use. Large-scale field measurements of transpiration are virtually impossible under prevailing field conditions so it is common to find measures of consumptive use (combined evaporation plus transpiration) more widely adopted and of greater value to the practicing hydrologist. Table 2-2 gives some values of consumptive use for several

TABLE 2-2 Consumptive Use for Crops in the Montrose, Colorado Area During the Irrigation or Growing Season[19]

Crop	Consumptive use, in.
Alfalfa	26.5
Corn	19.7
Small Grain	14.9
Grass Hay	23.3
Natural Vegetation	37.3

crops in the Montrose area of Colorado.[19] These values are presented only to indicate their order of magnitude in this area during the growing

or irrigation season. More complete information on consumptive use by various crops can be found elsewhere.[1,19,20]

For many small local projects it is not possible to carry out detailed field studies to determine the consumptive use of crops. In such cases, it is common to use either the Blaney-Criddle or Penman method for estimating seasonal consumptive use.[21,22] The Blaney-Criddle method will be briefly described here.

The seasonal consumptive use for a particular crop can be computed using the relation

$$U = k_s B \tag{2-31}$$

where U is the consumptive use of water during the growing season (in.), k_s a seasonal consumptive use coefficient applicable to a particular crop and is empirically derived, and B the summation of monthly consumptive use factors for a given season. The term B can be expressed as

$$B = \Sigma(tp/100) \tag{2-32}$$

where t is the mean monthly temperature (°F), and p the monthly daytime hours given as percent of the year. If monthly values for the consumptive use coefficient k are available, monthly consumptive use can be found by using

$$u = ktp/100 \tag{2-33}$$

where u is the monthly consumptive use (in.) and the other terms are as previously described. Selected values of p and k are available in the literature.[20,21] An example will illustrate the use of this equation.

Example 2-3 Determine the monthly consumptive use of an alfalfa crop grown in southern California for the month of July if average monthly temperature is 72°F, average daytime hours in percent of the year is 9.88 and the mean monthly consumptive use coefficient for alfalfa is 0.85.

Using Eq. 2-33,

$$\begin{aligned} u &= ktp/100 \\ &= 0.85 \times 72 \times 9.88/100 \\ &= 6.05 \text{ in. of water} \end{aligned}$$

Transpiration Reduction

Water conservation through transpiration reduction is being seriously studied, and certain preventative practices are presently in use. Methods of control include the use of chemicals to inhibit water consumption (analogous to the use of films to control surface evaporation except that chemicals are applied in the root zone); harvesting of plants; improved irrigation practices, and actual removal or destruction of certain vegetative types.[27]

In arid regions of the Southwest, certain plants known as *phreatophytes* (plants capable of tapping the water table or capillary fringe) transpire

enormous quantities of water each year without providing any particular apparent benefit (although this statement is open to question). Many of these plants such as the salt cedar grow in stream channels and tend to create flood-control problems by restricting channels in addition to using valuable underground water supplies. In New Mexico there have been as many as 43,000 acres of salt cedar along the Pecos River alone. Control of these phreatophytes could result in estimated savings of over 200,000 acre-ft of water in a critically water-short region of the United States.[23] Conservation through transpiration control may be important but the ecologic consequences of such control practices should be given careful consideration.

2-8 Evapotranspiration

In most cases of practical interest to the hydrologist, only total evaporation from an area—combined evaporation plus transpiration (consumptive use)—is of real interest. Various methods for determining evapotranspiration have been proposed, but there is no one system generally acceptable under all conditions. Basically there are three major types of approaches:

1. Theoretical, based on physics of the process.
2. Analytical, based on energy or water budgets.
3. Empirical.

The water-budget method was illustrated in Example 1-1. Its adequacy is dependent upon the accuracy with which the several terms in the budget equation can be evaluated. The energy budget can also be used to calculate field evapotranspiration in a manner similar to that previously described for lakes. For this application, however, the soil's thermal properties must be known, and temperature and vapor pressure gradients measured at two levels above the ground are needed in Bowen's ratio. For field plots, the amount of energy advected usually can be neglected.

Mass-transfer equations of the form previously discussed can also be used to estimate evapotranspiration. The Thornthwaite-Holzman equation is a good example of a mass-transfer equation which has often been employed for this purpose. However, Linsley et al. indicate there is some question as to the adequate verification of this model to estimate evapotranspiration losses.[24] The equation is expressed as

$$E = \frac{833\,\kappa^2 (e_1 - e_2)(V_2 - V_1)}{(T + 459.4)\log_e (z_2/z_1)^2} \tag{2-34}$$

where it is assumed that the atmosphere is adiabatic, and that the wind speed and moisture are distributed logarithmically in a vertical direction. Evaporation E is given in inches per hour; κ is von Kármán's constant

(0.4); e_1 and e_2 are vapor pressures (in. Hg); V_1 and V_2 are wind speeds (mph); and T is the mean temperature (°F) of the layer between the lower level z_1 and the upper level z_2. Considering the small differences between wind and vapor pressure to be expected at two levels so closely spaced, and since these gradients are directly related to the sought-after evaporation, highly exacting instrumentation is required to get reliable results.

A number of specific evapotranspiration equations are available, the Blaney-Criddle equation (Eq. 2-31) being representative. Reference 28 gives an excellent summary of the state-of-the-art in evapotranspiration research.

2-9 Interception

Part of the storm precipitation which occurs is intercepted by vegetation and other forms of cover on the drainage area. Interception can be defined as that segment of the gross precipitation input which wets and adheres to above ground objects until it is returned to the atmosphere through evaporation. Precipitation striking vegetation may be retained on leaves or blades of grass, flow down the stems of plants and become stemflow, or fall off the leaves to become part of the throughfall. The modifying effect which a forest canopy can have on rainfall intensity at the ground (the throughfall) can be put to practical use in watershed management schemes.

The amount of water intercepted is a function of (a) storm character, (b) species, age, and density of prevailing plants and trees, and (c) season of the year. Usually about 10 to 20 percent of the precipitation which falls during the growing season is intercepted and returned to the hydrologic cycle by evaporation. Water losses by interception are especially pronounced under dense closed forest stands—as much as 25 percent of the total annual precipitation. Schomaker has reported that the average annual interception loss by Douglas fir stands in western Oregon and Washington is about 24 percent.[3] A 10-yr-old loblolly pine plantation in the South showed losses on a yearly basis of approximately 14 percent, while Ponderosa pine forests in California, were found to intercept about 12 percent of the annual precipitation. Mean interception losses of approximately 13 percent of gross summer rainfall were reported for hardwood stands in the white Mountains of New Hampshire. Additional information given in Table 2-3 includes some data on interception measurements obtained in Maine from a mature spruce-fir stand, a moderately well-stocked white and gray birch stand, and an improved pasture.[3]

Lull indicates that oak or aspen leaves may retain as much as 100 drops of water.[1] For a well-developed tree, interception storage on the order of 0.06 in. of precipitation could therefore be expected on the basis of an

TABLE 2-3[1] Weekly Average Precipitation Catch of Standard U. S. Weather Bureau-Type Rain Gauges Located in a Spruce-Fir Stand, a Hardwood Stand, and a Pasture During the Winter of 1965–66

Measuring date[2]	Weekly average precipitation catch, in. of equivalent rain			Percent interception by forest cover	
	Spruce-fir	Birch	Pasture	Spruce-fir	Birch
11/ 9/65	.24	.33	.39	38	15
11/16/65	1.01	1.25	1.45	30	14
11/23/65	1.01	1.23	1.36	26	10
12/10/65[3]	1.41	1.65	1.79	21	8
12/17/65	.55	.81	.87	37	7
12/30/65	.66	.95	1.08	39	12
1/ 4/66	.20	.25	.26	23	4
1/12/66	.36	.55	.61	41	10
1/18/66	Trace	Trace	Trace	—	—
1/25/66	.25	.58	.59	58	2
2/ 1/66	1.38	1.91	1.96	30	3
2/ 8/66	.05	.07	.06	17	16
2/11/66	.29[4]	.02	Trace	—	—
2/15/66	.76	.81	.98	22	17
2/21/66	.17	.22	.22	23	0
3/ 2/66	.86	1.23	1.45	41	16
3/ 7/66	.76	.84	.97	22	13
3/15/66	0	0	0	—	—
3/29/66	.73	1.13	1.27	43	11
Total	10.69	13.83	15.31	30.2	9.5

1 After Schomaker (Ref. 3).

2 The period between measuring dates is seven days, except when precipitation occurred on the seventh day. In this event, measurement was postponed until precipitation ceased.

3 Measurements were delayed until a method was devised to melt frozen precipitation on the site.

4 This measurement in the spruce stand was the result of foliage drip during a thaw from previously intercepted snow.

average retention of about 20 drops per leaf. For light showers ($P_g < 0.01$ in.) 100 percent interception might occur, whereas for showers where $P_g > 0.04$ in., losses in the range of 10 to 40 percent are realistic.[5]

Figure 2-7 illustrates the general time distribution pattern of interception loss intensity. Most interception loss develops during the initial storm period and the rate of interception rapidly approaches zero thereafter. Total storm interception can be estimated by using[1,5]

$$L_i = S_i + KEt \tag{2-35}$$

where L_i is the volume of water intercepted in inches, S_i the interception storage which will be retained on the foliage against the forces of wind and gravity (usually varies between 0.01 and 0.05 in.), K the ratio of surface area of intercepting leaves to horizontal projection of this area, E the amount of water evaporated per hour during the precipitation period, and t the time in hours. Total interception by an individual plant is directly related to the amount of foliage, its character and orientation.

Equation 2-35 can be used to estimate total interception losses. For analyses of individual storms it is also necessary to know or assume distribution of this abstraction. A general equation for estimating such losses is not available, since most studies have been related to particular species or experimental plots strongly associated with a given locality. In addition, the loss function varies with the storm's character. If adequate experimental data are available, the nature of the variance of interception versus time might be inferred. Otherwise, common practice is to deduct the estimated volume entirely from the initial period of the storm ("initial abstraction").

2-10 Depression Storage

Precipitation which reaches the ground may infiltrate, flow over the surface, or become trapped in numerous small depressions from which the only escape is evaporation or infiltration. The nature of depressions as well as their size is largely a function of the original land form and local land-use practices. Because of extreme variability in the nature of depressions and the paucity of sufficient measurements, no generalized relationship with enough specified parameters for all cases is feasible. A rational model can be suggested, however, and information on the range of interception losses reported in the literature will be offered.

According to Linsley et al.[29] the volume of water stored by surface depressions at any given time can be approximated using

$$V = S_d(1 - e^{-kP_e}) \tag{2-36}$$

where V is the volume actually in storage at some time of interest, S_d the maximum storage capacity of the depressions, P_e the rainfall excess (gross rainfall minus evaporation, interception and infiltration), and k is a constant equivalent to $1/S_d$. The value of the constant can be determined by considering that if $P_e \approx 0$, essentially all of the water will fill depressions and dV/dP_e will equal one. This requires that $k = 1/S_d$. Estimates of S_d may be secured by making sample field measurements of the area under study. Combining such data with estimates of P_e permits a determination of V. The manner in which V varies with time must still be estimated if depression storage losses are to be abstracted from the gross rainfall input.

One assumption regarding dV/dt is that all depressions must be full before overland flow supply begins. Actually, this would not agree with reality unless the location of depressions was graded with the largest ones occurring downstream. If the depression storage is abstracted in this manner, total volume would be deducted from the initial storm period such as shown by the shaded area in Fig. 2-10. Such postulates have been used with satisfactory results under special circumstances.[6]

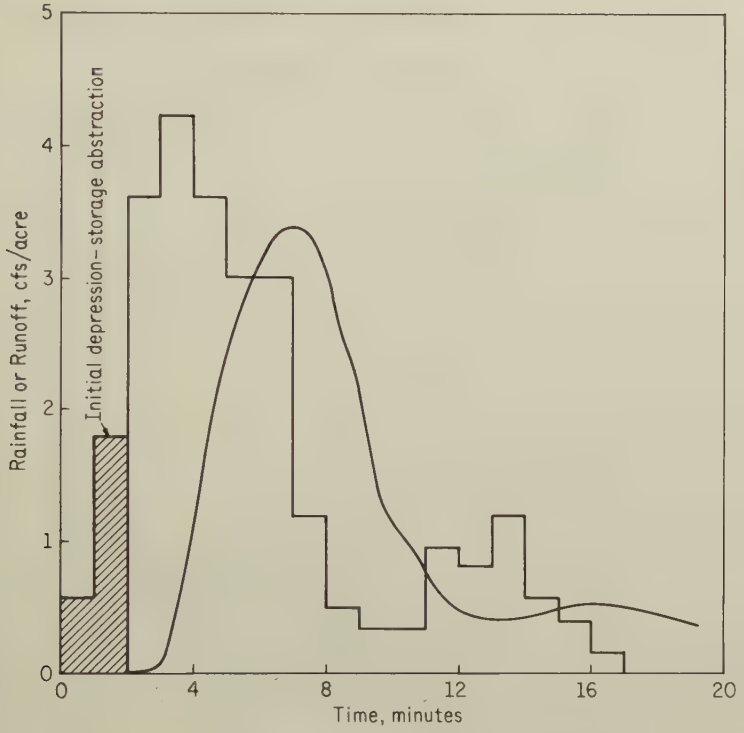


FIG. 2-10. Simple depression-storage abstraction scheme.

Depression-storage intensity can also be estimated using Eq. 2-36. If the overland flow supply rate σ plus depression storage intensity equal $(i - f)$, where i is the rainfall intensity reaching the ground and f is the infiltration rate, then the ratio of overland flow supply to overland flow plus depression-storage supply can be proved equal to

$$\frac{\sigma}{(i - f)} = 1 - e^{-kP_e} \tag{2-37}$$

This expression can be derived by adjudging

$$\frac{\sigma}{(i - f)} = \frac{i - f - v}{i - f} \tag{2-38}$$

and noting that v is equal to the derivative of Eq. 2-36 with respect to time. Then

$$v = \frac{d}{dt} S_d(1 - e^{-kP_e}) \tag{2-39}$$

$$v = (S_d k e^{-kP_e})\, dP_e/dt \tag{2-40}$$

It was shown that $k = 1/S_d$ so that

$$v = e^{-kP_e}(dP_e/dt) \tag{2-41}$$

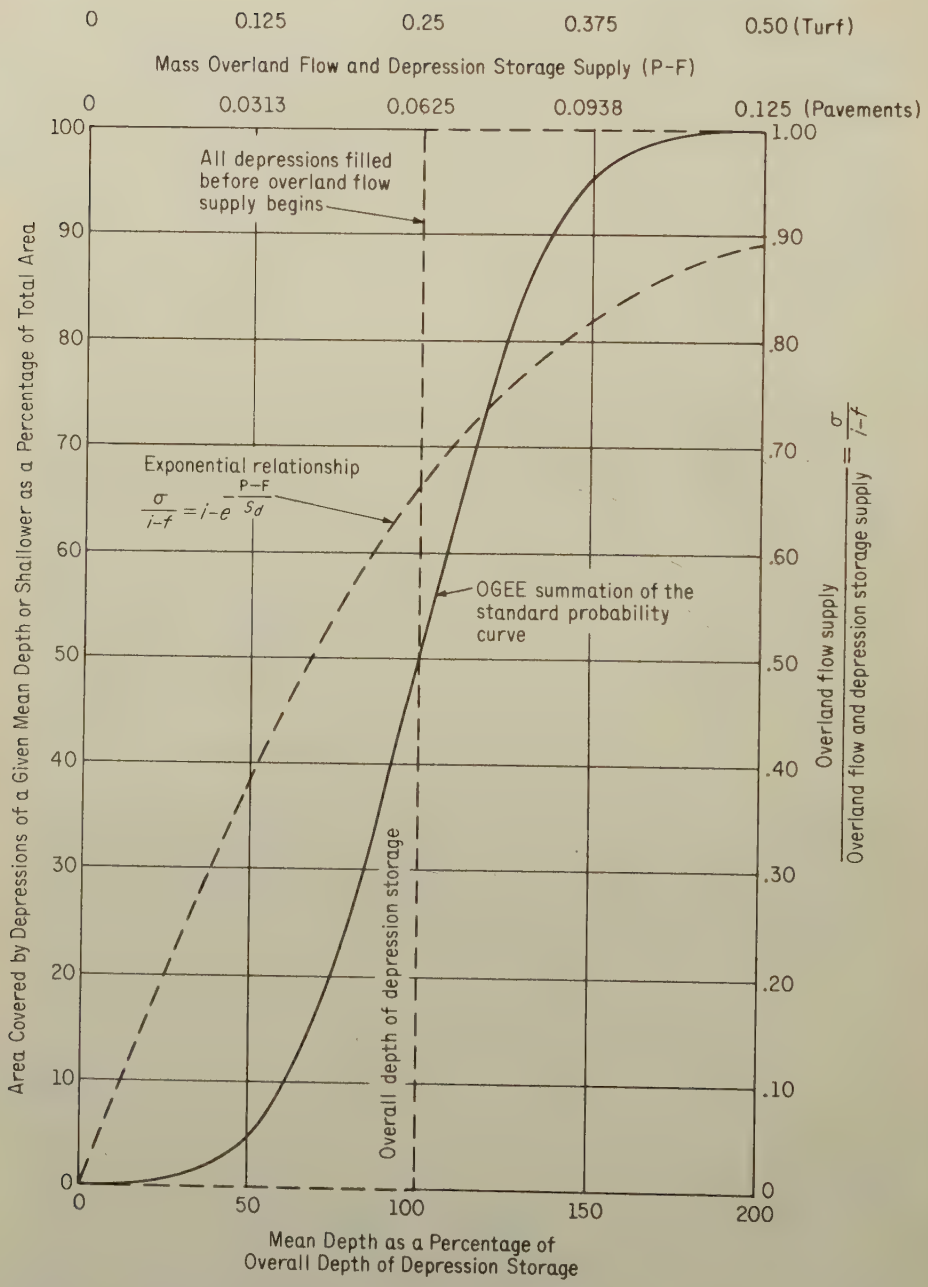


FIG. 2-11. Depth-distribution curve of depression storage. [After A. L. Tholin, and C. J. Kiefer, "The Hydrology of Urban Runoff," *Proc. ASCE, J. of San Eng. Div.*, Vol. 85, No. 5A2 (March 1959), Part 1, Proc. paper 198: pp. 47-106.]

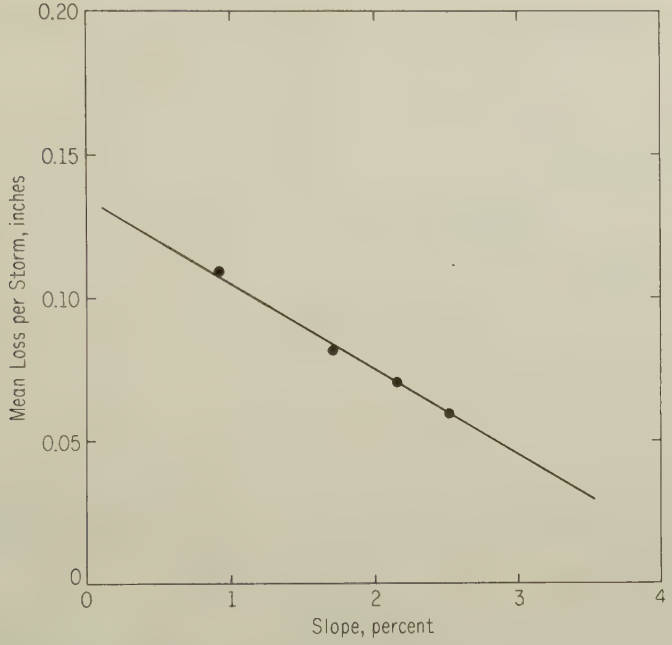


FIG. 2-12. Depression-storage loss versus slope for four impervious drainage areas. (After Viessman.)

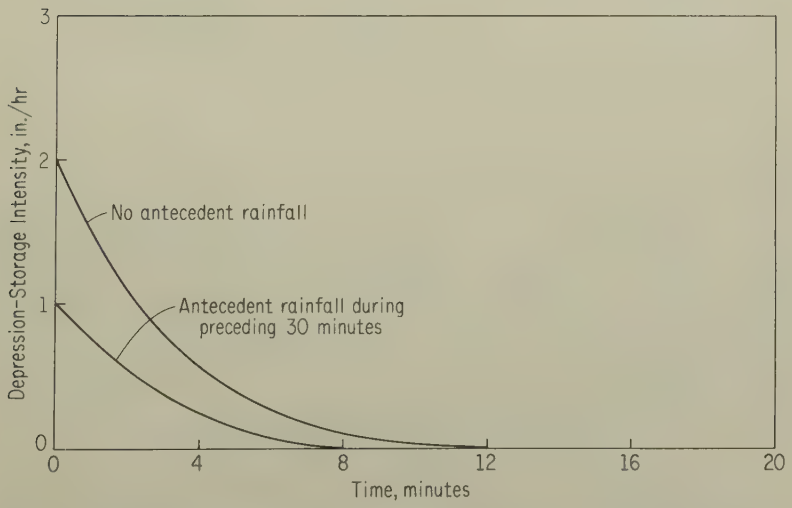


FIG. 2-13. Depression-storage intensity versus time for an impervious area. (After Turner.)

The excess precipitation P_e equals gross rainfall minus infiltrated water and since the derivative with respect to time can be replaced by equivalent intensity $(i - f)$, the intensity of depression storage becomes

$$v = e^{-kP_e}(i - f) \tag{2-42}$$

Inserting in Eq. 2-38,

$$\frac{\sigma}{(i - f)} = \frac{i - f - (i - f)e^{-kP_e}}{i - f} \tag{2-43}$$

and

$$\frac{\sigma}{(i - f)} = \frac{(i - f)[1 - e^{-kP_e}]}{(i - f)} \tag{2-44}$$

or

$$\frac{\sigma}{(i - f)} = [1 - e^{-kP_e}] \tag{2-45}$$

Figure 2-11 illustrates a plot of this function versus the mass overland flow and depression-storage supply $(P - F)$, where P is the accumulated mass infiltration.[7] In the drawing plot a mean depth of 0.25 in. for turf and 0.0625 in. for pavements was assumed. Maximum depths were 0.50 in. and 0.125 in. respectively. The figure also depicts the effect on estimated overland flow supply rate which is derived from choice of the abstraction model. For example, if it is assumed that all depressions must be full before overland flow begins, the graph shows that for a mean depth ≤ 0.25 in. for turf areas, $\sigma = 0$, whereas at mean depths ≥ 0.25 in., $\sigma = (i - f)$. For the exponential model, σ always will be greater than zero. Tholin and Kiefer have recommended that a relationship somewhat intermediate between those previously mentioned is likely more representative of fully developed urban areas.[7] A cumulative standard probability curve was selected for this representation and also described in Fig. 2-11.

Some values for depression-storage losses from intense storms reported by Hicks are 0.20 in. for sand, 0.15 in. for loam, and 0.10 in. for clay.[8] Tholin and Kiefer have used values of 0.25 in. in pervious urban areas and 0.0625 in. for pavements.[7] Studies of four small impervious drainage areas by Viessman yielded the information shown in Fig. 2-12, where mean depression-storage loss is highly correlated with slope. This is easily understood since a given depression will hold its maximum volume if horizontally oriented. Using very limited data from a small paved-street section, Turner devised the curves shown on Fig. 2-13.[9] Other sources of data related to surface storage are available in the literature.[1,2,10]

2-11 Infiltration

The role played by infiltration in distribution of precipitation input is an exceedingly important one. It can affect not only the timing but the

distribution and magnitude of surface runoff as well. For this reason, reliable estimates of infiltration must be incorporated in any watershed model.[26]

Infiltration is the flow of water into the ground through the earth's surface. The rate f at which it occurs is influenced by such factors as type and extent of vegetal cover, condition of the surface crust, temperature, rainfall intensity, physical properties of the soil, and water quality.

The rate at which water is transmitted through the surface layer is highly dependent upon condition of the surface. For example, inwash of fine materials may seal the surface so that infiltration rates are low even when the underlying soils are highly permeable. After water crosses the surface interface, its rate of downward movement will be controlled by the transmission characteristics of the underlying soil profile. Volume of storage available below ground is also a factor affecting infiltration rates.

Considerable research on infiltration has taken place but considering the infinite combinations of soil and other factors existing in nature, it is not hard to recognize that no adequately quantified general relationship exists.

Past research efforts were generally directed toward (1) development of empirical equations based on field observation, and (2) solution of equations based on the mechanics of saturated flow in porous media.[10] In the final analysis, more basic approaches will probably yield the most reliable methods, but these efforts have not been satisfactorily translated into practical hydrologic tools.

The infiltration process was thoroughly studied by Horton in the early 1930's.[2] An outgrowth of his work, shown graphically in Fig. 2-14, was

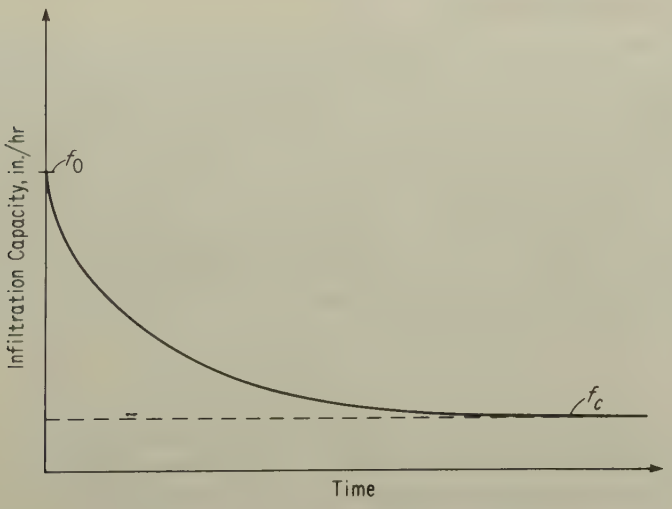


FIG. 2-14. An infiltration-capacity function.

the following relationship for determining infiltration capacity:

$$f = f_c + (f_0 - f_c)\, e^{-kt} \tag{2-46}$$

where f is the infiltration capacity (depth / time) at some time t, k is a constant representing rate of decrease in f capacity, f_c is a final or equilibrium capacity, and f_0 is the initial infiltration capacity. It indicates that if rainfall supply exceeds infiltration capacity, infiltration tends to decrease in an exponential manner. Although simple in form, difficulties in determining useful values for f_0 and k restrict the use of this equation.

Measuring Infiltration

Commonly used methods for determining infiltration capacity are hydrograph analyses and infiltrometer studies. Infiltrometers are usually classified as rainfall simulators or flooding devices. In the former, artificial rainfall is simulated over a small test plot and infiltration calculated from observations of rainfall and runoff, with consideration given to depression storage and surface detention.[1] Flooding infiltrometers are usually rings or tubes inserted in the ground. Water is applied and maintained at a constant level and observations made of the rate of replenishment required.

Estimates of infiltration based on hydrograph analyses have the advantage over infiltrometers of relating more directly to prevailing conditions of precipitation and field. However, they are no better than the precision with which rainfall and runoff are measured. Of particular importance in such studies is the areal variability of rainfall. Several methods have been developed and are in use. Reference 1 gives a good description of the most prominent of these.

Calculation of Infiltration

Infiltration calculations vary in sophistication from the application of reported average rates for specific soil types and vegetal covers to the use of conceptually sound differential equations governing the flow of water in unsaturated porous media. For small urban areas which respond rapidly to storm input, more precise methods are sometimes warranted. On large watersheds subject to peak-flow production from prolonged storms, average or representative values may be adequate.

Following Horton's work, numerous equations for infiltration rate have been proposed by Holtan and others.[11] In 1954 Philip developed an equation of the form[31]

$$\frac{dF}{dt} = \frac{k}{\mu}\left(1 + \left[\frac{(m - m_0)(P + H)}{F}\right]\right) \tag{2-47}$$

where μ = fluid viscosity
k = saturated permeability

F = total volume of infiltrated water
H = depth of water on soil surface
m_0 = initial moisture content of soil
P = capillary potential at wetting front
m = average moisture content to depth of wetting front at time t

Use of this equation or equations such as 2-46 is hampered because of the independent variable, time. As long as the rate of precipitation exceeds infiltration capacity, no problem arises. When supply rates drop below infiltration capacity, difficulty occurs. Several investigators have circumvented this problem by introducing soil moisture as the dependent variable.[10,11,30] The following equation proposed by Huggins and Monke is an example,[10]

$$f = f_c + A \left[\frac{S - F}{T_p}\right]^P \tag{2-48}$$

where A and P are coefficients, S is the storage potential of a soil overlying the impeding layer (T_p minus antecedent moisture); F is the total volume of water which infiltrates; and T_p is the total porosity of soil lying over the impeding strata. The variable F must be calculated for each time increment in the iteration process. At the beginning of a storm $F = 0$ and f is therefore known. In essence the continuity equation is solved for a block of soil with an inflow rate f (or smaller if the rainfall is less) and an outflow determined according to Eq. 2-49. Expression $dS/dt \cdot \Delta t$ then gives the change in storage of the soil. When added to the storage at the beginning of the time increment, total storage is obtained. Equation 2-48 is a modification of one originated by Holtan and Overton[11,30] and appears to have merit over the form of Eq. 2-46 if the rate of infiltration supply is less than infiltration capacity.

In order to use this relationship when the water-supply rate only intermittently exceeds infiltration capacity, the rate at which water drains from the "control zone," which determines soil moisture content ($S - F$), must be found. It is evaluated as follows:[10] (1) where moisture content of the control zone is less than field capacity, the drainage rate is considered zero; (2) drainage rate is assumed equal to infiltration rate when soil is saturated and infiltration rate becomes constant; and (3) if water content is between field capacity and saturation, the drainage rate is computed as

$$\text{Drainage rate} = f_c \left[1 - \frac{P_u}{G}\right]^3 \tag{2-49}$$

where P_u = unsaturated pore volume
G = maximum gravitational water, i.e., total porosity minus field capacity

Data from sprinkling infiltrometer studies of various watersheds of interest are used to estimate the coefficients in Eq. 2-48.[10]

Indexes

Infiltration indexes generally assume infiltration occurs at some constant or average rate throughout a storm. Consequently, initial rates are underestimated and final rates are overstated if an entire storm sequence with little antecedent moisture is considered. Best application is to large storms on wet soils or storms where infiltration rates may be assumed relatively uniform.[1]

The most common index is termed the ϕ *index* for which total volume of storm period infiltration is estimated and distributed uniformly across the storm pattern. Then volume of precipitation above the index line is equivalent to runoff (Fig. 2-15). A variation is the W index, which excludes surface storage and retention. Initial abstractions are often deducted from the early storm period to exclude initial depression storage and wetting.

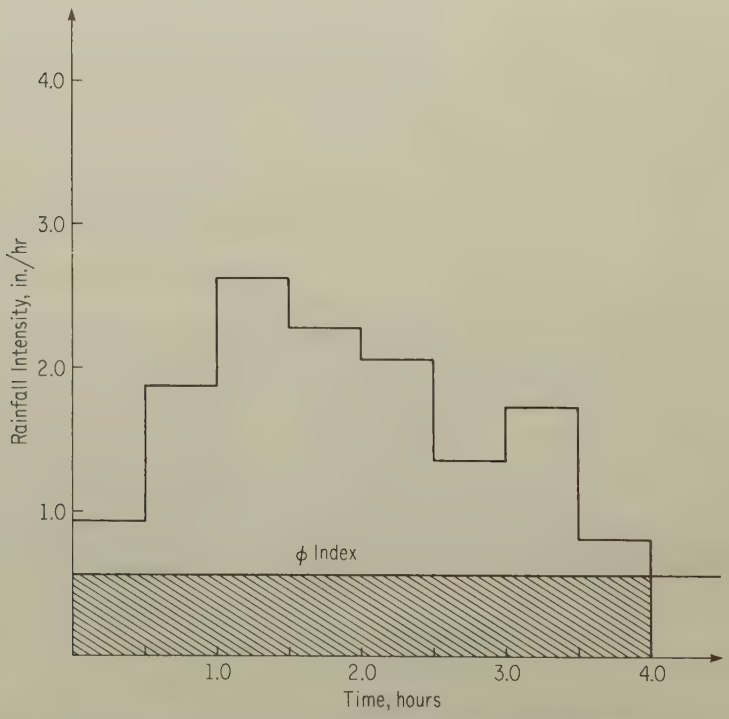


FIG. 2-15. Representation of a ϕ index.

References

1. Chow, Ven Te (ed.), *Handbook of Applied Hydrology.* New York: McGraw-Hill Book Company, 1964.

2. Horton, R. E., *Surface Runoff Phenomena: Part I Analysis of the Hydrograph*, Horton Hydrol. Lab. Pub. 101. Ann Arbor, Mich.: Edwards Bros., Inc., 1935.

3. Schomaker, C. E., "The Effect of Forest and Pasture on the Disposition of Precipitation," *Maine Farm Research* (July 1966).

4. Gunaji, N. N., et al., "Evaporation Reduction Investigation—Elephant Butte Reservoir, New Mexico," Engr. Exp. Sta., Tech. Report 25. Las Cruces, N.M.: New Mexico State University, 1965.

5. Kittredge, Joseph, *Forest Influences.* New York: McGraw-Hill Book Company, 1948.

6. Viessman, Warren, Jr., "A Linear Model for Synthesizing Hydrographs for Small Drainage Areas," paper presented at 48th Annual Meeting American Geophysical Union, Washington, D.C., April 1967.

7. Tholin, A. L., and C. J. Kiefer, "The Hydrology of Urban Runoff," *Trans. A.S.C.E.*, Vol. 125 (1960).

8. Hicks, W. I., "A Method of Computing Urban Runoff," *Trans. A.S.C.E.*, Vol. 109 (1944).

9. Turner, L. B., "Abstraction of Depression Storage From Storms on Small Impervious Areas," Master's thesis, University of Maine, Orono, Me., August 1967.

10. Huggins, L. F., and E. J. Monke, "The Mathematical Simulation of the Hydrology of Small Watersheds," Technical Report 1, Purdue University Water Resources Center, Lafayette, Ind., August 1966.

11. Holtan, H. N., "A Concept for Infiltration Estimates in Watershed Engineering, United States Department of Agriculture, Agricultural Research Service, (1961), pp. 41–51.

12. "Water-loss Investigations," Vol. 1, Lake Hefner Studies, U.S. Geologic Survey Professional Paper No. 269 (1954). (Reprint of U.S. Geological Survey Circ. 229, 1952).

13. Bowen, I. S., "The Ratio of Heat Losses by Conduction and by Evaporation from Any Water Surface," *Phys. Rev.*, Vol. 27 (1926).

14. Anderson, E. R., L. J. Anderson, and J. J. Marciano, "A Review of Evaporation Theory and Development of Instrumentation," Lake Mead Water Loss Investigation; Interim Report, Navy Electronmics Laboratory Report No. 159 (February 1950).

15. Sutton, O. G., "The Application to Micrometeorology of the Theory of Turbulent Flow Over Rough Surfaces," Royal Meteorological Society, *Quart. J.*, Vol. 75, No. 236 (October 1949).

16. Meyer, A. F., "Evaporation From Lakes and Reservoirs," Minnesota Resources Commission, St. Paul, Minn., June 1944.

17. Franzini, J. B., "Evaporation Suppression Research," *Water and Sewage Works*, May 1961.

18. La Mer, Victor K., "The Case for Evaporation Suppression," *Chem. Eng.*, June 10, 1963.

19. Blaney, H. F., "Water and Our Crops," *Water, the Yearbook of Agriculture.* Washington, D.C.: U.S. Department of Agriculture, 1955.

20. Blaney, H. F., "Monthly Consumptive Use Requirements for Irrigated Crops," *Proc. A.S.C.E., J. Irrigation and Drainage Div.*, Vol. 85, No. IR1 (March 1959).

21. Criddle, W. D., "Methods of Computing Consumptive Use of Water," *Proc. A.S.C.E., J. Irrigation and Drainage Div.*, Vol. 84, No. IR1 (January 1958).

22. Penman, H. L., "Estimating Evaporation," *Trans. Am. Geophys. Union*, Vol. 37, No. 1 (1956).

23. Hughes, E. H., "Research on Control of Phreatophytes," Proc. 9th Ann. Water Conf., New Mexico State University, Las Cruces, N.M., March, 1964.

24. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, *Hydrology for Engineers.* New York: McGraw-Hill Book Company, 1958.

25. Meier, W. L., Jr., "Analysis of Unit Hydrographs for Small Watersheds in Texas," Texas Water Commission, Bull. 6414, Austin, 1964.

26. Crawford, N. H., and R. K. Linsley, Jr., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Tech. Report 39, Department of Civil Engineering, Stanford University, July 1966.

27. Roberts, N. J., "Reduction of Transpiration," *J. Geophy. Res.*, Vol. 66, No. 10 (October 1961).

28. Rosenberg, N. J., H. E. Hart, and K. W. Brown, "Evapotranspiration—Review of Research," MP 20, Agricultural Experiment Station, University of Nebraska, Lincoln, 1968.

29. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, *Applied Hydrology.* New York: McGraw-Hill Book Company, 1949.

30. Overton, D. C., "Mathematical Refinement of an Infiltration Equation for Watershed Engineering." Washington, D.C.: U.S. Department of Agriculture, ARS 41–99, 1964.

31. Philip, J. R., "An Infiltration Equation with Physical Significance," *Soil Science*, Vol. 77 (1954).

32. Shands, A. L., "Mean Precipitable Water in the United States," U.S. Weather Bureau Tech. Papers No. 10 (1949).

33. Miller, D. W., J. J. Geraghty, and R. S. Collins, *Water Atlas of the United States.* Port Washington, N.Y.: Water Information Center, Inc., 1963.

34. U.S. Weather Bureau, Technical Papers 1–33. Washington, D.C.: Government Printing Office.

Problems

2-1. A mean draft of 130 mgd is to be developed from a 200-sq-mi catchment area. At the flow line the reservoir is estimated to be 4,300 acres. Annual rainfall is 35 in., the mean annual runoff is 13 in., and mean annual evaporation is 50 in. Find the net gain or loss in storage this represents. Compute the volume of water evaporated. State this figure in a form such as the number of years the volume could supply a given community.,

2-2. Discuss how you would go about collecting data for analysis of the water budget of a region. What agencies would you contact? What other sources of information would you seek out?

2-3. For some area of your choice, make a plot of mean monthly precipitation vs. time. Explain how this fits the pattern of seasonal water uses for the area. Will the form of precipitation be an important consideration?

2-4. A 9,000-sq-mi watershed received 18 in. of precipitation in a 1-yr period. The annual streamflow was recorded as 5,300 cfs. Roughly estimate the combined amounts of water evaporated and transpired. Qualify your answer.

2-5. Find the daily evaporation from a lake during which the following data were obtained: air temperature 92°F, water temperature 69°F, wind speed 18 mph, and relative humidity 26 percent.

2-6. Find the monthly consumptive use of an alfalfa crop for a month when the mean temperature is 76°F, average percentage daytime hours for the year is 10.2, and monthly consumptive use coefficient is 0.87.

2-7. Use Eq. 2-35 to estimate possible interception of a prevalent deciduous tree species in your area.

2-8. Use the precipitation pattern of Fig. 2-10 to estimate maximum cumulative depression storage of a paved parking lot 100 ft by 200 ft.

2-9. Integrate Eq. 2-46 over time and show that $k = (f_0 - f_c)$ divided by the total volume of infiltration above the f_c line of Fig. 2-14.

2-10. Refer to Fig. 2-15. Assume 23 percent runoff. What will be the value of the ϕ index?

2-11. Consider an urban area of 40 acres mostly residential. A rainfall occurs during a 30-min interval. If the total precipitation during this period is (a) 0.05 in., (b) 0.10 in., (c) 0.30 in., and (d) 1.3 in., during which of these storms will the estimation of runoff be most affected by estimation of losses? Why?

2-12. Which 10 states in the United States exhibit the largest mean annual surface water-evaporation rates?

2-13. During a given month a lake having surface area of 400 acres has an inflow of 20 cfs/day, an outflow of 16 cfs/day, a total seepage loss of 1 in. Total monthly precipitation is 1.8 in. and evaporation loss is 4.2 in. Estimate the change in storage.

3

STREAMFLOW

Surface-water hydrology deals with the transfer of water along the earth's surface. Satisfactory surface-water flow rate and quality are highly important to such fields as municipal and industrial water supply, flood control, streamflow forecasting, reservoir design, navigation, irrigation, drainage, water quality control, water-based recreation, and fish and wildlife management.

3-1 Some Elementary Precipitation-Streamflow Relationships

The relationship between precipitation and runoff is usually complex and is influenced by various storm-pattern, antecedent, and basin characteristics. Because of these complexities and the frequent paucity of adequate data, many approximate formulas have been developed to relate rainfall and runoff. The earliest of these were usually crude empirical statements, whereas the modern trend is to develop sound rational equations based on physical principles.

A common approach in connecting precipitation and runoff has been to plot precipitation versus runoff, find the slope of the trend line, and estimate the percentage of rainfall appearing as runoff. Runoff quantities determined this way are crude and subject to large errors. The degree of reliability is higher for drainage areas whose properties are least subject to seasonal or other types of variations, that is, an impervious area. Figure 3-1 illustrates the procedure. The resulting equation takes the form

$$Q = (1/s)(P - P_b) \tag{3-1}$$

where s is the slope of the line (P/Q) and P_b is a base precipitation value below which Q is zero. From Fig. 3-1 the relationship would be

$$Q = 0.57\,(P - 24)$$

where Q and P are annual runoff and precipitation respectively in inches. Considerable scatter of several data points from the assumed relationship

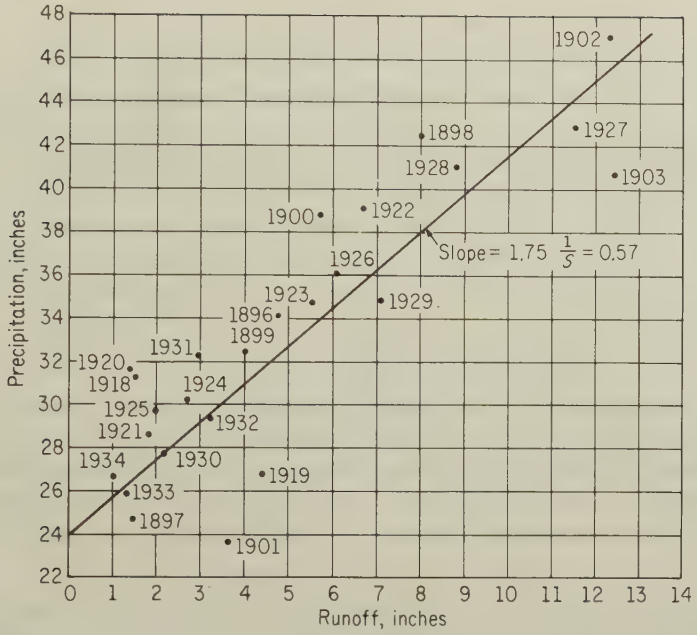


FIG. 3-1. Annual precipitation and annual runoff in the Neosho River Basin above Iola, Kansas. (U.S. Geological Survey Data.)

indicates this type of computation should be used with care. For rough approximations in preliminary planning studies, such methods are frequently helpful, however. Equations resembling 3-1 are improved if other parameters such as antecedent precipitation, soil moisture, season, and storm characteristics are included. Such relationships can be concluded using multiple-correlation techniques or graphical methods. Linsley et al. present a very complete treatment of methods for developing coaxial correlations involving several variables.[1]

Soil moisture relationships normally have a soil-moisture index as the variable, since direct measurements of actual antecedent soil moisture are not generally practical. Indexes which have been inserted are groundwater flow at beginning of the storm, antecedent precipitation, and basin evaporation.[2] Groundwater values should be weighted to reflect effects of precipitation occurring within a few days of the storm because soil-moisture changes from previous rains may be important. Pan-evaporation measurements can be employed to form soil-moisture figures, since evaporation is related to soil-moisture depletion.[3] Antecedent precipitation indexes (API) have probably received the widest use because precipitation is readily measured and relates directly to moisture deficiency of the basin.

A typical antecedent precipitation index is

$$P_a = aP_0 + bP_1 + cP_2 \tag{3-2}$$

where P_a is the antecedent precipitation index in inches, and P_0, P_1, and P_2 are the amounts of annual rainfall during the present year and for two years preceding the year in question. This index links annual rainfall and runoff values.[4] Coefficients a, b, and c are found by trial and error or other fitting techniques to produce the best correlation between runoff and antecedent precipitation index. The sum of the coefficients must be 1.

Kohler and Linsley[5] have proposed the following API for use with individual storms:

$$P_a = b_1P_1 + b_2P_2 + \cdots + b_tP_t \tag{3-3}$$

where the subscripts on P refer to precipitation which occurred that many days prior to the given storm, and the constants b (less than unity) are assumed to be a function of t. Values for the coefficients can be determined by correlation techniques. In daily evaluation of the index, b_t is considered to be related to t by

$$b_t = K^t \tag{3-4}$$

where K is a recession constant normally reported in the range 0.85 to 0.98. The initial value of the API (P_{a0}) is coupled to the API t days later (P_{at}) by

$$P_{at} = P_{a0}K^t \tag{3-5}$$

To evaluate the index for a particular day based on that of the preceding one, Eq. 3-5 becomes simply

$$P_{a1} = KP_{a0} \tag{3-6}$$

because $t = 1$.

Various empirical relationships have been proposed. Most are based on correlating two or three variables and at best yield only rough approximations. In many cases these bonds were developed without considering physical principles or dimensional homogeneity. An added shortcoming is that many formulas fit only a specific watershed and have little general utility. Empirical equations demand great caution and an understanding of their origin.

3-2 The Hydrograph

A knowledge of the size and time distribution of streamflows is essential to many aspects of water management and environmental planning.

A hydrograph is a continuous graph showing the properties of streamflow with respect to time, normally obtained by means of a continuous

strip recorder which indicates stage versus time (stage hydrograph), and is then transformed to a discharge hydrograph by application of a rating curve. The term *hydrograph* will generally be taken herein to indicate a discharge hydrograph.

The hydrograph is the result of the physiographic and hydrometeorological effects of the watershed. Typically it is a complex multiple-peak curve.

3-3 Components of the Hydrograph

A hydrograph has four component elements: (1) direct surface runoff, (2) interflow, (3) groundwater or base flow, and (4) channel precipitation. The rising portion of a hydrograph is known as the *concentration curve*; the region in the vicinity of the peak is called the *crest segment* and the falling leg is the *recession*.[1] The shape of a hydrograph depends upon precipitation-pattern characteristics and basin properties. Figure 3-2 illustrates the definitions presented.

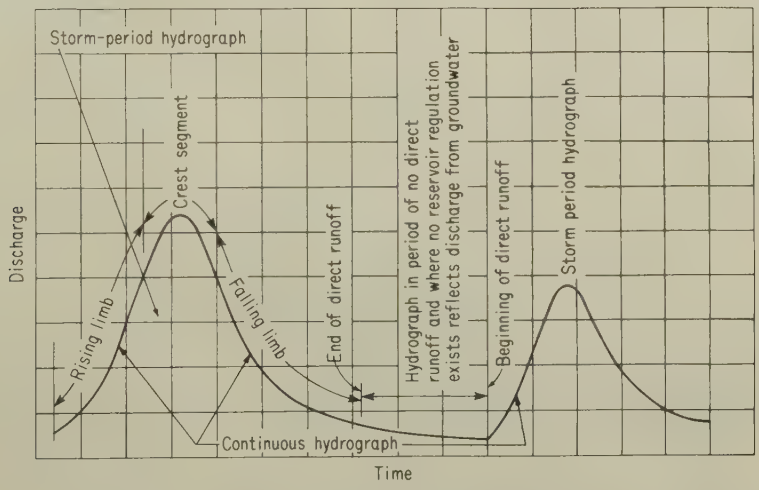


FIG. 3-2. Hydrograph definition.

3-4 Surface-Flow Phenomenon Within a Watershed

During a given rainfall, water is continually being abstracted by saturation of the upper levels of the soil surface; however, this saturation or infiltration is only one of many continuous abstractions. Rainfall is also intercepted by trees, plants, and roof surfaces, and at the same time is evaporated. Once rain falls and fulfills initial requirements of infiltration, natural depressions collect falling rain to form small puddles, creating *de-*

pression storage. In addition, minute depths of water forming *detention storage* build up on permeable and impermeable surfaces within the watershed. This stored water gathers in small rivulets which carry the water originating as *overland flow* into small channels, then into larger channels, and finally as *channel flow* to the watershed outlet.

In general, the channel of a watershed possesses a certain amount of *base flow* during most of the year. This flow comes from groundwater or spring contributions and may be considered as the normal day-to-day flow. Discharge from precipitation excess—that is, after abstractions—deducted from the original rainfall constitutes the direct-runoff hydrograph (DRH). Arrival of direct runoff at the outlet accounts for an initial rise in the DRH. As precipitation excess continues, enough time elapses for progressively distant areas to add to outlet flow. Consequently, the duration of rainfall dictates the proportionate area of the watershed amplifying the peak, and the intensity of rainfall during this period of time determines the resulting highest discharge.

If the rainfall maintains a constant intensity for a long enough period of time, a state of equilibrium discharge is reached, as depicted by Curve *A* in Fig. 3-3. Inflection point on Curve *A* indicates the time at which the

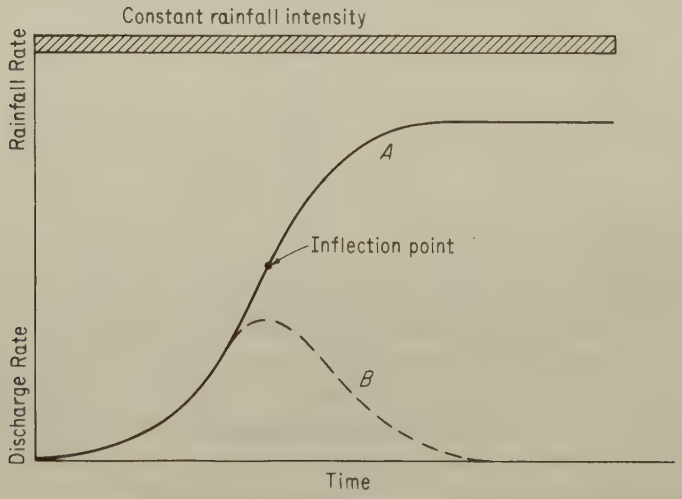


FIG. 3-3. Equilibrium discharge hydrograph.

entire drainage area contributes to the flow. At this time maximum storage of the watershed is only partially complete. As rainfall continues, maximum storage capacity is attained and equilibrium [inflow (rainfall) equals outflow (runoff)] reached. The condition of maximum storage and equilibrium is seldom if ever attained in nature. Extended rainfall may oc-

cur, but variations in intensity throughout its duration negates any possibility of a DRH of the theoretical shape for constant rainfall intensity.

A normal single-peak DRH generally possesses the shape shown by Curve *B* in Fig. 3-3. The time to peak magnitude of this hydrograph depends upon intensity and duration of rainfall, size, slope, shape, and storage capacity of the watershed. Once peak flow has been reached for a given isolated rain storm, the DRH begins to descend, its source of supply coming largely from water accumulated within the watershed such as detention and channel storage.

Processes involved in forming of the DRH can be better understood by visualizing the precipitation excess as partially disposed of immediately by surface runoff while a portion remains held within the watershed boundaries and is released later from storage. Thus shape and timing of the DRH are integrated effects of the duration and intensity of rainfall hydrometeorological factors, as well as the effect of the physiographic factors of the watershed upon the storage capacity.

3-5 Infiltration

The approach most used to get infiltration capacity of a particular soil-cover complex for small watersheds can be described by equations such as Eq. 2-48 or

$$f = f_c + (f_0 - f_c)e^{-kt} \tag{3-7}$$

where f = infiltration capacity, inches per hour
f_c, f_0 = constants for particular soil-cover complex
k = time constant for particular soil-cover complex
t = time from commencement of rainfall in hours

Many conditions of soil cover complex influence infiltration rates.[12] A few of the more important factors are chemical leaching, entrapped air, water ponding, depth to water table, cultivation, type and condition of cover, and weather. An example equation developed for good grass overlay takes the form

$$f = 0.53 + 2.47e^{-0.07t} \tag{3-8}$$

Typical values of an infiltration rate at the end of one hour are shown in Table 3-1. The relationship between the value of f_1 shown in Table 3-1 and the infiltration rate throughout a rainfall period is shown graphically in Fig. 3-4. The scope in values given in Table 3-1 is for a turf area and must be multiplied by a suitable factor for other types of cover complexes. A range of cover factors is listed in Table 3-2.

Total volume of infiltration and other abstractions from a given recorded rainfall are obtainable too from a discharge hydrograph if one is available. Separation of base flow from the discharge hydrograph results

TABLE 3-1 Typical f_1 Values*

Soil group	f_1 (in./hr)
High (sandy soils)	0.50–1.00
Intermediate (loams, clay, silt)	0.10–0.50
Low (clays, clay loam)	0.01–0.10

*After *ASCE Manual of Engineering Practice No. 28.*

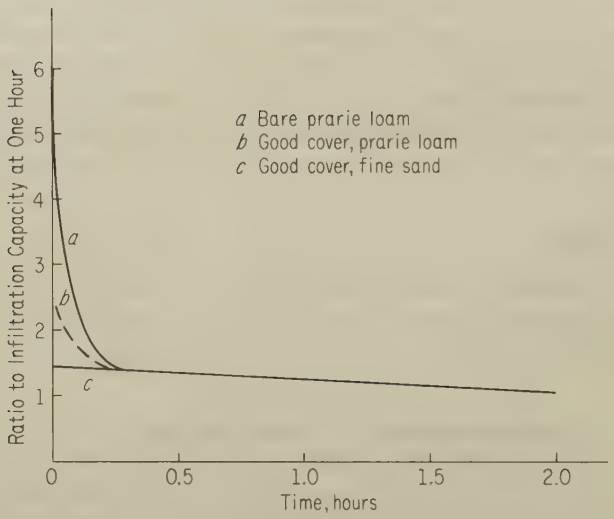


FIG. 3-4. Typical infiltration curves. (Ref. 14.)

TABLE 3-2 Cover Factors*

	Cover	Cover factor
Permanent forest and grass	Good (1″ humus)	3.0–7.5
	Medium (¼″–1″ humus)	2.0–3.0
	Poor (<¼″ humus)	1.2–1.4
Close-growing crops	Good	2.5–3.0
	Medium	1.6–2.0
	Poor	1.1–1.3
Row crops	Good	1.3–1.5
	Medium	1.1–1.3
	Poor	1.0–1.1

*After *ASCE Manual of Engineering Practice No. 28.*

in a direct-runoff hydrograph (DRH), which accounts for the direct surface runoff, that is, rainfall less abstractions. Direct surface runoff or precipitation excess in inches of rainfall uniformly distributed over a watershed can readily be calculated by picking values of DRH discharge at equal time increments throughout the storm and applying the formula[13]

$$P_e = \frac{(0.03719)(\Sigma q_i)}{(A)(n_d)} \tag{3-9}$$

where P_e = precipitation excess, in.
q_i = DRH ordinates at equal time intervals, cfs
A = drainage area, sq mi
n_d = number of time intervals per 24-hr period

For most cases the difference between original rainfall and direct runoff can be considered as infiltrated water. Exceptions may occur in areas of excessive subsurface drainage or tracts of intensive interception potential. The calculated value of infiltration can then be assumed as distributed according to an equation of the form of Eqs. 2-48 or 3-7, or it may be uniformly spread over the storm period. Choice of method employed depends upon the accuracy requirements and size of the watershed.

3-6 Hydrograph Separation

It is important to understand how the hydrograph can be subdivided into its component parts, and helpful to look at the effect on hydrograph shape of precipitation and watershed features. Figure 3-5 is used in the discussion.

Figure 3-5a illustrates the hydrograph of a permanent stream during a period between precipitation events, known as a *base-flow hydrograph* because groundwater sustains the flow. Four general conditions cause modification of the base-flow hydrograph shape. They were described by Horton[34] using the following sets of inequalities:

Set 1	$i < f$ $F < S_D$	Set 3	$i > f$ $F < S_D$
Set 2	$i < f$ $F > S_D$	Set 4	$i > f$ $F > S_D$

where Set 1 would produce a hydrograph similar to that shown in Fig. 3-5a except for a very small rise due to direct channel precipitation. No overland flow occurs since $i < f$, and since the soil's field capacity F is not reached, no interflow or added groundwater component develops. The entire effect of the storm would be to slightly reduce the soil-moisture deficiency S_D.

The conditions described by Set 2 still do not produce direct surface

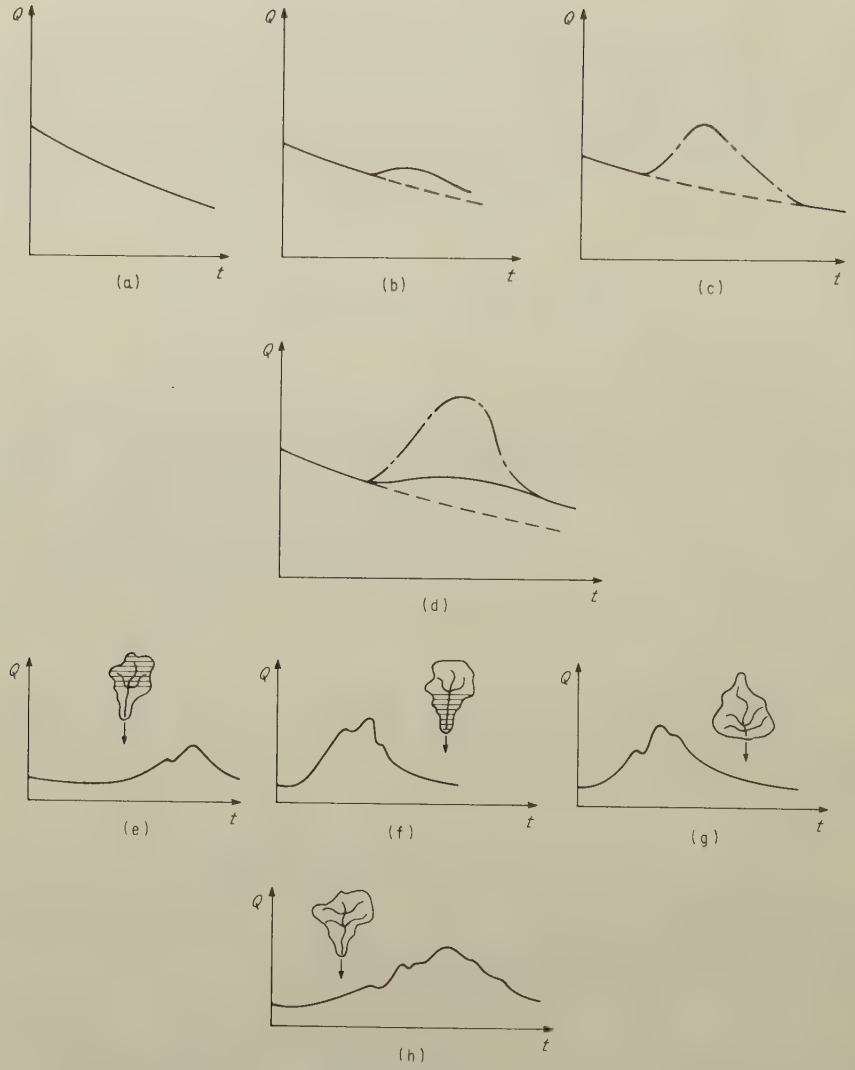


FIG. 3-5. Effects of storm and basin characteristics on hydrograph shape.

runoff, although the components of interflow and groundwater flow are added to channel precipitation. The initial hydrograph would be modified, since the field capacity of the soil is exceeded. Figure 3-5b illustrates this condition. Note that deviation of the hydrograph from the original base-flow curve is likely to be very small under these conditions.

Figure 3-5c illustrates a case where surface runoff becomes a component of flow because $i > f$. In this example, interflow and groundwater

flow are zero, as soil moisture deficiency still exists although at a reduced level. Channel precipitation likewise constitutes a component.

In the final set, all four components exist with rainfall intensity exceeding infiltration rate and the field capacity of the soil is reached. This case would be typical of a large storm event.

Figures 3-5e–h illustrate how hydrograph shape can be modified by areal variations in rainfall and rainfall intensity and by watershed configuration.[7] Minor fluctuations shown in these hydrographs are linked to variations in storm intensity. In Fig. 3-5e only the delaying effects pertinent to a storm over the upstream section of the area are indicated. Figure 3-5f shows the reverse of this condition. Figures 3-5g–h depict the comparative effects of basin geometry.

In most hydrograph analyses, interflow (Sec. 2-3) and channel precipitation are grouped with surface runoff rather than treated independently. Channel precipitation begins with inception of rainfall and ends with the storm. Its distribution with respect to time is highly correlated with storm pattern. The relative volume contribution tends to increase somewhat as the storm proceeds, since stream levels rise and water-surface area tends to increase. The fraction of watershed area occupied by streams and lakes is generally small, usually on the order of 5 percent or less, so the precentage of runoff related to channel precipitation is usually minor during important storms.

Interflow is that part of the subsurface flow which moves at shallow depths and reaches the surface channels in a relatively short period of time and therefore is commonly considered part of the direct surface runoff. Its distribution is commonly characterized by a slowly increasing rate up to the end of the storm period, followed by a gradual recession which terminates at the intersection of the surface-flow hydrograph and base-flow hydrograph. Figure 3-6 illustrates the approximate nature of the components of channel precipitation and interflow.

The base-flow component is composed of the water that percolates downward until it reaches the groundwater reservoir then flows to surface streams as groundwater discharge. The groundwater hydrograph may or may not show an increase during the actual storm period. Groundwater accretion resulting from a particular storm is normally released over an extended period measured in days for small watersheds and often in months or years for large drainage areas.

The surface-runoff component consists of water which flows overland until a stream channel is reached. During large storms it is the most significant hydrograph component. Figure 3-6 illustrates surface runoff and groundwater components of a hydrograph. As pointed out in Fig. 3-5, the relative magnitude of each component for a given storm is determined by a combination of many factors. Hydrographs are analyzed to provide

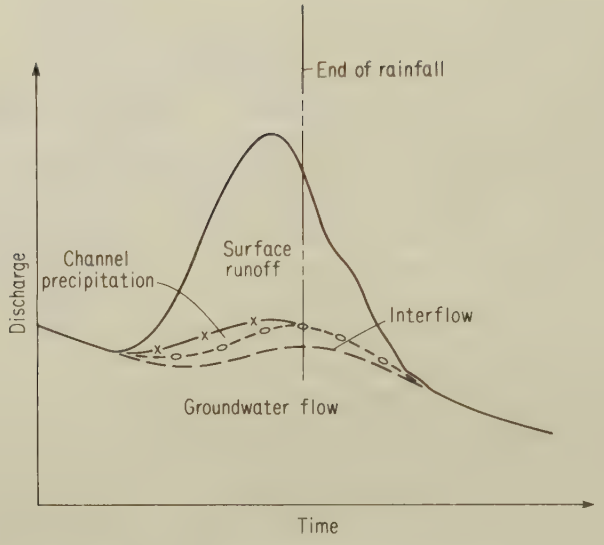


FIG. 3-6. Components of the hydrograph.

knowledge of the way precipitation and watershed characteristics interact to form them. The degree of hydrograph separation required depends upon the objective of the study. For most practical work, surface-runoff and groundwater components only are required. Research projects or more sophisticated analyses may dictate consideration of all components. When multiple storms occur within short periods, it is sometimes necessary to separate the overlapping parts of consecutive surface-runoff hydrographs.

3-7 Groundwater Recession

Several techniques are used to separate surface and groundwater flows, most of them based on analyses of groundwater recession or depletion curves. There is no added inflow to the groundwater reservoir, and if all groundwater discharge from the upstream area is intercepted at the stream-gauging point of interest, then groundwater discharge can be described by an equation of the form

$$q_t = q_0 K^t \tag{3-10}$$

where q_0 is a specified initial discharge, q_t is the discharge at any time t after flow q_0, and K is a recession constant.[8,9] Time units frequently used are days for large watersheds, hours or minutes for small basins. A plot of Eq. 3-10 yields a straight line on semilogarithmic paper by plotting t on the linear scale.

For most watersheds, groundwater-depletion characteristics are approximately stable, since they closely fit watershed geology. Nevertheless, the recession constant varies with seasonal effects such as evaporation and freezing cycles and other factors. Noting that $q_t\,dt$ is equivalent to $-dS$, where S is the quantity of water in storage, integration of Eq. 3-10 produces

$$S = \frac{(q_t - q_0)}{\log_e K} \tag{3-11}$$

This equation determines the quantity of water released from ground-

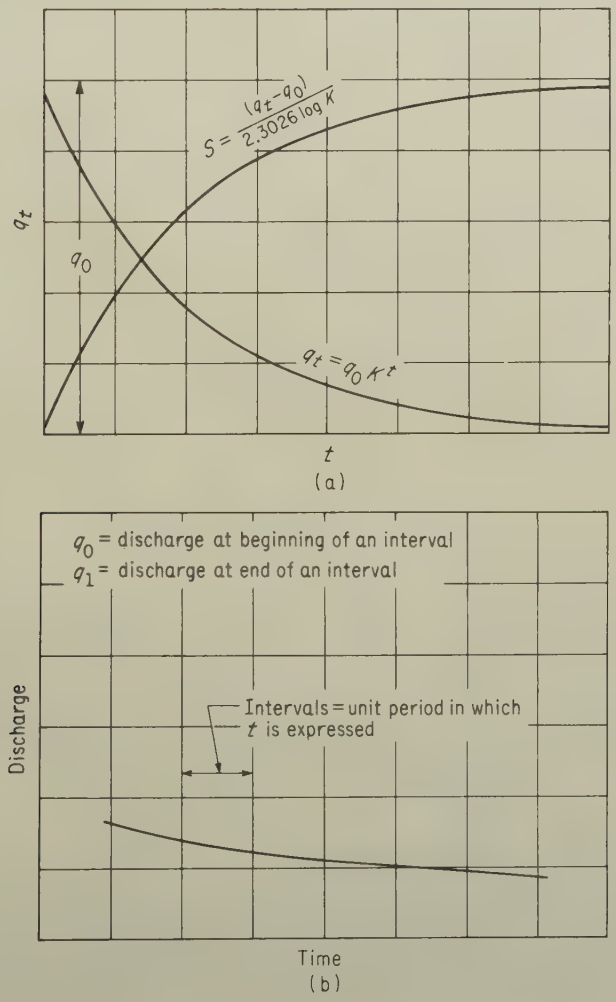


FIG. 3-7. Base-flow model.

water storage between the times of occurrence of two discharges of interest, or it can be used to calculate the volume of water still in storage at a time some chosen value of flow occurs. To get the latter, q_t is set equal to zero and q_0 becomes the reference discharge. Figure 3-7 plots Eqs. 3-10 and 3-11 and provides additional definition.

Groundwater-depletion curves can be analyzed by various methods to evaluate the recession constant K. One of these will be described. Data from a stream-gauging station are a prerequisite, and should reflect rainless periods with no upstream regulation such as a reservoir to affect flow at the gauging point. Otherwise an adjustment with its own errors is introduced.

From the streamflow data, plot a hydrograph (Fig. 3-7) to find values of discharge at the beginning and end of selected time intervals. Flows at the beginning of each interval are analogous to q_0 whereas those at the end are analogous to q_1. Next, select several time intervals and plot corresponding q_0's versus q_1's as shown on Fig. 3-8. The time period between

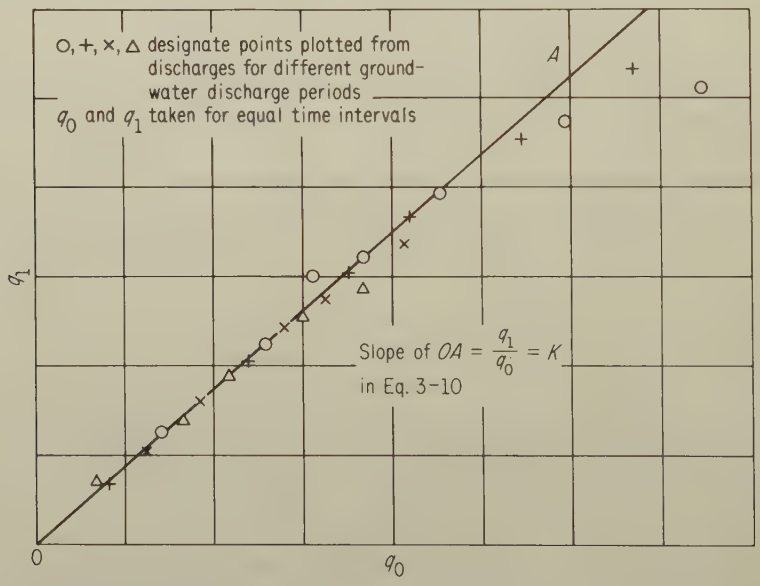

O, +, ×, △ designate points plotted from discharges for different groundwater discharge periods
q_0 and q_1 taken for equal time intervals
A
Slope of $OA = \frac{q_1}{q_0} = K$ in Eq. 3-10
q_1
q_0
0

FIG.3-8. Graphical method for determining recession constant K. (Soil Conservation Service)

consecutive values of q should be identical for each data set. Figures taken from recession curves of times which still reflect surface runoff will usually fall below and to the right of a 45° line drawn on the plot. These values will also be associated with larger numbers for q. Points taken from true groundwater-recession periods should approximately describe a straight line. Considering that $q_1 = q_0 = 0$ when $q_0 = 0$, a straight line

can be fitted graphically to the data points. Slope of this line is $q_1/q_0 = K$. Using this value, the depletion curve plots as a straight line on semi-logarithmic paper (t is the linear scale variable) or as a curve on arithmetic paper, Fig. 3-7a.

3-8 Separation Techniques

Several methods for base-flow separation are in use, unless the actual amount of base flow is known. During most serious storms, the maximum rate of discharge is just slightly affected by base flow, and inaccuracies in separation are fortunately not important.

The simplest base-flow separation technique is to draw a horizontal line from the point at which surface runoff begins, point A Fig. 3-9, to

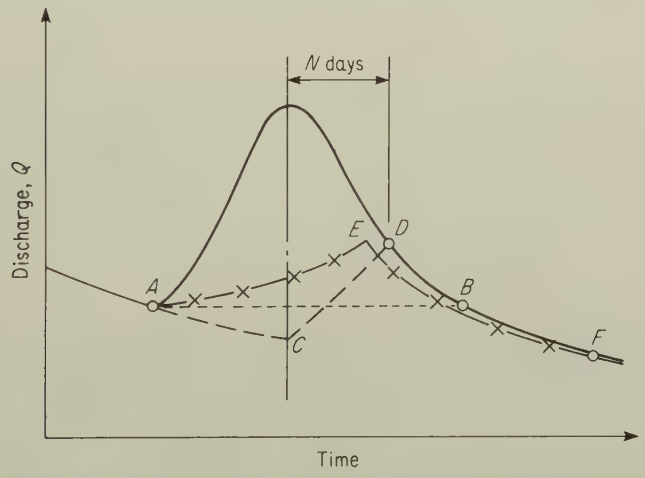


FIG. 3-9. Illustration of some hydrograph separation techniques.

an intersection with the hydrograph, point B. A second method projects the initial recession curve downward from A to C which lies directly below the peak rate of flow. Then point D on the hydrograph, representing N days after the peak, is connected to point C by a straight line defining the groundwater component. Choice of N is based on the formula

$$N = A^{0.2} \qquad (3\text{-}12)$$

where N is the time in days and A is the drainage area in square miles.[1]

A third procedure employs a base-flow recession curve which is fitted to the hydrograph, then computed in a time-decreasing direction. Point F, where the computed curve begins to deviate from the actual hydrograph, marks the end of direct runoff. The curve is projected backward arbitrarily to some point E and its shape from A to E arbitrarily assigned.

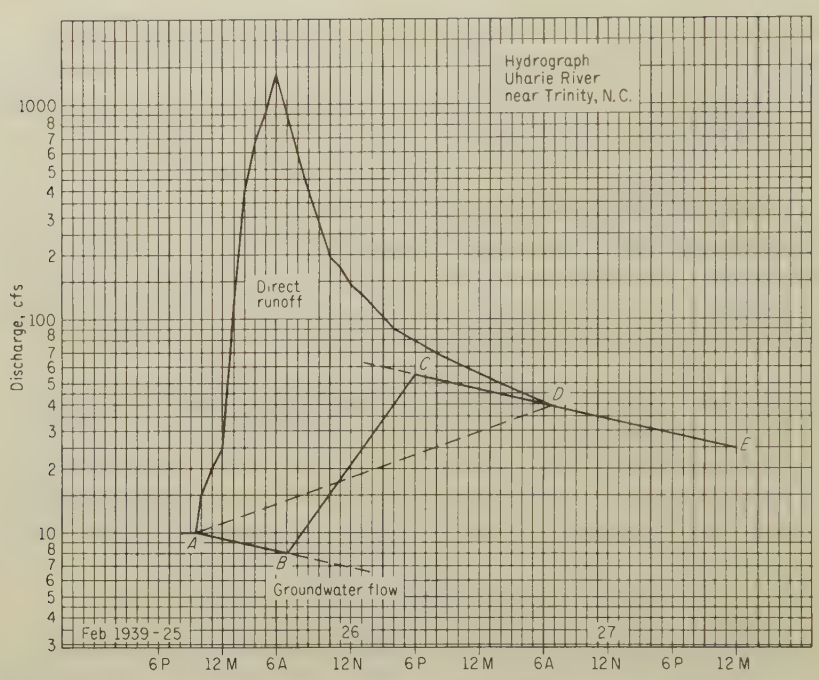


FIG. 3-10. Illustration of base-flow separation. (U.S. Department of Agriculture Soil Conservation Service.)

All of these methods are approximate and separation of hydrographs is a subjective procedure. Figure 3-10 illustrates two separation techniques to determine surface-runoff and groundwater-flow components. Line AD represents the simple procedure of connecting the point of beginning of direct runoff with an arbitrary point on the recession curve (an advantage over the horizontal line technique because the time base of direct runoff is much shorter). Curve $ABCD$ is constructed from the extension of the base-flow recession curve.

3-9 Time Base of the Hydrograph

The time base of a hydrograph is considered to be the time from which the concentration curve begins until the direct-runoff component essentially reaches zero. An equation for time base may take the form[7]

$$T_b = t_s + t_c \tag{3-13}$$

where T_b is the time base of the hydrograph, t_s the duration of runoff-producing rain, and t_c the time of concentration. The definition of the time of concentration used herein is the flow time from the most remote point in the drainage area to the outlet of interest. It can be estimated for

channels by the Chézy or Manning formulas. The hydrograph time base, related to both storm and basin characteristics, is a basic element of unit hydrograph theory.[10]

3-10 Unit Hydrograph Development

Ways to predict flood-peak discharges and discharge hydrographs from rainfall events have been studied intensively since the early 1930's. One method receiving considerable use and still employed to estimate peak discharge rates is called the *unit hydrograph method.* It was evolved from a method of unit graphs employed by Sherman[15] in 1932 and originally assumed that for a given duration rainfall, the hydrograph time base should remain constant.

The unit graph was defined as follows:[15]

> If a given one-day rainfall produces a 1-in. depth of runoff over the given drainage area, the hydrograph showing the rates at which the runoff occurred can be considered a unit graph for that watershed.

Application of a unit graph to design rainfall excess amounts other than 1 in. is accomplished simply by multiplying rainfall excess amount by the unit graph ordinates, since the runoff ordinates for a given duration are assumed to be directly proportional to rainfall excess.

Implicit in deriving the unit graph or unit hydrograph is the assumption that rainfall is distributed in the same temporal and spatial pattern for all storms. This is generally not true, and consequently variations in ordinates for different storms can be expected.

Construction of unit hydrographs for other than the derived duration is facilitated by a method known as the *S-hydrograph* developed by Morgan and Hullinhor.[16] The procedure, as explained in a later section, employs a unit hydrograph to form an S-hydrograph resulting from a continuous applied rainfall. Application of unit hydrograph theory to ungauged watersheds received early attention by Snyder[17] and also by Taylor and Schwartz,[18] who tried to relate features of the unit hydrograph to watershed characteristics. As a result of the attempted synthesis of data, these approaches are referred to as *synthetic unit-hydrograph methods.* The need to alter duration of a unit hydrograph encouraged studies to define the shortest possible storm duration—that is, an instantaneous unit rainfall. The concept of *instantaneous unit hydrographs* can be traced to Clark[19] in 1945.

Methods of solving unit hydrographs vary, and final answers are still subject to engineering judgment. The level of sophistication employed to unravel the problem depends largely upon the kind of issue in question. Several methods useful in the determination of unit hydrographs will be

TABLE 3-3 Local Climatological Data

Latitude 40° 49′ N Longitude 96° 42′ W Elevation (ground) 1150 ft Standard time used: CENTRAL MBAN #14971

	Temperature °F							Weather types on dates of occurence	Snow, ice pellets or ice on ground at 06 AM in.	Precipitation		Avg. station pressure in. --- elev. 1189 ft m.s.l.	Wind					Sunshine		Sky cover tenths		
						Degree days base 65°				Water equivalent in.	Snow, ice pellets in.		Resultant direction	Resultant speed mph	Average speed mph	Fastest mile		Hours and tenths	Percent of possible	Sunrise to sunset	Midnight to midnight	
Date	Maximum	Minimum	Average	Departure from normal	Average dew point	Heating	Cooling	1 Fog 2 Heavy fog X 3 Thunderstorm 4 Ice pellets 5 Hail 6 Glaze 7 Duststorm 8 Smoke, haze 9 Blowing snow								Speed mph	Direction					Date
1	2	3	4	5	6	7A	7B	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	45	29	37	−8		28	0		T	.04	.4					31	NW	4.4	34	8		1
2	50	31	41	−4		24	0		0	0	0					32	NW	12.2	96	0		2
3	49	29	39	−7		26	0		0	0	0					15	NE	12.7	99	5		3
4	47	29	38	−8		27	0		0	0	0					12	NE	5.5	43	7		4
5	47	31	39	−8		26	0		0	0	0					17	NE	8.3	64	8		5
6	62	26	44	−3		21	0		0	0	0					10	S	12.9	100	0		6
7	76	39	58	10		7	0		0	0	0					28	SW	13.0	100			7
8	74	51	63	14		2	0		0	0	0					36	NW	7.8	61	7		8
9	65	41	53	4		12	0		0	0	0					16	N	13.0	100	0		9
10	77	39	58	8		7	0	DATA	0	0	0					33	S	12.3	94	1		10
11	73	58	66	16		0	1	ARE NOT	0	0	0					27	S	12.0	91	6		11
12	73	48	61	10		4	0	ENTERED	0	0	0					26	NE	10.6	80	1		12
13	64	42	53	2		12	0	IN THIS	0	0	0					27	NE	12.1	92	2		13
14	70	35	53	1		12	0	COLUMN	0	0	0					23	SW	12.8	96	5		14
								BECAUSE														

15	83	50	67	15		0	2	RECORDS	0	0	0					33	S	12.5	94	6		15
16	69	54	62	9		3	0	OF	0	.05	0					19	S	0.5	4	10		16
17	74	50	62	9		3	0	WEATHER	0	.04	0					29	S	4.6	34	7		17
18	72	57	65	12		0	0	TYPES	0	T	0					30	S	1.6	12	10		18
19	80	59	70	16		0	5	MAY BE	0	0	0					30	S	6.2	46	8		19
20	69	50	60	6		5	0	INCOMPLETE	0	.22	0					23	S	1.4	10	9		20
21	73	50	62	7		3	0		0	T	0					20	SE	7.7	57	6		21
22	60	45	53	−2		12	0		0	.14	0					25	NE	3.1	23	10		22
23	67	42	55	0		10	0		0	0	0					17	NW	11.7	86	3		23
24	72	44	58	2		7	0		0	T	0					14	NE	9.4	69	7		24
25	63	50	57	1		8	0		0	T	0					19	NE	2.8	21	10		25
26	62	47	55	−2		10	0		0	.18	0					22	NE	1.0	7	10		26
27	59	42	51	−6		14	0		0	0	0					26	NW	9.6	69	5		27
28	60	36	48	−9		17	0		0	0	0					20	NW	13.3	96	1		28
29	64	35	50	−8		15	0		0	0	0					17	S	10.5	75	8		29
30	71	46	59	1		6	0		0	.11	0					17	SW	7.0	50	7		30
	Sum	Sum				Total	Total			Total	Total		For the month:					Total		Sum	Sum	
	1970	1285				321	8	Number of days		.78	.4					36	NW	252.5	Percent for month	167		
	Avg.	Avg.	Avg.	Dep.	Avg.	Dep.	Dep.	Precipitation ≤ .01 in. 7		Dep.						Date: 08		Possible		Avg.	Avg.	
	65.7	42.8	54.3	2.5		−81				−1.67								399.2	63	5.6		
						Season to date		Snow, ice pellets ≤ 1.0 in.														
	Number of days					Total	Total			Greatest in 24 hr and dates						Greatest depth on ground of snow, ice pellets or ice and date						
	Maximum temp.		Minimum temp.			5985	8	Thunderstorms		Precipitation		Snow, ice pellets										
	≥90°	≤32°	≤32°	≤0°		Dep.	Dep.	Heavy fog X		.22	20	.4		1		T		1				
	0	0	6	0		322		Clear 9 Partly cloudy 11 Cloudy 10														

SOURCE: U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Environmental Data Service, Lincoln, Nebraska, 901 North 17th Street, April 1971.

discussed. They are subdivided into (1) taking unit hydrographs obtained from field data and manipulating them by S-hydrograph methods, and (2) constructing synthetic unit hydrographs.

3-11 Derivation of Unit Hydrographs

Data collection preparatory to deriving a unit hydrograph for a gauged watershed can be extremely time consuming. Fortunately many watersheds have available records of streamflow and rainfall and these can be supplemented with office records of the Water Resources Division of the U.S. Geological Survey.[20] Rainfall records may be secured from "Climatological Data"[21] published for each State by the National Oceanic and Atmospheric Administration (NOAA). Sample data are given in Tables 3-3 and 3-4 and indicate only a portion of published information offered for selected watersheds. Hourly rainfall records for recording rainfall stations are published as a *Summary of Hourly Observations* for the location. Summaries are listed for approximately 300 first-order stations in the United States.

To develop a unit hydrograph, it is desirable to get as many rainfall records as possible within the study area to insure that the amount and distribution of rainfall over the watershed is accurately known. Preliminary selection of storms to use in deriving a unit hydrograph for a watershed should be restricted to

1. Storms occurring individually, that is, simple storm structure.
2. Storms having uniform distribution of rainfall throughout the period of rainfall excess.
3. Storms having uniform spatial distribution over the entire watershed.

These restrictions place both upper and lower limits on size of the watershed to be employed. An upper limit of watershed size of approximately 1,000 sq mi is overcautious although general storms over such areas are not unrealistic. The lower limit of watershed extent depends upon numerous other factors and cannot be precisely defined. A general rule of thumb is to assume about 1,000 acres. Fortunately, other hydrologic techniques help resolve unit hydrographs for watersheds outside of this range. (See Chapter 6.)

Preliminary screening of suitable storms for unit hydrograph formation must meet more restrictive criteria before further analysis:

1. Duration of rainfall event should be approximately 10 to 30 percent of time of rise of runoff event.
2. Direct runoff for the selected storm should range from 0.5 to 1.75 in.
3. A suitable number of storms should be analyzed to obtain an

TABLE 3-4 Typical Small Watershed Data*

Kickapoo Creek near Kickapoo, Ill. (5-5630) Lat. 40°48′00″, Long. 89°48′00″; Drainage Area, 120 sq mi; Mean Annual Precipitation, 34.02 in.

Water year	Date	Peak discharge, cfs	Antecedent base discharge, cfs	Direct runoff, in.	Assoc. precipitation, in.	Storm duration, days	Antecedent precipitation, in.		Annual runoff, in.
							5-day	30-day	
1945	Jun. 16	3,240	74	0.58	2.17	2	0.17	2.48	7.82
1946	Jan. 5	5,840	17	a1.90	1.64	2	.29	1.82	7.82
1947	Jun. 17	3,800	39	.56	1.54	2	.41	5.80	8.42
1948	Mar. 19	9,340	182	1.12	1.98	1	.44	2.65	6.74
1949	Feb. 13	6,150	24	a1.42	.77	1	0	2.95	5.84
1950	Jul. 17	7,300	10	1.15	3.89	1	.07	3.61	10.97
1951	Jul. 22	18,400	34	1.39	3.59	1	.45	6.98	12.99
1952	Jun. 14	3,800	62	.62	2.46	2	1.28	4.90	10.89
1953	Jul. 21	2,420	20	.17	.72	2	.68	5.01	5.66
1954	Jun. 1	3,950	167	.55	1.79	1	2.43	2.72	5.31
1955	Apr. 24	8,590	120	.90	1.89	1	1.72	3.10	11.86
1956	May 27	1,300	2.0	.14	3.52	3	.06	2.25	1.23
1957	Jun. 14	7,310	52	.70	1.25	2	1.08	4.75	4.14
1958	Jul. 2	12,100	29	1.38	3.54	2	.01	6.58	6.73
1959	Apr. 26	6,000	26	.49	1.85	2	.01	1.97	4.81
1960	Mar. 30	b10,000	28	1.97	2.04	1	.01	1.28	12.34

*Courtesy U.S. Geological Survey, Water Resources Division.
[a] Runoff increased by snowmelt.
[b] Approximate value.

average of the ordinates for a selected unit hydrograph duration (approximately five events). Modifications may be made to adjust unit hydrograph durations by means of S-hydrographs or lagging procedures.

4. Direct-runoff ordinates for each storm should be reduced so each event represents 1 in. of direct runoff.
5. The final unit hydrograph of a specific duration for the watershed is obtained by averaging ordinates of selected events and adjusting the result to obtain 1 in. of direct runoff.

Constructing the unit hydrograph in this way produces the integrated effect of runoff resulting from a representative set of equal-duration storms. Extreme rainfall intensity is not reflected in the determination. If intense storms are needed, a study of records should be made to ascertain their influence upon the discharge hydrograph by comparing peaks obtained utilizing the derived unit hydrograph and actual hydrographs from intense storms. If large discrepancies appear, a safety factor must be introduced.

Essential steps in developing a unit hydrograph for an isolated storm follow.

1. Analyze the stream-flow hydrograph to permit separation of surface runoff from groundwater flow, accomplished by the methods developed in Sec. 3-8.
2. Measure the total volume of surface runoff (direct runoff) from the storm producing the original hydrograph equal to the area under the hydrograph after groundwater base flow has been removed.
3. Divide the ordinates of direct-runoff hydrograph by total direct-runoff volume in inches and plot these results versus time as a unit graph for the basin.
4. Finally, the effective duration of the runoff-producing rain for this unit graph must be found from the hydrograph (time history of rainfall intensity) of the storm used.

Other procedures than those listed are required for complex storms or in developing synthetic unit graphs when few data are available. Unit hydrographs can also be transposed from one basin to another under certain circumstances. An example will illustrate the derivation and application of a unit hydrograph.

Example 3-1 Using the hydrograph given in Fig. 3-11, derive a unit hydrograph for the 3-sq-mi drainage area. Then from this unit hydrograph derive a hydrograph of direct runoff for the rainfall sequence given in Table 3-6.

Solution:

1. Separate the base or groundwater flow to get the total direct runoff hydrograph. A common method is to draw a straight line AC which begins

when the hydrograph starts an appreciable rise and ends where the recession curve intersects the base flow curve. The important point here is to be consistent in methodology from storm to storm.

2. Determine the duration of effective rainfall (rainfall that actually produces surface runoff). Effective rainfall volume must be equivalent to the volume of direct surface runoff. Usually the unit time of effective rainfall will be 1 day, 1 hr, 12 hr, or some other interval appropriate for the size of drainage area studied. As stated previously, the unit storm duration should not exceed about 25 percent of the drainage-area lag time. The effective portion of the rainstorm for this example is given in Fig. 3-11 along with its duration. Effective volume is 1.4 in.

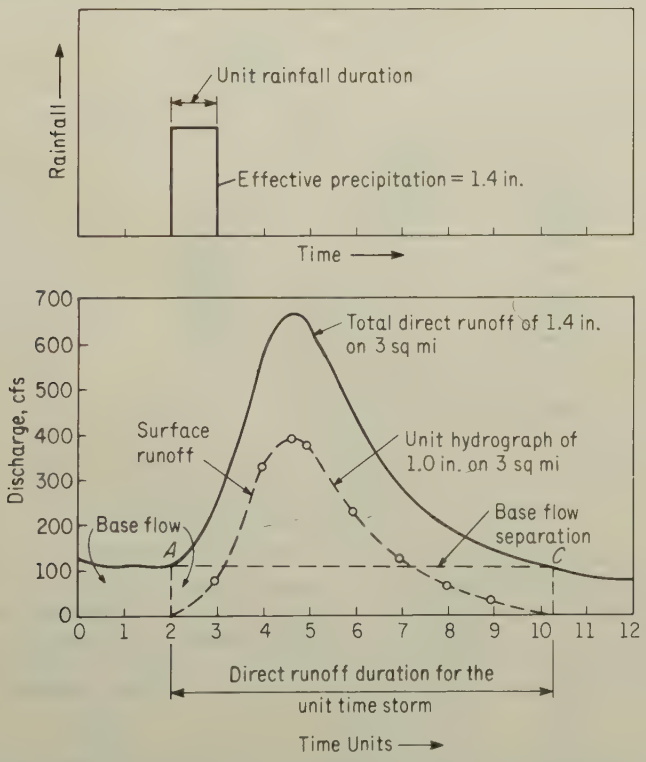


FIG. 3-11. Illustration of the derivation of a unit hydrograph from an isolated storm.

3. Project the unit hydrograph base length to the abscissa to get the base-flow-separation line AC. Unit hydrograph theory assumes that for all storms of equal duration, regardless of intensity, the period of surface runoff is the same.
4. Tabulate the ordinates of direct runoff at peak rate of flow and at sufficient other positions to determine the hydrograph shape. Note that the direct-runoff ordinate is the one above the base-flow-separation line.

5. Compute ordinates of the unit hydrograph by using

$$\frac{Q_s}{V_s} = \frac{Q_u}{1} \tag{3-14}$$

where Q_s = magnitude of a hydrograph ordinate of direct runoff having a volume equal to V_s in inches at some instant of time after start of runoff

Q_u = ordinate of the unit hydrograph having a volume of one inch at same instant of time

TABLE 3-5 Determination of a Unit Hydrograph from an Isolated Storm

1	2	3	4	5
Time unit	Total runoff, cfs	Base flow, cfs	Total direct runoff, (2) − (3), cfs	Unit hydrograph ordinate, (4) ÷ 1.4, cfs
1	110	110	0	0
2	122	122	0	0
3	230	120	110	78.7
4	578	118	460	328
4.7	666	116	550	393
5	645	115	530	379
6	434	114	320	229
7	293	113	180	129
8	202	112	90	64.2
9	160	110	50	35.7
10	117	105	12	8.6
10.5	105	105	0	0
11	90	90	0	0
12	80	80	0	0

TABLE 3-6 Unit Hydrograph Application

1	2	3	4*		
Time unit sequence	Rain unit number	Effective rainfall, in.	Hydrograph ordinates for rainfall units 1-3		
			1	2	3
1	1	0.7	55.1	—	—
2	2	1.7	229	134	—
2.7	3	1.2	275	—	—
3	—	—	265	558	94.5
3.7	—	—	—	668	—
4	—	—	161	644	393
4.7	—	—	—	—	472
5	—	—	90.5	389	455
6	—	—	44.9	219	275
7	—	—	25.0	109	155
8	—	—	6.0	60.7	77
9	—	—	—	14.6	42.8
10	—	—	—	—	10.3

*Values in column 4 obtained by multiplying values in column 3 by unit hydrograph ordinates.

In this example the values are obtained by dividing the direct runoff ordinates by 1.4. Table 3-5 outlines the computation of the unit hydrograph ordinates.

6. Using values from Table 3-5, plot the unit hydrograph as shown on Fig. 3-11.
7. Utilizing the derived ordinates of the unit hydrograph, determine the ordinates of the hydrographs for each consecutive unit rainfall period from Table 3-6.
8. Find the synthesized hydrograph for unit storms 1-3 by plotting the three hydrographs and summing ordinates as indicated in Fig. 3-12.

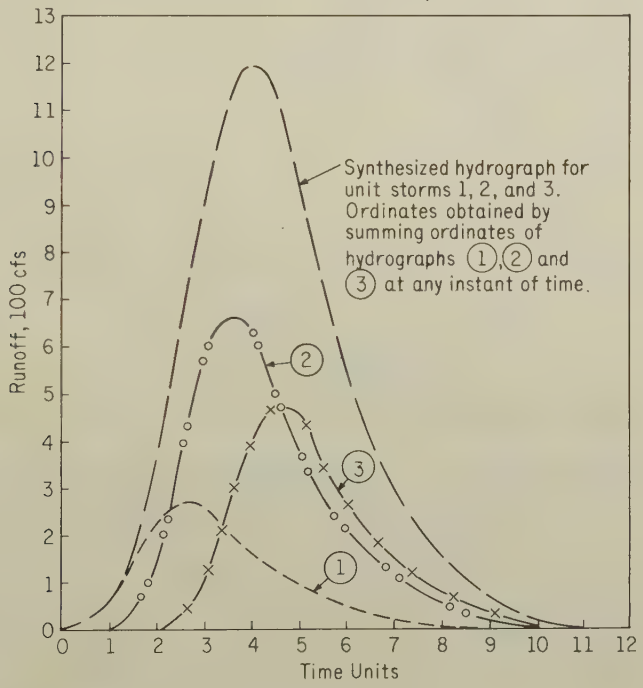

Runoff, 100 cfs
Time Units
Synthesized hydrograph for unit storms 1, 2, and 3. Ordinates obtained by summing ordinates of hydrographs (1), (2) and (3) at any instant of time.
1
2
3

FIG. 3-12. Synthesized hydrograph derived by the unit hydrograph method.

3-12 S-Hydrographs and Lagging Methods

Unit hydrographs developed according to procedures outlined in the previous section represent 1 in. of direct runoff resulting from a rainfall excess occurring in a specified period of time. Application to storms of larger or smaller durations will often require a different method to define additional unit hydrographs. The method of "lagging" is one possibility. It is based on the assumption that linear response of the watershed is not influenced by previous storms—that is, one can superimpose storms and the results are directly additive. For example, if a 1-hr unit hydrograph

is available for a given watershed, a unit hydrograph resulting from a 2-hr unit storm is obtained by plotting two 1-hr unit hydrographs, with the second unit hydrograph lagged 1 hr, adding ordinates, and dividing by two. This is demonstrated in Fig. 3-13 where the dashed line represents

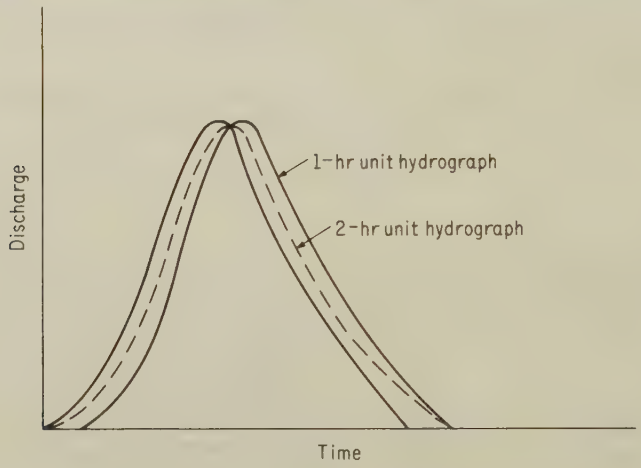


FIG. 3-13. Unit hydrograph lagging procedure.

the resulting 2-hr unit hydrograph. Thus the one inch of rainfall contained in the original 1-hr duration has been distributed over a 2-hr period.

Modifications of the original unit hydrograph duration can be made so that two 1-hr unit hydrographs are used to form a 2-hr unit hydrograph; two 2-hr unit hydrographs result in a 4-hr diagram and so on. Care must be taken not to mix durations in the lagging procedure, since errors are introduced; hence a 1-hr and a 2-hr unit hydrograph do not represent a 3-hr unit hydrograph. Lagging procedure is therefore restricted to multiples of the original duration according to the expression

$$D^1 = 2^n D \tag{3-15}$$

where D^1 = possible durations of unit hydrograph
D = original duration of unit hydrograph
n = 1,2,3,...

The S-hydrograph method overcomes restrictions imposed by the lagging plan and allows construction of any duration-unit hydrograph. By observing the lagging system just described, it is apparent that for a 1-hr unit hydrograph, the 1-in. rainfall excess has an intensity of 1 in./hr, whereas the 2-hr unit hydrograph is produced by a rainfall intensity of 0.5 in./hr. Continuous lagging of either one of these unit hydrographs is comparable to a continuously applied rainfall at either 0.5 in./hr or 1 in./hr intensity, depending upon which unit hydrograph was chosen.

As an example, using the 1-hr unit hydrograph, continuous lagging represents direct runoff from a constant rainfall of 1 in./hr as shown in Fig. 3-14a. Cumulative addition of the initial unit hydrograph ordinates at time intervals equal to the unit storm duration results in an S-hydrograph (see Fig. 3-15). Graphically, construction of an S-hydrograph is readily accomplished with a pair of dividers. Maximum discharge of the S-hydrograph occurs at a time equal to the time base of the initial unit hydrograph as shown in Fig. 3-14a.

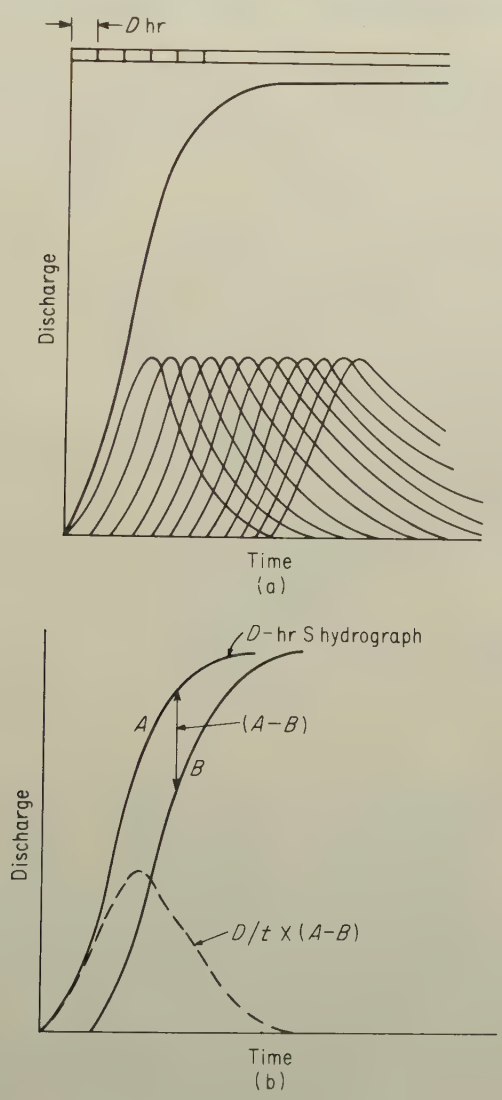


FIG. 3-14. S-hydrograph method.

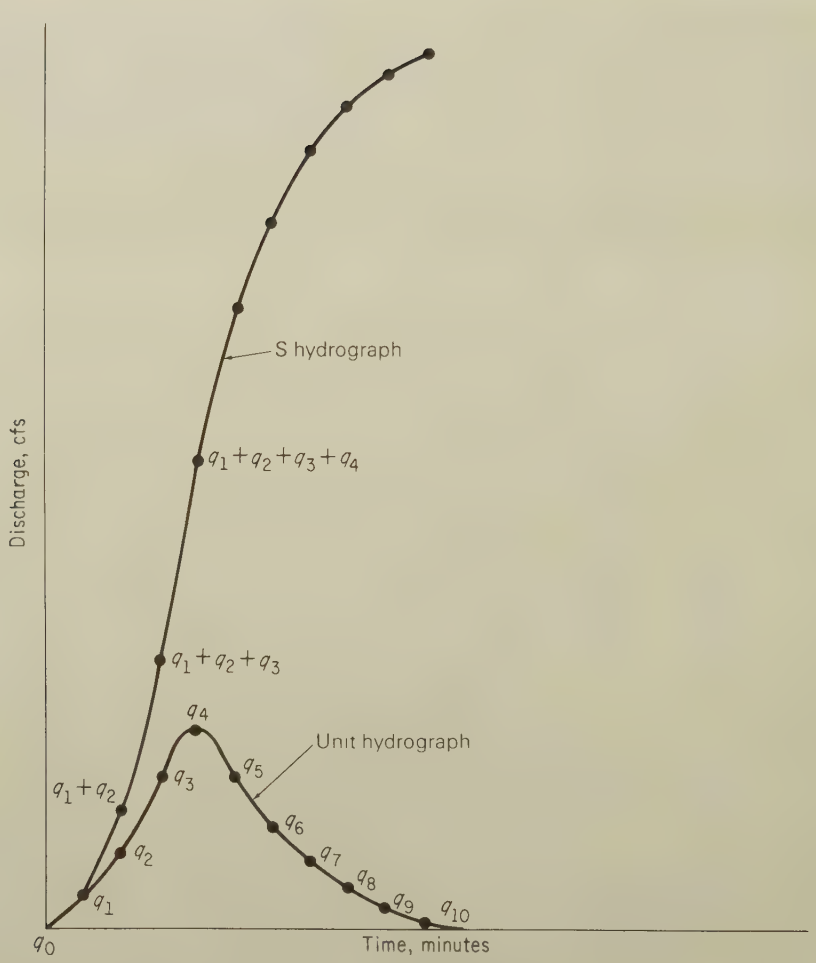


FIG. 3-15. S hydrograph.

To pictorially construct a 2-hr unit hydrograph, simply lag the first S-hydrograph by a second S-hydrograph a time interval equal to the desired duration. The difference in S-hydrograph ordinates must then be divided by two. Any duration-t unit hydrograph may be obtained in the same manner once another duration D unit hydrograph is known. Simply form a D-hr S-hydrograph, lag this S-hydrograph t hr and multiply the difference in S-hydrograph ordinates by D/t. Accuracy of the graphical procedure depends upon the scales chosen to plot the hydrographs. Tabular solution of the S-hydrograph method is also employed, but hydrograph tabulations must be at intervals of the original unit hydrograph duration.

3-13 Matrix Methods of Unit Hydrographs*

Methods used to construct design-storm hydrographs from known unit hydrographs require multiplying the design rainfall excess of each time period by the proper unit hydrograph ordinate. Lagging the hydrographs for each time period and adding the ordinates permits construction of the required storm hydrograph. A design-storm hydrograph does not include base flow at this stage of development. Base flow may be added to procure a more realistic design storm by studying base-flow behavior and applying a representative value or values of discharge throughout the storm period.

The process of getting a design-storm hydrograph with base flow excluded is graphically demonstrated for a three-period storm in Fig. 3-16. Equations representing the design-storm hydrograph of Fig. 3-16c are given as Eqs. 3-16 through 3-21.

$$p_1 u_1 = q_1 \tag{3-16}$$

$$p_2 u_1 + p_1 u_2 = q_2 \tag{3-17}$$

$$p_3 u_1 + p_2 u_2 + p_1 u_3 = q_3 \tag{3-18}$$

$$p_3 u_2 + p_2 u_3 + p_1 u_4 = q_4 \tag{3-19}$$

$$p_3 u_3 + p_2 u_4 = q_5 \tag{3-20}$$

$$p_3 u_4 = q_6 \tag{3-21}$$

The relationship between the original unit hydrograph, design rainfall, and storm hydrograph time periods follows the form

$$n = j + i - 1 \tag{3-22}$$

where n = number of storm hydrograph ordinates
j = number of unit hydrograph ordinates
i = number of periods of rainfall excess

In matrix notation Eqs. 3-16 through 3-21 can be written more concisely as

$$PU = Q \tag{3-23}$$

where

$$P = \begin{bmatrix} p_1 & 0 & 0 & 0 & 0 & 0 \\ p_2 & p_1 & 0 & 0 & 0 & 0 \\ p_3 & p_2 & p_1 & 0 & 0 & 0 \\ 0 & p_3 & p_2 & p_1 & 0 & 0 \\ 0 & 0 & p_3 & p_2 & 0 & 0 \\ 0 & 0 & 0 & p_3 & 0 & 0 \end{bmatrix}$$

*See Appendix A for brief review of matrix algebra.

$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ 0 \\ 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$

The reverse of the process shown in Fig. 3-16 is the basis of the matrix method. Thus having an existing storm hydrograph Q less base flow, with known rainfall excess P, allows computation of unit hydrograph U subject to restrictions imposed by matrix algebra. A solution of Eq. 3-23 can be of the form

$$P^{-1} PU = P^{-1} Q \tag{3-24}$$

or

$$U = P^{-1} Q \tag{3-25}$$

The answer is valid if, and only if, an inverse can be found for the precipitation matrix, but an inverse exists only if it is a square matrix with a nonvanishing determinant. As can be seen from Eq. 3-23 the precipitation matrix does not fulfill this criteria. However, by use of the transpose of the precipitation matrix, a square matrix is formed as follows

$$P^T PU = P^T Q \tag{3-26}$$

and the solution for the unit hydrograph matrix becomes

$$U = (P^T P)^{-1} P^T Q \tag{3-27}$$

Resolving Eq. 3-27 produces a trial unit hydrograph with duration equal to the selected time interval of precipitation elements. Reproduction of the initial storm hydrograph based upon this unit hydrograph, however, will generally not be exact. To minimize the error, adjustments can be made to unit hydrograph ordinates to obtain a "best" solution. This process is usually performed by reducing the square of the error or dif-

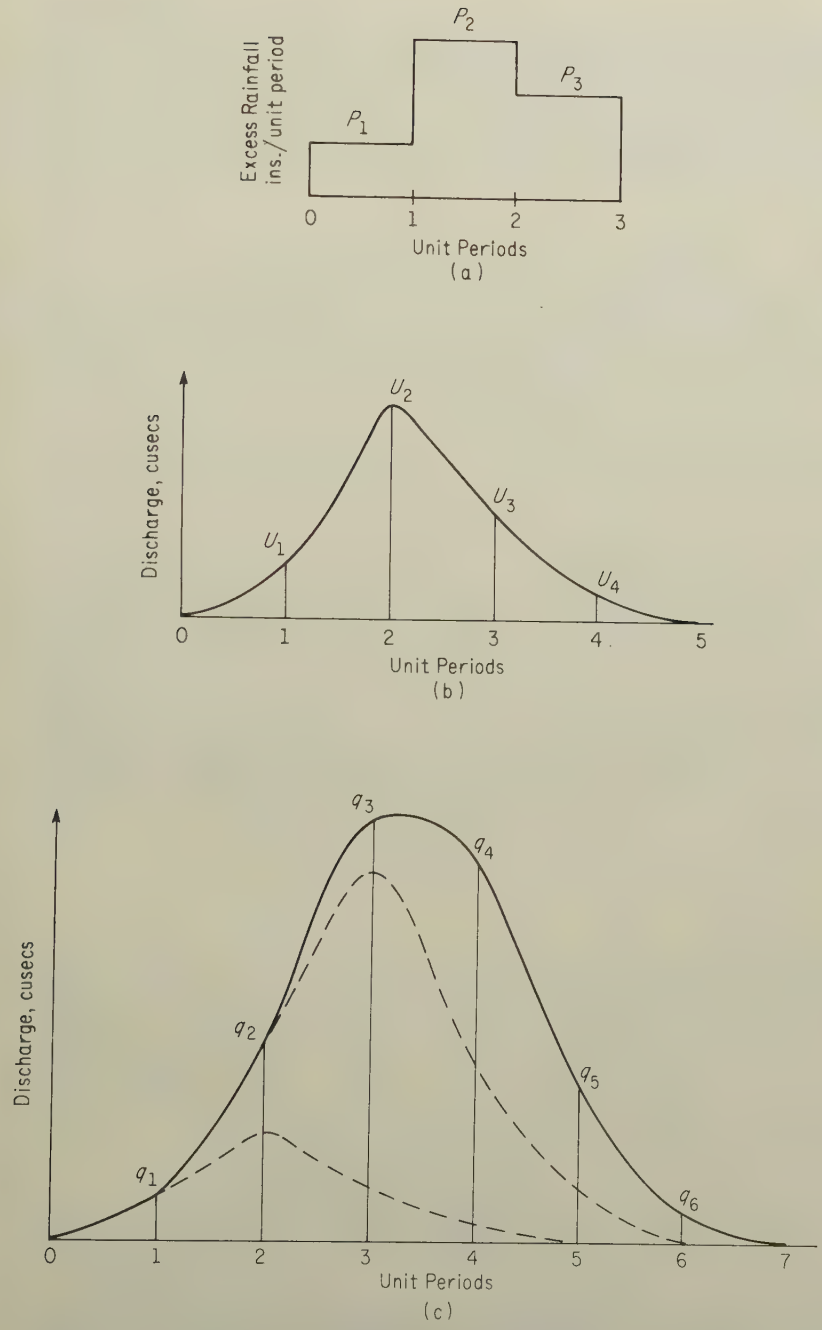


FIG. 3-16. Determination of a design storm hydrograph: (a) excess rainfall, (b) unitgraph, and (c) surface runoff hydrograph.

ference between the original and the derived storm hydrograph ordinates. Further refinements in this method are presented by Newton and Vineyard.[22] Generally, solution of the unit hydrograph by matrix methods is performed on a digital computer. Most computer installations have packaged programs for work with large systems and for determining the inverse and transpose of matrices needed.

3-14 Synthetic Methods of Unit Hydrographs

As previously noted, the linear characteristics exhibited by unit hydrographs for a watershed are a distinct advantage in constructing more complex storm-discharge hydrographs. Generally however, basic streamflow and rainfall data are not available to allow construction of a unit hydrograph except for relatively few watersheds, so techniques have evolved which allow generation of "synthetic unit hydrographs."

3-15 Snyder's Method

One technique employed by the U.S. Corps of Engineers[23] is based on methods developed by Snyder[17] and expanded by Taylor and Schwarz.[18] Unfortunately it does not provide a simple method of constructing the entire time distribution of the discharge hydrograph but allows computation of lag time, unit hydrograph duration, peak discharge, and hydrograph time widths at 50 and 75 percent of peak flow. Using these points, a sketch of the unit hydrograph is obtained and checked by Eq. 3-9 to see if it contains 1 in. of direct runoff (see Fig. 3-17).

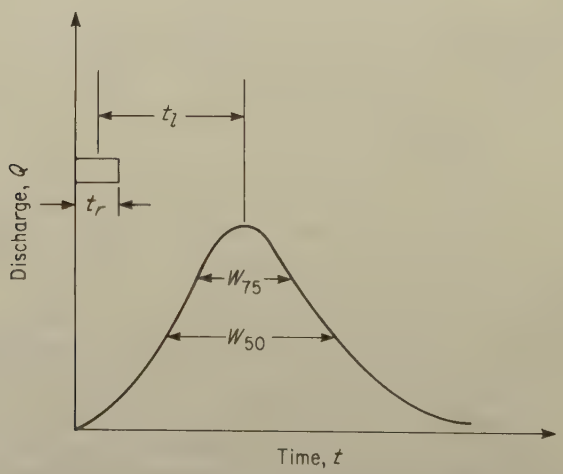


FIG. 3-17. Snyder's synthetic unit hydrograph.

Time To Peak

Snyder's method of synthetic unit hydrographs relies upon correlation of the dependent variables of lag time and peak discharge with various physiographic watershed characteristics. Lag-time relationship derived by Snyder for watersheds from 10 to 10,000 sq mi located in the Appalachian Highlands is

$$t_l = C_t(LL_{ca})^{0.3} \tag{3-28}$$

where t_l = lag time, hr
C_t = coefficient representing variations of watershed slopes and storage
L = length of the main stream channel, mi
L_{ca} = length along main channel to a point opposite watershed centroid, mi

It is assumed that lag time is a constant for a particular watershed—that is, uninfluenced by variations in rainfall intensities or similar factors. The use of L_{ca} accounts for watershed shape, and C_t takes care of wide variations in topography, from plains to mountainous regions. Values of C_t for Synder's study ranged from 1.8 to 2.2 When snow-pack accumulations influence peak discharge, values of C_t will be between one-sixth to one-third of Snyder's values.

Duration

The duration of rainfall excess in Snyder's synthetic unit hydrograph is a function of lag time as shown by Eq. 3-29.

$$t_r = t_l/5.5 \tag{3-29}$$

where t_r = duration of unit rainfall excess, hr
t_l = lag time from centroid of unit rainfall excess to peak of unit hydrograph

This synthetic technique always results in an initial unit hydrograph duration equal to $t_l/5.5$. However, since changes in lag time do occur with changes in duration of the unit hydrograph, the following equation was built to allow lag time and peak discharge adjustments for other unit hydrograph durations.

$$t_{lR} = t_l + 0.25\,(t_R - t_r) \tag{3-30}$$

where t_{lR} = adjusted lag time, hr
t_l = original lag time, hr
t_R = desired unit hydrograph duration, hr
t_r = original unit hydrograph duration = $t_l/5.5$, hr

Peak Discharge

Assuming a given duration rainfall produces 1 in. of direct runoff, outflow volume is some relatively constant percentage of inflow volume,

and a simplified approximation of outflow volume is $t_l \times Q_P$, then the equation for peak discharge can be written

$$Q_P = 640 C_P A / t_l \tag{3-31}$$

where Q_P = peak discharge, cfs
C_P = coefficient accounting for flood wave and storage conditions
A = watershed size, sq mi
t_l = lag time, hr

Thus peak discharge can be calculated given lag time and coefficient of peak discharge C_P. Values for C_P range from 0.4 to 0.8 and generally indicate retention or storage capacity of the watershed.

Time Base

The time base of a synthetic unit hydrograph for the study area was found to follow the relationship

$$T = 3 + t_l/8 \tag{3-32}$$

where T = base time of synthetic unit hydrograph, days
t_l = lag time, hr

Equation 3-32 gives reasonable estimates for large watersheds but will produce excessively large values for smaller areas. A general rule of thumb for small areas is to use three to five times the time to peak as a base value when sketching a unit hydrograph.

Hydrograph Construction

From Eqs. 3-9, 3-28, 3-29, 3-31, and 3-32 we plot critical points for the unit hydrograph and sketch a synthetic unit hydrograph, remembering that total direct runoff amounts to 1 in. An analysis by the U.S. Corps of Engineers (see Fig. 3-18) gives additional assistance in plotting time widths for points on the hydrograph located at 50 and 75 percent of peak discharge. As a general rule of thumb, the time width at W_{50} and W_{75} ordinates should be proportioned each side of the peak in a ratio of 1 to 2 with the short time side on the left of the synthetic unit hydrograph peak. As noted earlier, for smaller watersheds, Eq. 3-32 gives unrealistic values for the base time. If this occurs, a value can be estimated by multiplying total time to the peak by a value of from 3 to 5. This ratio can be modified based upon the amount and time rate of depletion of storage water within the watershed boundaries.

Application of Synder's synthetic unit hydrograph method to areas other than the original study area should be preceded by a reevaluation of coefficients C_t and C_P in Eqs. 3-28 and 3-31. This analysis can be accomplished by use of unit hydrographs in the region under study which

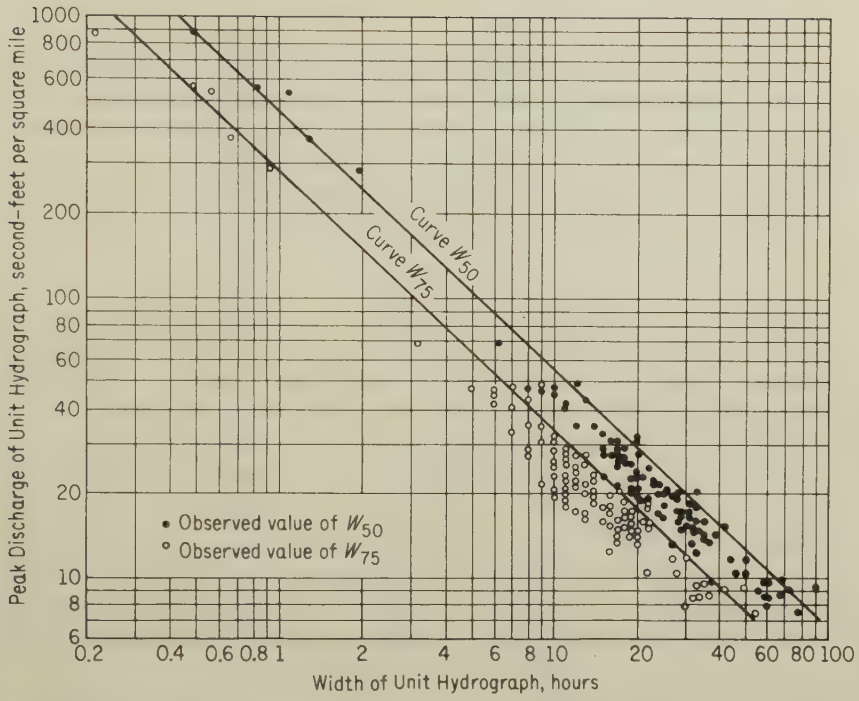


FIG. 3-18. Unit-hydrograph width at 50 and 75 percent of peak flow. (U.S. Corps of Engineers.)

have the proper lag-time-rainfall-duration ratio, that is, $t_r = t_l/5.5$. If another rainfall duration is selected, variations of C_t and C_P can be expected.

3-16 SCS Method

A method developed by the Soil Conservation Service for constructing synthetic unit hydrographs is based on a dimensionless hydrograph (Fig. 3-19), this dimensionless graph is the result of an analysis of a large number of natural unit hydrographs from a wide range in size and geographic locations. The method requires only the determination of the time to peak and the peak discharge by Eqs. 3-33 and 3-34 and Fig. 3-19.[33] Parameters t_p and Q_P are computed as follows:

$$t_P = D/2 + t_l \tag{3-33}$$

where t_p = time from beginning of rainfall to peak discharge, hr
D = duration of rainfall, hr
t_l = lag time from centroid of rainfall to peak discharge, hr

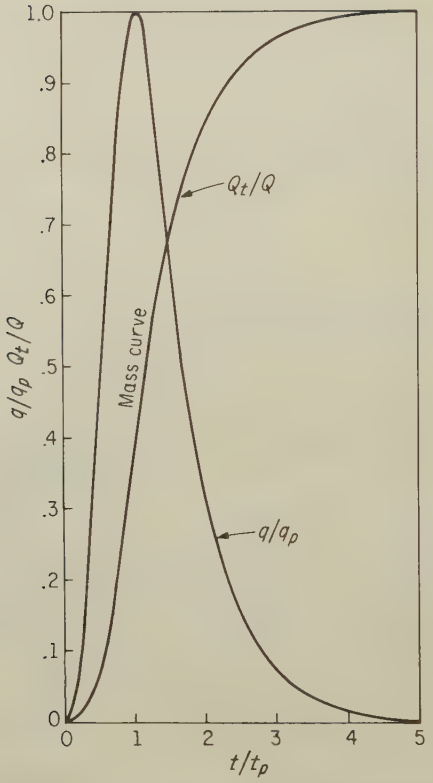


FIG. 3-19. Dimensionless unit hydrograph and mass curve. (After Mockus, Ref. 33.)

and

$$Q_P = \frac{484A}{t_p} \tag{3-34}$$

where Q_P = peak discharge, cfs
A = drainage area, sq mi
t_p = time to peak

A relationship of t_l to size of watershed can be used to estimate lag time. Relationships from two geographic regions are

$$t_l = 1.44A^{0.6} \quad \text{Texas} \tag{3-35}$$
$$t_l = 0.54A^{0.6} \quad \text{Ohio} \tag{3-36}$$

By finding a value of t_l, a synthetic unit hydrograph of chosen duration D is obtained from Fig. 3-19.

3-17 Gray's Method

A method of generating synthetic unit hydrographs for midwestern watersheds has been developed by Gray[24] An approximate upper limit of watershed size for application of this method to the geographic areas of Central Iowa, Missouri, Illinois, and Wisconsin is 94 sq mi. The method is based on dimensionalizing the incomplete gamma distribution and results in a dimensionless graph of the form

$$Q_{t/P_R} = \frac{25.0\,(\gamma')^q}{\Gamma(q)}\,(e^{-\gamma' t/P_R})\left(\frac{t}{P_R}\right)^{q-1} \tag{3-37}$$

where Q_{t/P_R} = percent flow/$0.25P_R$ at any given t/P_R value; the factors q and γ are shape and scale parameters, respectively; Γ denotes the gamma function of q which is equal to $(q-1)!$* e is the base of the natural logarithms; P_R is the period of rise; and t is time in minutes. The relationship for γ' is defined as $\gamma' = \gamma P_R$ and $q = 1 + \gamma'$.

This form of the dimensionless unit hydrograph (Fig. 3-20) allows computation of the discharge ordinates for the unit hydrograph at times equal to 1/4 intervals of the period of rise P_R, that is, the time from the centroid of rainfall to the time of peak discharge of the unit hydrograph.

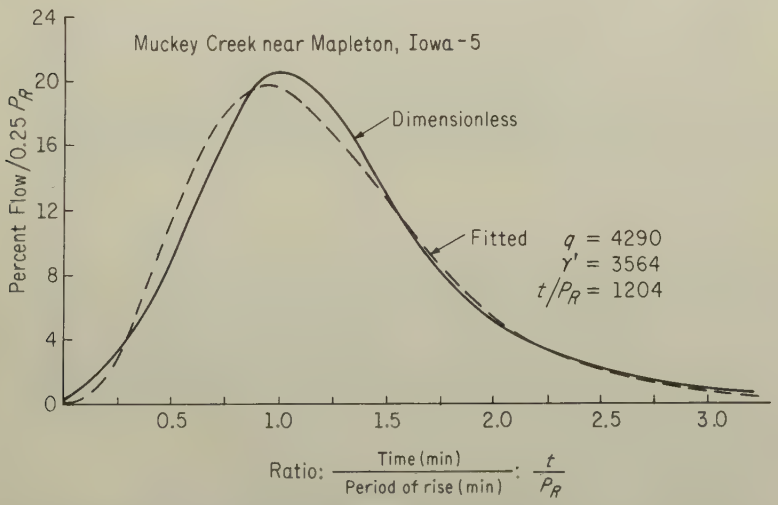


FIG. 3-20. Dimensionless graph and fitted two-parameter gamma distribution for watershed 5. (After Ref. 24, p. 41.)

*If q is not a integer $\Gamma(q) = \Gamma(N + Z) = (N - 1 + Z)(N - 2 + Z) \ldots (1 + Z)\,\Gamma(1 + Z)$ where N equals the integer.

Correlations with physiograph characteristics of the watershed can be developed to get the values of both P_R and γ'.

As an example, the storage factor P_R/γ' has been linked with watershed parameter $L/\sqrt{S_c}$, where L is the length of the main channel of the watershed in miles measured from the outlet to the uppermost part of the watershed (Fig. 3-21); S_c is defined as an average slope in percent obtained

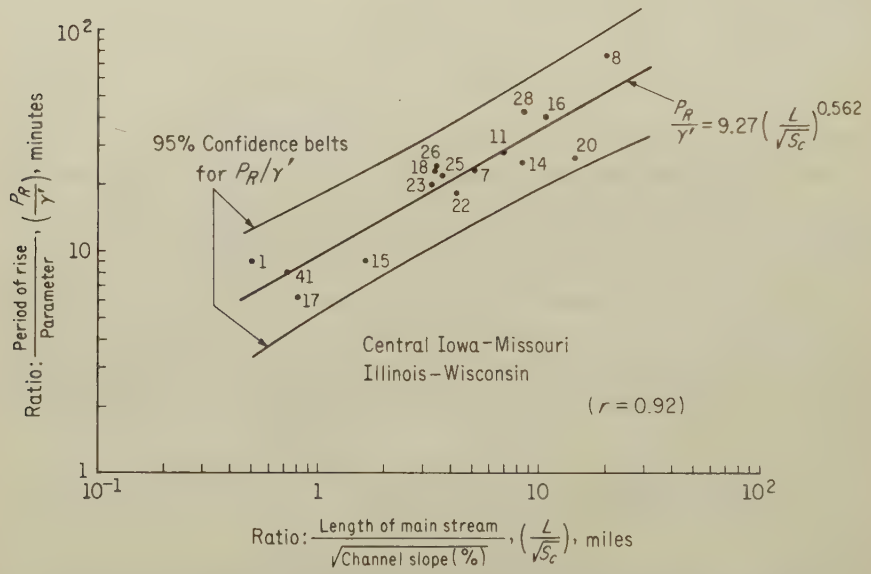


FIG. 3-21. Relation of storage factor, P_R/γ', and watershed parameter, $L\sqrt{S_c}$, for watersheds in central Iowa-Missouri-Illinois-Wisconsin. (After Ref. 24, p. 45.)

by plotting the main-channel profile and drawing a straight line through the outlet elevation such that the positive and negative areas between the stream profile and the straight line are equal. The storage factor P_R/γ' can also be correlated with the period of rise P_R as shown in Fig. 3-22. These two correlations allow solution of Eq. 3-37 and produce a synthetic unit hydrograph of duration $P_R/4$ for an ungauged area.

The solution proceeds as follows:

1. Determine L, S_c, and A for the ungauged watershed.
2. Determine parameters P_R, γ', and q.
   (a) With $L/\sqrt{S_c}$, enter Fig. 3-21 and select P_R/γ'.
   (b) With P_R/γ', enter Fig. 3-22 and obtain P_R. Compute γ' as the ratio $P_R/P_R/\gamma'$.
   (c) Substitute γ' obtained in step 2b into the equation, $q = 1 + \gamma'$, and solve for q.

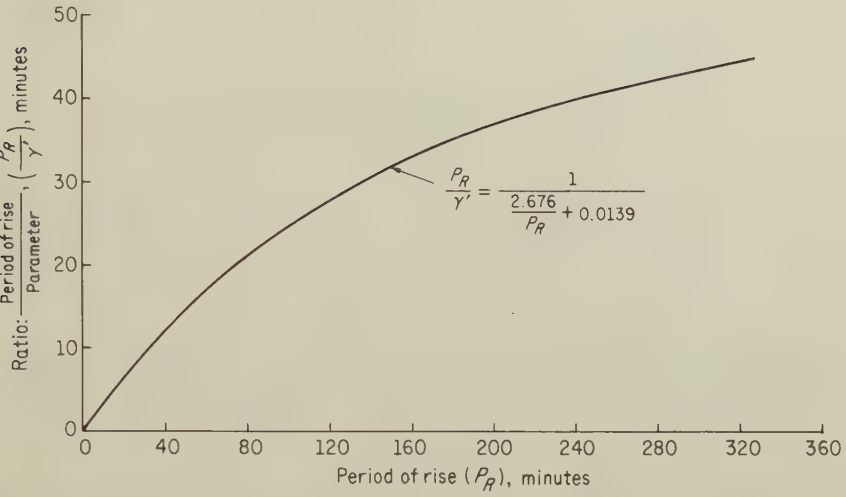


FIG. 3-22. Relation of storage factor P_R/γ', and period of rise P_R. (After Ref 24, p. 48.)

3. Compute the ordinates for the dimensionless graph using the equation

$$Q_{t/P_R} = \frac{25.0(\gamma')^q}{\Gamma(q)} (e^{-\gamma' t/P_R}) \left(\frac{t}{P_R}\right)^{q-1} \qquad \textbf{(3-37)}$$

compute percent flow/$0.25P_R$ for values of t/P_R = 0.125, 0.375, 0.625 . . . and every succeeding increment of t/P_R = 0.250 until the sum of the percentage flows approximates 100 percent (col. 4, Table 3-7). Also compute the peak percentage by substituting $t/P_R = 1$.

4. Compute the unit hydrograph
   (a) Compute the necessary conversion factor to convert the volume of direct runoff under the dimensionless graph to 1 in. of precipitation excess over the entire watershed.
      (1) Volume of unit hydrograph = V

$$V = 1 \text{ in.} \times A \text{ sq mi} \times 640 \frac{\text{acre}}{\text{mi}^2} \times \frac{1}{12 \text{ in./ft}} \times 43{,}560 \frac{\text{ft}^2}{\text{acre}}$$

      (2) Volume of dimensionless graph = V_D

$$V_D = \Sigma \text{ cfs} \times 0.25 \times P_R \times 60 \frac{\text{sec}}{\text{min}}$$

      (3) Solve for Σ cfs by equating V and V_D, since they must be equal.

(b) Convert dimensionless-graph ordinates to unit hydrograph ordinates

$$Q_u = \frac{\text{Percent flow}/0.25P_R}{100} \quad \Sigma \text{ cfs}$$

(c) Translate time base of dimensionless graph to absolute time units by multiplying $t/P_R \times P_R$ for each computed point. Remember that runoff does not commence until the centroid of rainfall, or at a time $P_R/8$.

An example problem demonstrates the solution of Gray's method for a Missouri watershed. A plot representing the derived hydrograph is shown in Fig. 3-23. Note again that direct runoff commences at the centroid of

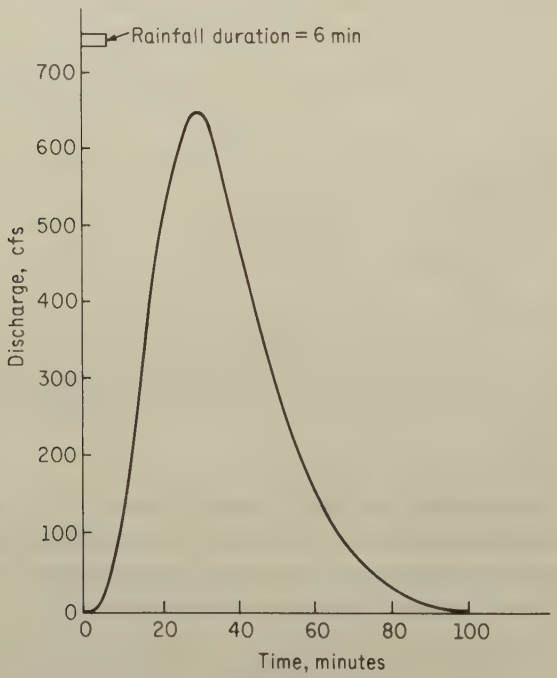


FIG. 3-23. Derived hydrograph of Example 3-2 using Gray's method.

rainfall. Thus it is necessary to add $D/2$ or $P_R/8$ to col. 2, Table 3-7, to obtain the proper times of unit hydrograph ordinates.

Example 3-2 For the given data, use Gray's method to construct a unit hydrograph.

Green Acre Branch, drainage area = 0.62 sq mi, length = 0.98 mi, S_c = 1.45 percent.

Procedure:

I.

(a) Fig. 3-21; $L/\sqrt{S} = 0.813$ mi $P_R/\gamma' = 8.25$ min

(b) Fig. 3-22; $P_R/\gamma' = 8.25$ min $P_R = 24.9$ min

(c) $q = 1 + \gamma' = 4.02$ $\gamma' = 3.02$

II.

(a)

TABLE 3-7 Tabulation for Example 3-2

(1) t/P_R	(2) Time, min	(3) Percent flow/ $.25P_R$	(4) Cumulated flow	(5) UHG, cfs
0.000	0	0	0	0
0.125	3.11	0.448	0.448	17.3
0.375	9.33	5.800	6.248	224
0.625	15.55	12.700	18.948	490
0.875	21.77	16.350	35.298	631
1.000	24.90	16.850	—	651
1.125	27.99	16.250	51.548	628
1.375	34.21	14.200	65.748	548
1.625	40.43	11.100	76.848	428
1.875	46.65	7.970	84.818	308
2.125	52.87	5.550	90.368	214
2.375	59.29	3.560	93.928	138
2.625	65.51	2.28	96.208	88.0
2.875	71.73	1.410	97.618	54.4
3.125	77.95	0.864	98.482	33.3
3.375	84.17	0.500	99.982	19.3

III.

(a) 1. $V = 1 \times 0.62 \times 640 \times 43{,}560/12 = 14.4 \times 10^5$ ft^3

2. $V_D = 0.25 \times 24.9 \times 60 \times \Sigma$ cfs $= 373.5 \Sigma$ cfs

3. Σ cfs $= 3{,}860$

(b) Column 5 is tabulated by multiplying 3,860 times values in col. 3 divided by 100.

(c) Column 2 is secured by multiplying 24.9 times values in col. 1.

(d) Column 3 comes from solution of Eq. 3-37.

A sample Fortran IV program is shown in Fig. 3-24 demonstrating the solution of Gray's method for a drainage area of 14.22 sq mi. Results of the physiographic correlations for this area produced values of $P_R = 128$ min and $\gamma' = 4.4139$, which results in $q = 5.4139$, and $\Gamma(q) = 47$. These values are used as inputs to the computer program, although routines could be developed to allow internal computation of all values by simply inputting physiographic parameters of the watershed.

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 72306        11/14/47

0001                TPR = 0.125
0002                QT = 0.
0003                T = 16.
0004                GAMMA = 47.
0005                V = 1.*14.22*640.*43560./12.
0006                VD = .25*128.*60.
0007                CFS = V/VD
0008                WRITE(6,11)
0009             11 FORMAT (8X,'TPR',8X,'ACCUM TIME',8X,'Q',8X,'QT',8X,'UNIT GRAPH')
0010                A = 4.41379
0011                B = 5.41379
0012                DO 10 I = 1,50
0013                Q = ((25.0*(A**B)*(TPR**(B-1.)))/(GAMMA*2.71828**(A*TPR)))
0014                QT = QT+Q
0015                IF (100.-QT) 20,20,21
0016             21 UH = CFS*Q/100.
0017                WRITE(6,25) TPR,T,Q,QT,UH
0018             25 FORMAT(6X,F6.3,9X,F6.1,8X,F6.3,6X,F5.1,7X,F8.0)
0019                TPR = TPR + 0.250
0020             10 T = T + 32.
0021             20 CONTINUE
0022                U = ((25.0*(A**B))/(GAMMA*2.71828**A))
0023                WRITE(6,6) U
0024                U = U*CFS/100.
0025                WRITE(6,7) U
0026              6 FORMAT(6X,'U = ',F8.1)
0027              7 FORMAT(6X,'U = ',F10.1)
0028                STOP
0029                END
```

TPR	ACCUM TIME	Q	QT	UNIT GRAPH
0.125	16.0	0.098	0.1	17.
0.375	48.0	4.147	4.2	714.
0.625	80.0	13.114	17.4	2256.
0.875	112.0	19.208	36.6	3305.
1.125	144.0	19.320	55.9	3324.
1.375	176.0	15.540	71.4	2674.
1.625	208.0	10.776	82.2	1854.
1.875	240.0	6.722	88.9	1157.
2.125	272.0	3.875	92.8	667.
2.375	304.0	2.100	94.9	361.

```
 2.625          336.0          1.084          96.0          186.
 2.875          368.0          0.537          96.5           92.
 3.125          400.0          0.257          96.8           44.
 3.375          432.0          0.120          96.9           21.
 3.625          464.0          0.055          97.0            9.
 3.875          496.0          0.024          97.0            4.
 4.125          528.0          0.011          97.0            2.
 4.375          560.0          0.005          97.0            1.
 4.625          592.0          0.002          97.0            0.
 4.875          624.0          0.001          97.0            0.
 5.125          656.0          0.000          97.0            0.
 5.375          688.0          0.000          97.0            0.
 5.625          720.0          0.000          97.0            0.
 5.875          752.0          0.000          97.0            0.
 6.125          784.0          0.000          97.0            0.
 6.375          816.0          0.000          97.0            0.
 6.625          848.0          0.000          97.0            0.
 6.875          880.0          0.000          97.0            0.
 7.125          912.0          0.000          97.0            0.
 7.375          944.0          0.000          97.0            0.
 7.625          976.0          0.000          97.0            0.
 7.875         1008.0          0.000          97.0            0.
 8.125         1040.0          0.000          97.0            0.
 8.375         1072.0          0.000          97.0            0.
 8.625         1104.0          0.000          97.0            0.
 8.875         1136.0          0.000          97.0            0.
 9.125         1168.0          0.000          97.0            0.
 9.375         1200.0          0.000          97.0            0.
 9.625         1232.0          0.000          97.0            0.
 9.875         1264.0          0.000          97.0            0.
10.125         1296.0          0.000          97.0            0.
10.375         1328.0          0.000          97.0            0.
10.625         1360.0          0.000          97.0            0.
10.875         1392.0          0.000          97.0            0.
11.125         1424.0          0.000          97.0            0.
11.375         1456.0          0.000          97.0            0.
11.625         1488.0          0.000          97.0            0.
11.875         1520.0          0.000          97.0            0.
12.125         1552.0          0.000          97.0            0.
12.375         1584.0          0.000          97.0            0.
U =      19.9
U =      3431.8
```

FIG. 3-24. Sample Fortran IV program for Gray's method.

3-18 Simulation Methods in Hydrology

Hydrologic problems are often complex and/or unique. In such cases, physical models can sometimes be built and operated to determine characteristics of the system.[25] Such models are occasionally effective, but are usually costly and lack versatility.

All systems are analyzed with the aid of mathematical methods. These fall into two categories: (1) the analytic or direct approach, and (2) the trial-and-error or nonanalytic approach. Analytic solutions are the most desirable, since they yield the wanted result directly. Unfortunately, equations describing hydrologic systems are often so complex and numerous that direct solutions are impossible, so that nonanalytic or trial-and-error solutions are often useful. The principal advantage of the trial-and-error method is that relatively simple equations are usually involved. However, a separate trial is required for each combination of system parameters.

Crawford and Linsley have stated that simulation is an indirect approach to the study of the behavior or response of a system.[26] Systems may be simulated by physical models, analog models, or digital models. Digital simulation is a more recent method used to analyze large and complex systems.[26,27] It has the important advantage of high speed and does not require the extensive "hardware" often needed for physical or analog models. The digital program itself becomes the model and its parameters can be changed to allow for experiment or to represent any particular condition of interest. Simulation of a hydrologic system through use of models has many virtues. The model can be much more easily operated and observed than the real system. Another important advantage is that it is possible to compress real-time scales on the order of years to time scales on the order of minutes. As a result, long periods of real time can be successfully studied.

Digital simulation expresses physical systems in mathematical terms which involve various parameters. These mathematical models are then improved and verified by simulating the systems reaction to known inputs and outputs, continuing until the model is considered to be a reliable representation of the prototype system. The procedure is analogous to that of verifying a physical model. Once this has been accomplished, the model can be used for a variety of purposes such as project planning and project operation,[27] where model parameters and system input are varied to determine the effects of changes on the system output. The variations may demonstrate seasonal effects, impact of land-management practices, and many other situations of practical interest.

Crawford and Linsley state that the goal of digital simulation of a hydrologic system is to develop a practical hydrologic model which is a skeleton of the hypothetical "absolute knowledge" model.[26] A prime example

of its application is development of the Stanford Watershed Models (Mark I, II, III, and IV).[2,26] These models use various time durations of rainfall input to generate outflow hydrographs for a watershed. A general digital computer model builds the values of model parameters by adjusting such components as evapotranspiration losses, infiltration capacities, and groundwater recession rates to represent a particular basin. Chapter 10 treats simulation methods in greater detail.

3-19 Statistical and Probability Approaches to Runoff Determination

In Chapters 4 and 5 some statistical and probability methods and techniques adopted in analysing various hydrologic data are discussed.

A hydrologist is much concerned with the problem of estimating streamflows. Two approaches are available. First is an indirect approach in which runoff is computed on the basis of observed or expected precipitation. The second is founded on direct analyses of runoff records themselves without resort to precipitation data. Such investigations are usually frequency studies conducted to indicate the likelihood of certain runoff events taking place. A knowledge of the frequency of runoff events is helpful in determining risks associated with proposed designs or anticipated operating schemes. Unfortunately, most existing runoff records are usually short-term, and as a result they limit utility for reliable frequency analyses. Few adequate records are available earlier than about 1900. To overcome this deficiency, serious effort is being placed on the application of sequential generation techniques to develop synthetic records of any desired length. These generated sequences can then be examined despite the brevity of the historic traces. Frequency analyses are directed toward studies of maximum (flood) and minimum (drought) flows.[28–30] Figure 3-25 illustrates a typical drought frequency analysis.

Time-series analyses are particularly pertinent to the problem of estimating trends, cycles, and fluctuations in hydrologic data. They also permit derivation of generating processes by which synthetic records of runoff can be developed.

3-20 Streamflow Forecasting

Surface-water hydrology is basic to the design of many engineering works, and important in water quality and other management schemes. In addition, the ability to reliably forecast flows for short periods into the future is of signal value in operating storage and other works and in planning proper actions during times of flood.[1,32] A good example is operation of a reservoir with an uncontrolled inflow but having a means of regu-

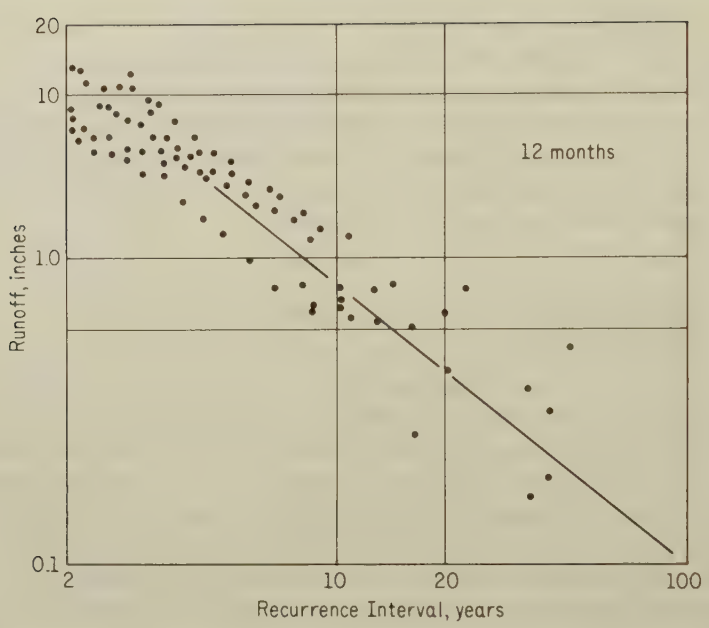


FIG. 3-25. Low-flow frequency data consolidated for Five Rivers. (After William Whipple, Jr., "Regional Drought Frequency Analysis," *ASCE Proc.*, Paper No. 4837, *Irrigation and Drainage Division Journal*, Vol. 92, No. IR2, June 1966, pp. 11–31.)

lating the outflow. If information on the nature of the inflow is determinable in advance, then the reservoir can be operated by some decision rule to minimize downstream flood damage. Such operations can be computerized to continually improve estimates based on incoming data and thus offer direction on the nature of the releases to be made. For river forecasts to be reliable, adequate, dependable data on various watershed and meteorologic conditions are needed on a continuing basis. Modern monitoring stations capable of telemetering data to computer control centers provide an important support function for forecasting. The methods used to forecast flows are basically the same ones employed in design; namely, precipitation-runoff equations, unit hydrographs, watershed models, and flow-routing techniques.

References

1. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, *Applied Hydrology*. New York: McGraw-Hill Book Company, 1949.

2. Chow, Ven Te (ed.), *Handbook of Applied Hydrology*. New York: McGraw-Hill Book Company, 1964.

3. Linsley, R. K., Jr., and W. C. Ackermann, "Method of Predicting the Runoff From Rainfall," *Trans. ASCE*, Vol. 107 (1942).

4. Butler, S. S., *Engineering Hydrology.* Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1957.

5. Kohler, M. A., and R. K. Linsley, Jr., "Predicting the Runoff From Storm Rainfall," U.S. Weather Bureau Research Paper 34, 1951.

6. Horton, R. E., "Erosional Development of Streams and Their Drainage Basins: Hydrophysical Approach to Quantitative Morphology," *Bull. Geol. Soc. Am.*, Vol. 56 (1945).

7. DeWiest, Roger J. M., *Geohydrology.* New York: John Wiley & Sons, Inc., 1965.

8. "Hydrology," Engineering Handbook Section 4. Washington, D.C.: U.S. Department of Agriculture, Soil Conservation Service.

9. Barnes, B. S., "Discussion of Analysis of Runoff Characteristics by O. H. Meyer," *Trans. ASCE*, Vol. 105 (1940).

10. Sherman, L. K., "Streamflow From Rainfall by the Unit-Graph Method," *Eng. News-Rec.*, Vol. 108 (1932).

11. Horton, R. E., "An Approach Toward a Physical Interpretation of Infiltration Capacity," Proceedings, Soil Science Soc. of America, Vol. 5 (1940), pp. 399–417.

12. Meinzer, O. E., *Hydrology.* New York: Dover Publications Inc., 1942.

13. Mitchell, W. D., "Unit Hydrographs in Illinois," Illinois Waterways Division, 1948.

14. American Society of Civil Engineers, "Hydrology Handbook," Manuals of Engineering Practice No. 28. New York: ASCE, 1957.

15. Sherman, L. K., "Stream-flow from Rainfall by the Unit-Graph Method," *Eng. News-Rec.*, Vol. 108 (Apr. 7, 1932), pp. 501–505.

16. Morgan, Rand, and D. W. Hullinhors, "Unit Hydrographs for Gauged and Ungauged Watersheds," U.S. Engineers Office, Binghamton, New York, July 1939.

17. Snyder, F. F., "Synthetic Unit Graphs," *Trans. Am. Geophys. Union,* Vol. 19 (1938), pp. 447–454.

18. Taylor, A. B., and H. E. Schwarz, "Unit Hydrograph Lag and Peak Flow Related to Basin Characteristics," *Trans. Am. Geophys. Union,* Vol. 33 (1952), pp. 235–246.

19. Clark, C. O., "Storage and the Unit Hydrograph," *A.S.C.E. Trans.*, Vol. 110 (1945), pp. 1419–1446.

20. "Water Supply Papers," U.S. Geological Survey, Water Resources Division, Washington, D.C.: Government Printing Office.

21. "Hourly Precipitation Data," National Oceanic and Atmospheric Agency. Washington, D.C.: Government Printing Office.

22. Newton, D. W., and J. W. Vinyard, "Computer-Determined Unit Hydrograph from Floods," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 93, No. HY5 (1967), pp. 219–236.

23. "Flood-Hydrograph Analysis and Computations," U.S. Army Corps of Engineers, Engineering and Design Manuals, Em1110-2-1405. Washington, D.C.: Government Printing Office, Aug. 31, 1959.

24. Gray, D. M., "Synthetic Unit Hydrographs for Small Drainage Areas," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 87, No. HY4 (July 1961).

25. Grace, R. A., and P. S. Eagleson, "Scale Model of Urban Runoff From

Storm Rainfall," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 93, No. HY3, Paper 5249 (May 1967).

26. Crawford, N. H., and R. K. Linsley, Jr., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Tech. Report 39, Dept. of Civil Engineering, Stanford University, July 1966.

27. Beard, Leo R., "Hydrologic Simulation in Water-Yield Analysis," *Proc. A.S.C.E., J. Irrigation and Drainage Div.*, Vol. 93, No. IR1 (March 1967).

28. Beard, L. R., "Statistical Methods in Hydrology," U.S. Army Engineer District, Sacramento, Calif., 1962.

29. DeCoursey, Donn G., "A Runoff Hydrograph Equation." Washington, D.C.: U.S. Department of Agriculture, Agricultural Research Service, February 1966, pp. 41–116.

30. Whipple, William W., Jr., "Regional Drought Frequency Analysis," *Proc. A.S.C.E., J. Irrigation and Drainage Div.*, Vol. 92, No. IR2 (1966).

31. Philip, J. R., "An Infiltration Equation with Physical Significance," *Soil Science*, Vol. 77 (1954).

32. Quick, Michael C., "River Flood Flows: Forecasts and Probabilities," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 91, No. HY3 (May 1965).

33. Mockus, V., "Use of Storm and Watershed Characteristics in Synthetic Hydrograph Analysis and Application," U.S. Soil Conservation Service, 1957.

34. Horton, Robert E., *Surface Runoff Phenomena.* Ann Arbor, Mich.: Edwards Brothers, Inc., 1935.

Problems

3-1. Given the following storm pattern, unit storm, and unit hydrograph, determine the composite hydrograph.

Unit storm = 1 unit of rainfall for 1 unit of time				
Actual storm: Time units	1	2	3	4
Pattern: Rainfall units	1	3	5	3

Unit hydrograph—triangular with base length = 4 time units; time of rise = 1 time unit, and maximum ordinate = ½ rainfall unit height.

3-2. Given a unit rainfall duration of one time unit, an effective precipitation of 1.5 in., and the following hydrograph and storm sequence. Determine (a) the unit hydrograph and (b) the hydrograph for the given storm sequence.

Storm Sequence				
Time units	.1	2	3	4
Precipitation, in.	0.3	1.0	1.9	1.2

Hydrograph														
Time units	1	2	3	4	4.5	5	6	7	8	9	10	11	12	13
Flow, cfs	100	98	220	512	620	585	460	330	210	180	105	87	60	54

3-3. Do Prob. 3-2 if the storm sequence is as follows:

Storm Sequence				
Time units	1	2	3	4
Precipitation, in.	0.7	1.6	1.5	0.8

3-4. For a local drainage basin of your choice, plot annual precipitation in inches vs. runoff in inches. Does the relationship appear to be strong? Under what conditions might you use this?

3-5. Select a raingage record of interest. Use the annual values as data to calculate the coefficients of an antecedent precipitation index of the form of Eq. 3-2.

3-6. Given an initial infiltration capacity rate of 4.8 in./hr, a final rate of 0.40 in./hr, and a time constant k of 0.35, find the infiltration capacity at 10 min, 30 min, 1 hr, 2 hr, and 6 hr. Plot these values.

3-7. Refer to Fig. 3-2. Replot this hydrograph and use two different techniques to separate base flow.

3-8. Obtain streamflow data for a water course of interest. Plot the hydrograph for a major runoff event and estimate the base flow.

3-9. For the event of Prob. 3-8, tabulate the precipitation causing the surface runoff and determine the duration of runoff producing rain. Estimate the time of concentration and use Eq. 3-13 to estimate the time base of the hydrograph. Compare this with the time base computed from the hydrograph.

3-10. Using U.S. Geological Survey records, or other data, select a streamflow hydrograph for a large preferably single-peaked runoff event. Separate base flow and determine a unit hydrograph for the area.

3-11. For the unit hydrograph of Prob. 3-1 construct an S-hydrograph.

3-12. For the unit hydrograph computed in Prob. 3-2 construct an S-hydrograph.

3-13. Use the S-hydrograph of Prob. 3-12 to find a 3 time-unit unit hydrograph.

3-14. Given a watershed of 100 sq mi, assume that $C_t = 1.9$, the length of main stream channel is 20 mi, and the length to a point opposite the centroid is 8 mi. Use Snyder's method to find (a) time lag, (b) duration of the synthetic unit hydrograph, and (c) peak discharge.

3-15. Apply Snyder's method to the determination of a synthetic hydrograph for a drainage area of your choice.

3-16. Use Fig. 3-19 to determine a unit hydrograph for a rainfall duration of 2 hr if the drainage area is 80 sq mi and Eq. 3-35 is applicable.

3-17. Solve Prob. 3-16 using Eq. 3-36 instead of Eq. 3-35.

3-18. For the given data, use Gray's method to determine a unit hydrograph. Drainage area = 0.75 sq mi, length = 0.5 mi, $S_c = 1.3$ percent.

3-19. A drainage area contains 27 sq mi. The length of the main channel is 8 mi and the representative watershed slope is 2.8 percent. Use Gray's method to determine a unit hydrograph.

4

POINT-FREQUENCY ANALYSIS

Many hydrologic processes are so complex that they can be interpreted and explained only in a *probabilistic* sense. Hydrologic events appear as uncertainties of nature and are the result, it must be assumed, of an underlying process with random or *stochastic* components. The information to investigate these processes is contained in records of hydrologic observations. Methods of statistical analysis provide ways to reduce and summarize observed data, to present information in precise and meaningful form, to determine the underlying characteristics of the observed phenomena, and to make predictions concerning future behavior.

Statistical analysis involves two basic sets of problems, one *descriptive*, the other *inferential*. The former is a straightforward application of statistical methods, requiring few decisions and representing little risk. The inferential problem, however, entails decisions bearing some risk, and requires an understanding of the methods employed and the dangers involved in predicting and estimating. The most common inferential problem is to describe the whole class of possible occurrences when only a portion of them has been observed. The whole class is the *population* and the portion observed is the *sample*.

The variables in the process under study are *continuous* if they may take on all values in the range of occurrence, including figures differing only by an infinitesimal amount; they are *discrete* if they are restricted to specific, incremental values. Distribution of the variables over the range of occurrence is defined in terms of the frequency or probability with which different values have occurred or might occur. Historical sequences of floods, droughts, streamflow, and rainfall are assumed to result from underlying processes that are *stationary*—that is, their characteristics remain unchanged by long-term trends or other effects. Usually length of record

as well as design life for an engineering project are relatively short compared with geologic history and tend to temper, if not to justify, the assumption of stationarity.[1,2]

4-1 Concepts of Probability

The laws of probability underlie any study of the statistical nature of repeated observations or trials. The process which produces or generates the various observations in a hydrologic sequence can be described as *probabilistic* or *stochastic*. The probability of a single event, say E_1, is defined as the relative number of occurrences of the event after a long series of trials. Thus $P(E_i)$, the probability of event E_1, is n_1/N for n_1 occurrences of the same event in N trials if N is sufficiently large. The number of occurrences n_1 is the *frequency*, and n_1/N the *relative frequency*.

Often the probabilities and the rules governing their manipulation are known intuitively or from experience. In the familiar coin-tossing experiment, $P(\text{Heads}) = P(\text{Tails}) = \frac{1}{2}$. Each outcome of a single toss (a trial) has a finite probability and the sum of the probabilities of all possible outcomes is 1. Also, the outcomes are *mutually exclusive*—that is, if one occurs, say a head, then a tail cannot occur. In two successive tests, there are four possible outcomes—HH, TT, HT, TH—each with a probability of $\frac{1}{4}$. In this case, because each trial is independent of the other one, probabilities for each outcome are found by $P(\text{First Trial}) \times P(\text{Second Trial}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Again, the sum of the probabilities of the possible outcomes is 1. Note that the probability of getting exactly one head and one tail during the experiment (without regard to the order) is $P(\text{HT}) + P(\text{TH}) = \frac{1}{2}$.

Summarizing the rules of probability indicated by coin-tossing, we find:

1. The probability of an event is nonnegative and never exceeds one.

$$0 \leq P(E_i) \leq 1 \tag{4-1}$$

2. The sum of the probabilities of all possible outcomes in a single trial is 1.

$$\sum_i P(E_i) = 1 \tag{4-2}$$

3. The probability of a number of *independent* and *mutually exclusive* events is the sum of the probabilities of the separate events.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) \tag{4-3}$$

The probability statement, $P(E_1 \cup E_2)$, signifies a union of probabilities and is read "the probability of E_1 *or* E_2."

4. The probability of two *independent* events occurring simultaneously or in succession is the product of the individual probabilities.

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2) \tag{4-4}$$

$P(E_1 \cap E_2)$ is called the *intersection* or *joint probability* and is read "the probability of E_1 *and* E_2."

Consider the following example of events that are not independent or mutually exclusive: An urban drainage canal reaches flood stage each summer with relative frequency of .10; power failures in industries along the canal occur with probability of .20; experience shows that when there is a flood the chances of a power failure are raised to .40. The probability statements are

P(Flood) = $P(F)$ = .10 P(Power failure) = $P(P)$ = .20
P(No flood) = $P(\bar{F})$ = .90 P(No power failure) = $P(\bar{P})$ = .80
P(Power failure given that a flood occurs) = .40

The last statement is called a *conditional probability*. It signifies the joint occurrence of events and is usually written $P(P \mid F)$. Rules 3 and 4 above no longer are strictly applicable. If Rule 3 applied, $P(F \cup P) = P(F) + P(P) = 0.3$. If the events remained independent, the joint probabilities would be

$$P(F \cap P) = .1 \times .2 = .02$$
$$P(F \cap \bar{P}) = .1 \times .8 = .08$$
$$P(\bar{F} \cap P) = .9 \times .2 = .18$$
$$P(\bar{F} \cap \bar{P}) = .9 \times .8 = .72$$

The probability of a flood or a power failure during the summer would be the sum of the first three joint probabilities above:

$$P(F \cup P) = P(F \cap P) + P(F \cap \bar{P}) + P(\bar{F} \cap P) = .28$$

The events are dependent, however, from the statement of conditional probability: When a flood occurs with $P(F) = .1$, a power failure will occur with probability .4, and true joint probability is $P(F) \times P(P \mid F) = .1 \times .4 = .04 = P(F \cap P)$. The union probability is then $P(F \cup P) = P(F) + P(P) - P(F \cap P) = .1 + .2 - .04 = .26$. Note the contrast:

$P(F \cup P)$ = .30 for mutually exclusive events
$P(F \cup P)$ = .28 for joint but independent events
$P(F \cup P)$ = .26 otherwise

The new, more general rule for the union of probabilities is

5. $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ (4-5)

and a sixth rule should be added for conditional probabilities:

6. $P(E_1 \mid E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)}$ (4-6)

A very important concept of independence is expressed in a variation of Rule 6, viz., $P(E_1 \mid E_2) = P(E_1)$ if events E_1 and E_2 are inde-

pendent. This repeats the explanation above that $P(E_1) \times P(E_2) = P(E_1 \cap E_2)$ for independent events.

The example of flooding can be extended to show some interesting features about probabilities and risks associated with hydrologic phenomena. $P(F) = .10$ implies a 10 percent chance each year for the flood level to be reached or exceeded. In the long run, the level would be reached on the average once in 10 yr. Thus the average return period T in years is defined as

$$T = \frac{1}{P(F)} = \frac{1}{1 - P(\bar{F})} \tag{4-7}$$

and the following general probability relationships hold:

1. The probability that F will occur in any year:

$$P(F) = \frac{1}{T} \tag{4-8}$$

2. The probability that F will not occur in any year:

$$P(\bar{F}) = 1 - P(F) = 1 - \frac{1}{T} \tag{4-9}$$

3. The probability that F will not occur for n successive years:

$$P_1(\bar{F}) \times P_2(\bar{F}) \cdots P_n(\bar{F}) = P(\bar{F})^n = \left(1 - \frac{1}{T}\right)^n \tag{4-10}$$

4. The probability, called *risk*, R, that F will occur at least once in n successive years:

$$R = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - \{P(\bar{F})\}^n \tag{4-11}$$

Table 4-1 shows return periods associated with various levels of risk.

Example 4-1 What return period must a highway engineer use in his design of a critical underpass drain if he is willing to accept only a 10 percent risk that flooding will occur in the next 5 years?

Solution:

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

$$0.10 = 1 - \left(1 - \frac{1}{T}\right)^5$$

$$T = 48.1 \text{ years}$$

TABLE 4-1 Return Periods Associated with Various Degrees of Risk and Expected Design Life

Risk, %	Expected design life, years							
	2	5	10	15	20	25	50	100
75	2.00	4.02	6.69	11.0	14.9	18.0	35.6	72.7
50	3.43	7.74	14.9	22.1	29.4	36.6	72.6	144.8
40	4.44	10.3	20.1	29.9	39.7	49.5	98.4	196.3
30	6.12	14.5	28.5	42.6	56.5	70.6	140.7	281
25	7.46	17.9	35.3	52.6	70.0	87.4	174.3	348
20	9.47	22.9	45.3	67.7	90.1	112.5	224.6	449
15	12.8	31.3	62.0	90.8	123.6	154.3	308	616
10	19.5	48.1	95.4	142.9	190.3	238	475	950
5	39.5	98.0	195.5	292.9	390	488	976	1,949
2	99.5	248	496	743	990	1,238	2,475	4,950
1	198.4	498	996	1,492	1,992	2,488	4,975	9,953

4-2 Probability Distributions

Random variables, either discrete or continuous, are characterized by the distribution of probabilities attached to the specific values which the variable may take on. A random variable throughout its range of occurrence is generally designated by a capital letter, and a specific value or outcome of the random process is designated by a small letter. For example $P(X = x_1)$ is the probability that random variable X takes on the value x_1. A shorter version is $P(x_1)$. Figure 4-1 shows the probability

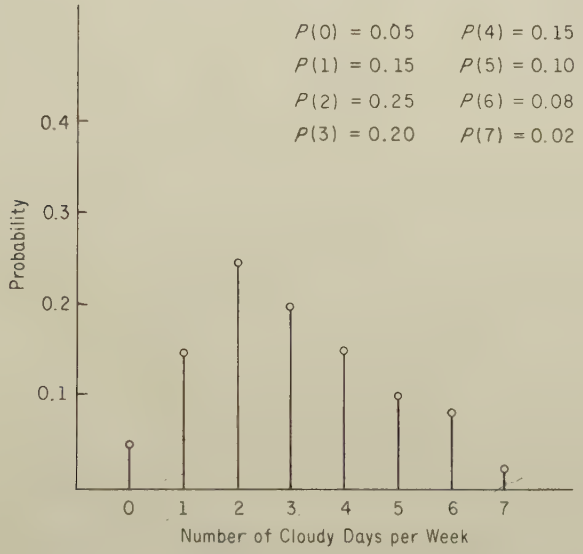


FIG. 4-1. Probability distribution of cloudy days per week.

distribution of the number of cloudy days in a week. It is a discrete distribution because the number of days is exact; in the record from which the relative frequencies were taken, a day had to be described as cloudy or not. Observe that each of the seven events has a finite probability and the sum is one, i.e.,

$$\sum_i P(x_i) = 1$$

Another convenient description of probabilities is the *cumulative distribution function*, CDF, defined as the probability that any outcome in X is less than or equal to a stated, limiting value x. CDF is denoted $F(x)$. Thus

$$F(x) = P(X \leq x) \tag{4-12}$$

and the function increases monotonically from a lower limit of zero to an

upper bound of unity. Figure 4-2 is the CDF of the number of cloudy days in a week derived from Fig. 4-1 by taking cumulative probabilities. The function shows the probability is 90 percent that the number of cloudy days in the week will be 5 or less. Conversely, there is a 10 percent

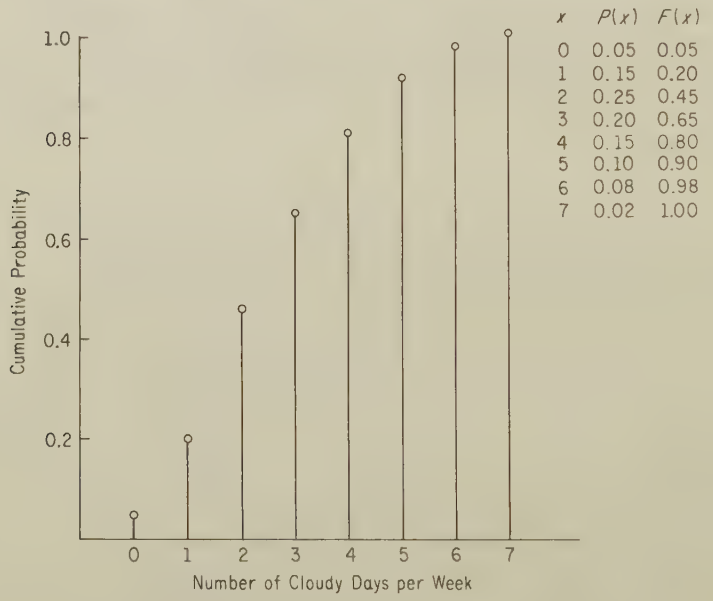


FIG. 4-2. Cumulative distribution of cloudy days per week.

probability that it will be cloudy for 6 or 7 days. This complementary cumulative probability is sometimes called $G(x)$, where[3]

$$G(x) = 1 - F(x) = P(X \geq x) \tag{4-13}$$

Continuous variables present a slightly different picture. Figure 4-3 is the *histogram* of an 85-yr record of annual streamflows. The observations were grouped into nine intervals ranging from 0 to 900 cfs and the number falling in each interval was plotted as frequency on the left ordinate. A convenient alternative is to plot relative frequency as shown by the right ordinate. The CDF for the streamflow record is shown in Fig. 4-4.

As the number of observations increase, the continuous distribution will be developed by reducing the size of the intervals. In the limit, the broken curves of Figs. 4-3 and 4-4 will appear as those in Fig. 4-5. There is a subtle difference between the ordinates of Figs. 4-3 and 4-5a. Since relative frequency is synonymous with probability, it is convenient to reconstitute the histogram so that the area in each interval represents probability, and the total area contained is thus unity. To do this, the ordinate

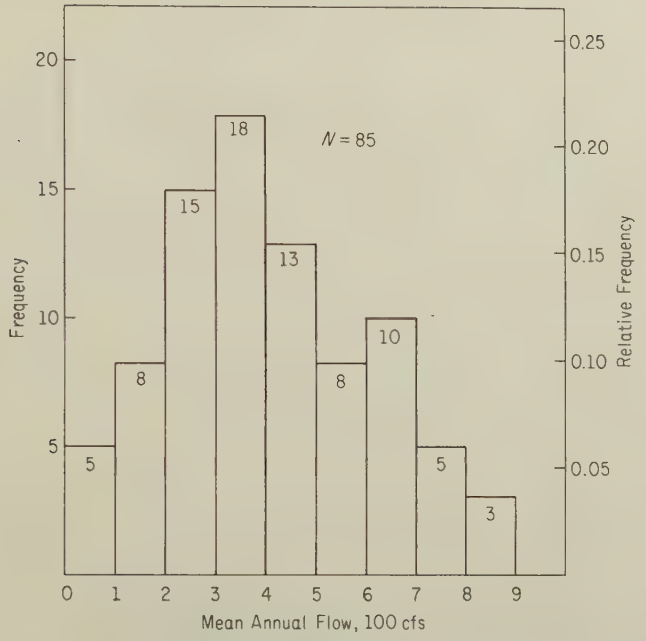


FIG. 4-3. Frequency distribution of mean annual flows.

in each interval, say f/N for relative frequency or probability, is divided by the interval width, Δx. The ratio $f/N\Delta x$ is literally the probability per unit length in the interval and therefore represents the average density of probability. The probability f/N in the interval is represented on the CDF (before the limiting process) as $\Delta F(x)$, or $F(x + \Delta x/2) - F(x - \Delta x/2)$. We then can define

$$f(x) = \lim_{\Delta x \to 0} \frac{\Delta F(x)}{\Delta x} = \frac{dF(x)}{dx} \tag{4-14}$$

which is called the *probability density function*, PDF.[3] The function is the density (or intensity) of probability at any point and $f(x)dx$ is described as the differential probability.

For continuous variables, $f(x) \geq 0$, since negative probabilities have no meaning. Also, the function has the property that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \tag{4-15}$$

which again is the requirement that the probabilities of all outcomes sum to 1. Further, the probability that x will fall between the limits a and b is written

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx \tag{4-16}$$

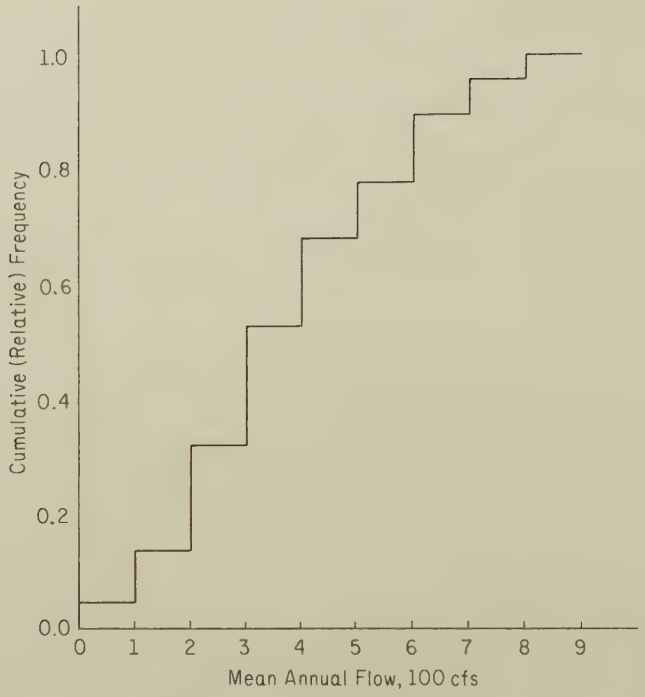


FIG. 4-4. Cumulative frequency distribution mean annual flows.

Note that the probability that x takes on a particular value, say a, is zero, i.e.

$$\int_a^a f(x)\,dx = 0$$

which emphasizes that finite probabilities are defined only as areas under the PDF between finite limits.

The CDF can now be defined in terms of the PDF as

$$P(-\infty \leq X \leq x) = P(X \leq x) = F(x) = \int_{-\infty}^{x} f(u)\,du \qquad (4\text{-}17)$$

where u is used as a dummy variable to avoid confusion with the limit of integration. Area under the CDF has no meaning, only the ordinates, or the difference in ordinates. For example, $P(x_1 \leq X \leq x_2)$, which is equivalent to Eq. 4-16, can be evaluated as $F(x_2) - F(x_1)$.

For discrete distributions that cannot be summarized in integral form, there are analogous arithmetic statements corresponding to the properties given in Eqs. 4-15, 4-16, and 4-17. In particular, the distribution of sampled data taken from a continuous distribution is a special case of discrete distributions and can be given in the form of arithmetic summations.[3] Thus

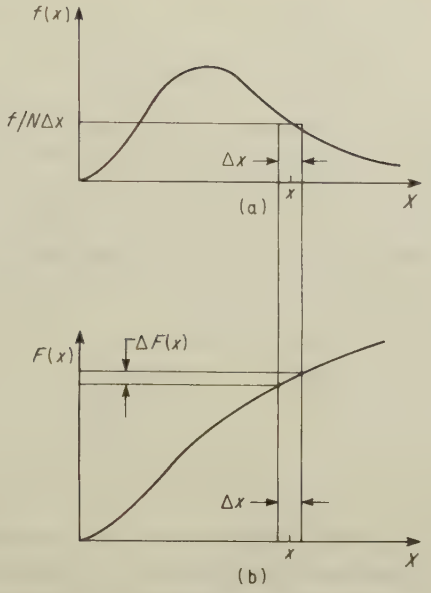


FIG. 4-5. Continuous probability distributions: (a) PDF; and (b) CDF.

$$\sum_i f(x_i) = 1 \tag{4-18}$$

$$P(a \leq X \leq b) = \sum_{x_i \geq a}^{x_i \leq b} f(x_i) \tag{4-19}$$

$$P(X \leq x_k) = \sum_{i=1}^{k} f(x_i) \tag{4-20}$$

For a finite number of observations in the sample, $f(x_i)$ is the probability of x_i for each outcome in the sample space and therefore $P(x_i) = P(x_1) = P(x_2) = \cdots = 1/N$. Hence $f(x_i)$ can be replaced with $P(x_i)$ in Eqs. 4-18, 4-19, and 4-20.

4-3 Moments of Distributions

The mathematical properties of statistical distributions can be defined completely in terms of the moments of the distribution. Fortunately, the moments represent parameters which usually have physical or geometrical significance. Students will recognize the analogy between statistical moments and the moments of areas studied in solid mechanics.

The rth moment about the origin is defined as[4]

$$\mu_r' = \int_{-\infty}^{+\infty} x^r f(x)\, dx \tag{4-21}$$

or

$$\mu'_r = \sum_{i=1}^{n} x_i^r f(x_i) = \frac{1}{n} \sum_{i=1}^{n} x_i^r \tag{4-22}$$

The first moment about the origin is the *mean*, or as it commonly is known, the average. It determines the distance from the origin to the centroid of the distribution frequency function; its importance as a parameter is so complete that it is written simply μ instead of μ'_1.

Moments can be defined about other axes than the origin; the axis used extensively in defining higher moments is the *mean* or, as given above, the first moment about the origin. Thus

$$\mu_r = \int_{-\infty}^{\infty} (x - \mu)^r f(x)\, dx \tag{4-23}$$

or

$$\mu_r = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^r \tag{4-24}$$

Whenever μ'_r or μ_r are defined for $r = 1 \ldots$, the distribution $f(x)$ is completely defined. It seldom is necessary to compute more than the first three moments; several important distributions require only two. The moments are used to specify the parameters and descriptive characteristics of distributions that follow in the next article. Because various characteristics of distributions are described by combinations of the moments about the mean and origin the following relationships are occasionally helpful:[1,4]

$$\mu_1 = 0 \tag{4-25}$$

$$\mu_2 = \mu'_2 - \mu^2 \tag{4-26}$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu + 2\mu^3 \tag{4-27}$$

4-4 Distribution Characteristics

Characteristics of statistical distributions are described by the parameters of probability functions which, in turn, are expressed in terms of the moments. The principal characteristics are *central tendency*: the grouping of observations or probability about a central value; *variability*: the dispersion of the variate or observations; and, *skewness*: the degree of assymetry of the distribution. The theoretical functions shown in Fig. 4-6 exhibit approximately the same grouping about a central value, but f_2 has much greater variability than f_1, and f_2 possesses a pronounced right-skew while f_1 is symmetrical.

In introducing the parameters of distributions, the usual sequence of statistical problems will be followed—that is, we shall derive the parameters from the distribution of sample data and use these as estimates of

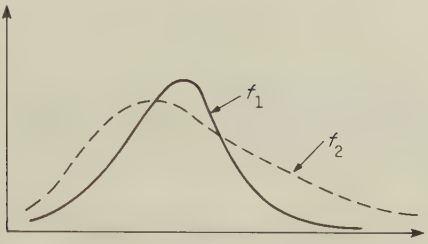


FIG. 4-6. Symmetrical and skewed probability distributions.

the parameters of the population distribution. Thus the summation forms are used to compute moments. For example, the mean of sample data is designated $\bar{x}$ and it is used as the best estimate of the population mean. By convention, Greek letters are used to denote population parameters.

Central Tendency

The familiar arithmetic average, the *mean*, is the most reliable measure of central tendency. It is the first moment about the origin and is designated

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{4-28}$$

The statistic, $\bar{x}$, is the best estimate of the population mean, μ.

Means other than the arithmetic mean—for example, the geometric mean or harmonic mean—are needed in some instances, but the one defined in Eq. 4-28 is sufficient for most purposes. Two additional measures of central tendency are the *median*, which is the middle value of the observed data and divides the distribution into equal parts, and the *mode*, which in discrete variables is the value occurring most frequently and in continuous variables is the peak value of probability density.

Variability

Dispersion can be represented by the total range of values or by the average deviation about the mean; however, the parameter of statistical importance is the mean squared deviation as measured by the second moment about the mean. The parameter is termed the variance and is designated by

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2 \tag{4-29}$$

But the population mean is not known precisely and therefore it is necessary to compute instead

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{4-30}$$

As the best estimate of σ^2, the quantity s^2 is found using $(n-1)$ in place of n in Eq. 4-29. The reasoning for this substitution involves loss of a degree of freedom by using $\bar{x}$ instead of μ, but a proof is beyond the scope of this text.

The square root of the variance is a statistic known as the *standard deviation* (σ or s) in which form variability is measured in the same units as the variate and the mean, and hence is easier to interpret and manipulate. The *coefficient of variation* (C_v), defined as σ/μ or $s/\bar{x}$ is an expression useful in comparing relative variability.

Skewness

A fully symmetrical distribution would exhibit the property that all odd moments equal zero. A skewed distribution, however, would have excessive weight to either side of the center and the odd moments would exist. The third moment is used to define skewness:

$$\alpha = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^3 \tag{4-31}$$

The best estimate of the population skewness must be computed by

$$a = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}(x_i - \bar{x})^3 \tag{4-32}$$

A *coefficient of skewness* (C_s) is represented by α/σ^3 or a/s^3. For symmetrical distributions, the third moment is zero and $C_s = 0$, for right skewness (that is, the long tail to the right side) $C_s > 0$, and for left skewness $C_s < 0$.

Example 4-2 Determine the parameters and compare the distributions of annual rainfall for the records shown in Table 4-2.

Solution:

Parameter	Anniston	Los Angeles	Richmond
Mean, $\bar{x}$ (Eq. 4-28)	57.2 in.	14.9 in.	41.5 in.
Std. dev., s (Eq. 4-30)	15.5 in.	5.9 in.	6.7 in.
Coef. var., $C_v(s/\bar{x})$	0.27	0.40	0.16
Coef. skew., $C_s(a/s^3$; Eq. 4-32)	1.69	−0.17	0.17

Comments: (1) Anniston's record shows a high annual average and a fairly large variability. In particular, Anniston's distribution has a pronounced right skew, caused principally by two very large observed values in this short period of record. (2) Los Angeles has a small annual average but a very large variability and a slightly negative skewness. (3) Richmond has the most uniform distribution —a relatively small variability and only a slight positive skewness.

TABLE 4-2 Annual Rainfall for Selected Cities

Year	Annual rainfall, in.		
	Anniston, Ala.	Los Angeles, Calif.	Richmond, Va.
1928	48	9	43
1927	49	19	44
1926	55	19	38
1925	98	9	31
1924	43	8	47
1923	53	6	49
1922	56	15	52
1921	47	20	31
1920	69	11	51
1919	57	9	40
1918	61	18	41
1917	64	8	43
1916	99	23	37
1915	54	17	36
1914	40	23	34
1913	47	17	38
1912	58	10	36
1911	44	18	37
1910	44	5	43
1909	64	24	34
1908	44	19	53
1907	51	15	49
1906	71	21	47

A Note on Computation

In the past, special forms of the moment equations were developed as an aid for computation by hand or with a desk-top calculator. These had easier expressions for summing and raising to powers, or short-cuts that relied on the frequency of points falling within class intervals. Since the advent of modern electronic computing facilities, however, the need for these forms has largely disappeared. Standard library programs are available for nearly all statistical computations. An example is STIL (Statistical Interpretive Language) which has been adapted to all IBM computers. The input process is extremely simple. Electronic computation is easier and freer of mistakes than laborious hand work.

4-5 Types of Probability Distributions

Many well-defined theoretical probability distributions have been used to describe hydrologic processes, and the common ones have been applied successfully to other groups of statistical phenomena. It should be emphasized, however, that any theoretical distribution is not an exact representation of the natural process but only a description which approximates the underlying phenomenon and has proved useful in describing the

observed data. Table 4-3 summarizes the common distributions, giving the PDF, mean, and variance of the functions. The distributions presented in the table have enjoyed wide application and have been derived and discussed in many standard textbooks on statistics. In the material to follow only special aspects of the most used distributions are given. In describing the relationship of distribution parameters to statistical moments we are following the scheme known as the *method of moments* for fitting theoretical distributions to sample data—that is, the best estimates of population moments are calculated from the sample data and are assumed to be equivalent to the moments of the theoretical distribution. In this way a general solution is obtained for the fitted distribution.

The use of discrete probability distributions is restricted generally to those random events in which the outcome can be described either as a success or failure. Furthermore, the successive trials are independent and the probability of success remains constant from trial to trial.[3,4] In a sense common discrete distributions are counting or enumerating techniques. Table 4-1 for example was based on such discrete probabilities as the example that followed illustrated.

Binomial distribution is frequently used to approximate other distributions and vice versa. For example, staying with discrete values, when n is large and p small (such that $np < 5$ preferably) the binomial approaches the Poisson distribution. This is a single-parameter distribution ($\lambda = np$), and is very useful in describing arrivals in queueing theory. When p approaches one-half and n grows large, the binomial becomes indistinguishable from the normal distribution described below.

4-6 Continuous Frequency Distributions

Most hydrologic variables are assumed to come from a continuous random process, and the common continuous distributions are used to fit historical sequences, as in frequency analysis, for example. Other applications are important too for continuous distributions. The elementary uniform distribution is the basis for computing random numbers so important in simulation studies. The whole body of material in the area of testing and estimating depends on derived distributions like Student's "t," chi-squared, and the F distribution. The explanations which follow concern the more common distributions applied in fitting hydrologic sequences. The reader is referred to standard texts for more detailed treatment.[3-6]

Normal Distribution

The normal distribution is a symmetrical, bell-shaped frequency function, also known as the Gaussian distribution or the natural law of errors.

TABLE 4-3 Table of Common Distributions

Distribution of random variable X	Probability distribution function	Range	Mean	Variance
Binomial	$P(x) = \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}$	$0 \le x < n$	np	$np(1-p)$
Poisson	$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	$0 \le x \le \cdots$	λ	λ
Uniform	$f(x) = \frac{1}{b-a}$	$a \le x \le b$	$(b+a)/2$	$(b-a)^2/12$
Exponential	$f(x) = \frac{1}{a} e^{-x/a}$	$0 \le x \le \infty$	a	a^2
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$	$-\infty \le x \le \infty$	μ	σ^2
Log normal ($y = \ln x$)	$f(y) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-(y-\mu_y)^2/2\sigma_y^2}$	$-\infty \le y \le \infty$ ($0 \le x \le \infty$)	μ_y	σ_y^2
Gamma	$f(x) = \frac{x^\alpha e^{-x/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}$	$0 \le x \le \infty$	$\beta(\alpha+1)$	$\beta^2(\alpha+1)$
Extreme value	$f(x) = \alpha \exp\{-\alpha(x-u) - e^{-\alpha(x-u)}\}$	$-\infty \le x \le \infty$	$u + \frac{0.5772}{\alpha}$	$\frac{\pi^2}{6\alpha^2}$

It describes many processes that are subject to random and independent variations. The whole basis for a large body of statistics involving testing and quality control is the normal distribution, and, while it often does not perfectly fit sequences of hydrologic data, it has wide application, for example, in dealing with transformed data which subsequently follow the normal distribution, and in estimating sample reliability by virtue of the central limit theorem.

The normal distribution has two parameters, the mean, μ, and the standard deviation, σ, for which $\bar{x}$ and s, derived from sample data, are substituted. By a simple transformation, the distribution can be written as single-parameter function only. Thus when $z = (x - \mu)/\sigma$, $dx = \sigma\, dz$, the PDF becomes

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \tag{4-33}$$

and the CDF

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2}\, du \tag{4-34}$$

The variable z is called the standard unit; it is normally distributed with zero mean and unit standard deviation. Tables of areas under the standard normal curve, as given in Appendix C, Table 1, serve all normal distributions after standardization of the variables. Given a cumulative probability, for example, the deviate z is found in the table of areas and x is found from the inverse transform:

$$x = \mu + z\sigma \quad \text{or} \quad x = \bar{x} + zs \tag{4-35}$$

Log-Normal Distribution

Many hydrologic variables exhibit a marked right skewness, partly due to the influence of natural phenomena having values greater than zero, or some other lower limit, and being unconstrained, theoretically, in the upper range. In such cases, frequencies will not follow the normal distribution, but, fortunately, the variables often are functionally normal and their logarithms follow a normal distribution.[7] Thus the PDF shown in Table 4-3 for the log-normal comes from substituting $y = \ln x$ in the normal. With μ_y and σ_y as the mean and standard deviation, respectively, the following relationships have been found to hold between the characteristics of the untransformed variate x and the transformed variate y:[1,7]

$$\mu = e^{\mu_y + \sigma_y^2/2} \tag{4-36}$$

$$\sigma^2 = \mu^2(e^{\sigma_y^2} - 1) \tag{4-37}$$

$$\alpha = (e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2)\, C_y^3 \tag{4-38}$$

$$C_v = (e^{\sigma_y^2} - 1)^{1/2} \tag{4-39}$$

$$C_s = 3C_v + C_v^3 \tag{4-40}$$

Also $\mu_y = \ln M$, where M is the median value *and* the geometric mean of the x's.

The log normal is especially useful because the transformation opens the extensive body of theoretical and applied uses of the normal distribution. Since both the normal and log-normal are two-parameter distributions, it is necessary only to compute the mean and variance of the untransformed variate x and solve Eqs. 4-36 and 4-37 simultaneously. Information on three-parameter or truncated log-normal distributions can be found in the literature.[1,7]

Gamma (and Pearson Type III)

The gamma distribution has wide application in mathematical statistics and has been used increasingly in hydrologic studies now that computing facilities make it easy to evaluate the *gamma function* instead of relying on the painstaking method of using tables of the Incomplete Gamma Function which lead to the CDF, $P(X < x)$. In greater use is a special case of *gamma*: The *Pearson Type III*. This distribution has been widely adopted as the standard method for flood frequency analysis in a form known as the *Log Pearson III* in which the transform $y = \log x$ is used to reduce skewness.[8-10] Although all three moments are required to fit the distribution, it is extremely flexible in that a zero skew will reduce the Log Pearson III distribution to a log-normal and the Pearson Type III to a normal. Tables of the cumulative function are available and will be explained in a later section.[10,11] A very important property of gamma variates as well as normal variates (including transformed normals) is that the sum of two such variables retains the same distribution. This feature is important in generating synthetic hydrologic sequences.[12,13]

Gumbel's Extremal Distribution

The theory of extreme values considers the distribution of the largest (or smallest) observations occurring in each group of repeated samples. The distribution of the n_1 extreme values taken from n_1 samples, with each sample having n_2 observations, depends on the distribution of the $n_1 n_2$ total observations. Gumbel was the first to employ extreme value theory for analysis of flood frequencies.[14] Chow has shown that the Gumbel distribution is essentially a log-normal with constant skewness.[15] The CDF of the density given in Table 4-3 is

$$P(X \leq x) = F(x) = \exp\{-\exp[-\alpha(x - u)]\} \tag{4-41}$$

a convenient form to evaluate the function. Parameters α and u are given as functions of the mean and standard deviation in Table 4-3 and tables of

the double exponential are available.[16] Gumbel also has proposed another extreme value distribution which appears better to fit instantaneous (maximum annual) drought flows.[17,18]

4-7 Frequency Analysis

The statistical methods presented in the earlier sections are used most frequently in describing hydrologic data such as rainfall depths and intensities, peak annual discharge, flood flows, low-flow durations, and the like. In the remainder of this chapter, the fitting of distributions will be restricted to point data—that is, observations at a single station of measurement. Time series, time-dependent information, and correlated variables for regional frequency studies are treated in the next chapter.

Two methods of fitting distributions and using them for frequency analysis will be described: One is a straightforward plotting technique for the cumulative distribution, and the other considers the use of frequency factors. The cumulative distribution provides a rapid means of determining the probability of the event equal to or less than some specified quantity. This attribute is used to obtain recurrence intervals for observed data. As a general rule, frequency analysis should be avoided when working with records shorter than 10 yr, and in estimating frequencies of expected hydrologic events greater than twice the record length.

4-8 Plotting Distributions

Summarizing cumulative distribution functions is simplified by employing probability paper, so constructed that the CDF for certain well-balanced distributions plots as a straight line. Normal probability paper, for example, has an origin at the midpoint, or the line of 50 percent probability, then distorts the scales equally and increasingly on either side to infinity. Thus a symmetrical distribution whose CDF is a perfect S-curve when plotted on cartesian coordinates becomes a straight line on probability paper. The other scale of the special paper is kept in arithmetic units. Similarly, the log-normal distribution becomes a straight-line CDF by using the same probability scale with the other axis in logarithmic values. In both cases the mean of the normal variate occurs at the 50 percent point, and variations at equal deviations above and below the mean will plot as a straight line. A special paper for the Gumbel extreme value distribution uses recurrence interval rather than its reciprocal, the exceedence probability.

The probability of the event can be obtained by use of a "plotting position." When annual maximum values are being analyzed, for example, the recurrence interval is defined as the mean time in years, with N future

trials, for the mth largest value among annual maxima to be exceeded once on the average. The mean number of exceedences for this condition can be shown to be

$$\bar{x} = N \frac{m}{n + 1} \tag{4-42}$$

where $\bar{x}$ = mean number of exceedences
N = number of future trials
n = number of values
m = rank of descending values, with largest equal to one

If the mean number of exceedences $\bar{x} = 1$, $N = T$, and

$$T = \frac{n + 1}{m} \tag{4-43}$$

indicating that the recurrence interval is equal to the number of years of record plus one, divided by rank of the event.

Several plotting position formulas are available[19] and are noted in Table 4-4. The range in recurrence intervals obtained for ten years of

TABLE 4-4 Plotting Position Formulas

Method	Solve for $P(X > x)$	For $m = 1$ and $n = 10$	
		P	T
California	m/n	0.10	10
Hazen	$(2m - 1)/2n$	0.05	20
Beard	$1 - (0.5)^{1/n}$	0.067	14.9
Weibull	$\frac{m}{n + 1}$	0.091	11
Chegadayev	$\frac{m - 0.3}{n + 0.4}$	0.067	14.9
Blom	$\frac{m - 3/8}{n + 1/4}$	0.061	16.4
Tukey	$\frac{3m - 1}{3n + 1}$	0.065	15.5

record can be observed in the right-hand column. These formulas do not account for the sample size, that is, the shortness or record length. A general formula which does account for sample size was given by Gringorten[20] and has the general form

$$T = \frac{n + 1 - 2a}{m - a} \tag{4-44}$$

where n = number of years of record
m = rank
a = $0 < a < 1$, depending upon n

n	10	20	30	40	50
a	0.448	0.443	0.442	0.441	0.440
n	60	70	80	90	100
a	0.440	0.440	0.440	0.439	0.439

The technique in all cases is to arrange the data in increasing or decreasing order of magnitude and to assign order number m to the ranked values. The most efficient formula for computing plotting positions for unspecified distributions,[19] and the one now commonly used for most sample data, is

$$P = \frac{m}{n + 1} \tag{4-45}$$

When m is ranked from lowest to highest, P is an estimate of the probability of values being equal to or less than the ranked value, that is, $P(X \leq x)$; when the rank is from highest to lowest, P is $P(X \geq x)$. For probabilities expressed in percentages the value is $100m/(n + 1)$.

4-9 Frequency Factors

Chow[21] has proposed use of

$$x = \bar{x} + Ks \tag{4-46}$$

as the general equation for hydrologic frequency analysis, where K is the *frequency factor*. It literally is the number of standard deviations above and below the mean to attain the probability point of interest. For two-parameter distributions it varies only with probability, or the equivalent in frequency analysis, the recurrence interval, T. It varies with the coefficient of skewness in skewed distributions, and it can be affected greatly by the number of years of record. For the normal distribution, and for the transformed variate in a log-normal distribution, the deviate z given in standard normal tables is synonymous with the frequency factor.

Log Pearson III

Frequency factors for the Pearson Type III (logarithmic or arithmetic) are shown in Appendix C for various recurrence intervals (or exceedence probabilities) and skew coefficients. As outlined by the Water Resources Council,[10] the fitting technique involves transforming annual floods to logarithmic values ($y_i = \log x_i$) and finding the mean, standard deviation, and skew coefficients of the logarithms. Flood magnitudes (Q) are estimated from

$$\log Q = \bar{y} + Ks \tag{4-47}$$

which is the same form as Eq. 4-46. Note that $K = \phi(T, C_s)$, a function of both recurrence interval and skewness. Because the skewness coeffi-

cient has a much greater variability than the mean or standard deviation, Beard[9] has recommended that only average regional coefficients of skew be employed in flood analysis for a single station unless the record exceeds 100 years. In practice this may be impractical to attain, and it will be best to compute all parameters and compare results with any other records, experience, or regional studies. Use of logarithms to reduce the skewness of an already skewed distribution helps. Hazen recommended that the computed skewness for Pearson III analysis be multiplied by a factor of $(1 + 8.5/n)$ to obtain an adjusted skewness.[22] Chow has developed K vs. T curves for the distribution.[1]

Example 4-3 For rainfall data developed in Example 4-2, fit distribution functions to the records of Richmond, Va. and Los Angeles, Calif.

Solution:

Both distributions exhibit small skewness and are approximately normal. For the purposes of illustration, the Richmond data are fitted with the normal and the Los Angeles data with a Pearson Type III.

1. The data are arrayed for plotting as follows:

m	Richmond	Los Angeles	$100\,m/(n+1)$
1	53	24	4.2
2	52	23	8.3
3	51	23	12.5
4	49	21	16.7
5	49	20	20.8
6	47	19	25.0
7	47	19	29.2
8	44	19	33.3
9	43	18	37.5
10	43	18	41.7
11	43	17	45.8
12	41	17	50.0
13	40	15	54.7
14	38	15	58.3
15	38	11	62.5
16	37	10	66.7
17	37	9	70.8
18	36	9	75.0
19	36	9	79.7
20	34	8	83.3
21	34	8	87.5
22	31	6	91.7
23	31	5	95.8

The points are plotted in Figs. 4-7 and 4-8 as exceedence probability (left-hand scale) vs. inches of rainfall.

2. The theoretical normal of best fit is a straight line through $(\bar{x} - s)$ at 15.9 percent, $\bar{x}$ at 50 percent, and $(\bar{x} + s)$ at 84.1 percent. Thus for Richmond:

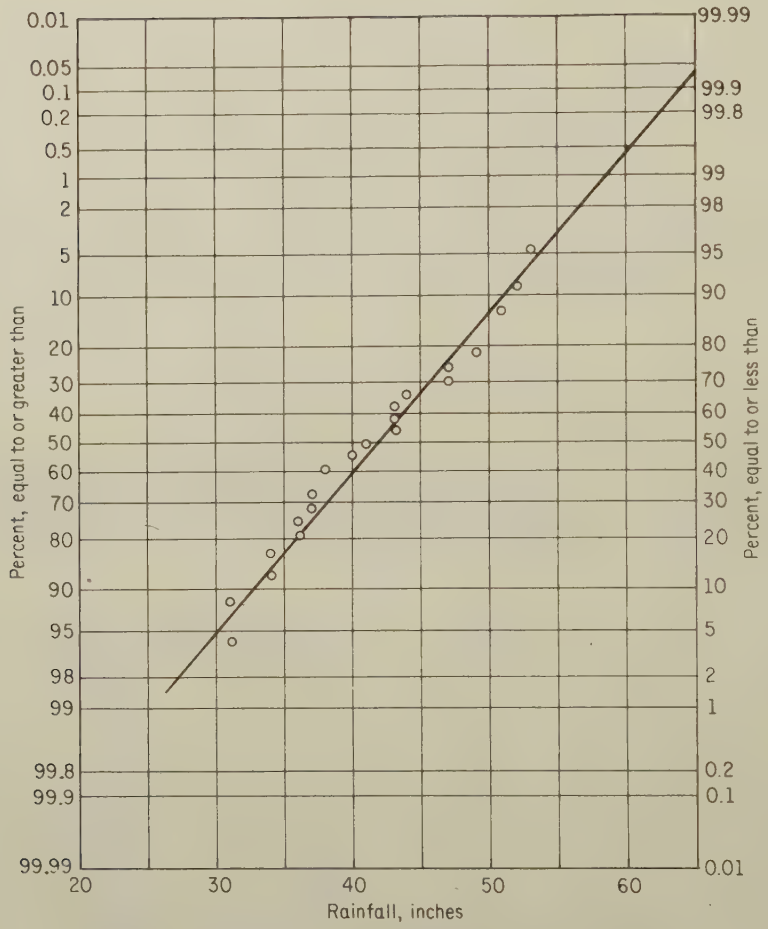


FIG. 4-7. Annual rainfall for Richmond, Virginia, 1906–1928.

X	Plotting Position (right-hand scale)
$\bar{x} - s = 41.5 - 6.7 = 34.8$	15.9
$\bar{x} = 41.5$	50.0
$\bar{x} + s = 41.5 + 6.8 = 48.3$	84.1

3. The plotting positions (read as percent chance) come from Table II, Appendix C, for the computed skewness. Thus for Los Angeles,

% chance	99	95	90	80	50	20	10	4	2	1	.5
$K(C_s = -0.17)$	−2.45	−1.69	−1.30	−0.83	0.03	0.85	1.26	1.69	1.96	2.20	2.41
$x = 14.9 + K(5.9)$	0.4	4.9	7.2	10.0	15.1	19.9	22.3	24.9	26.5	27.9	29.1

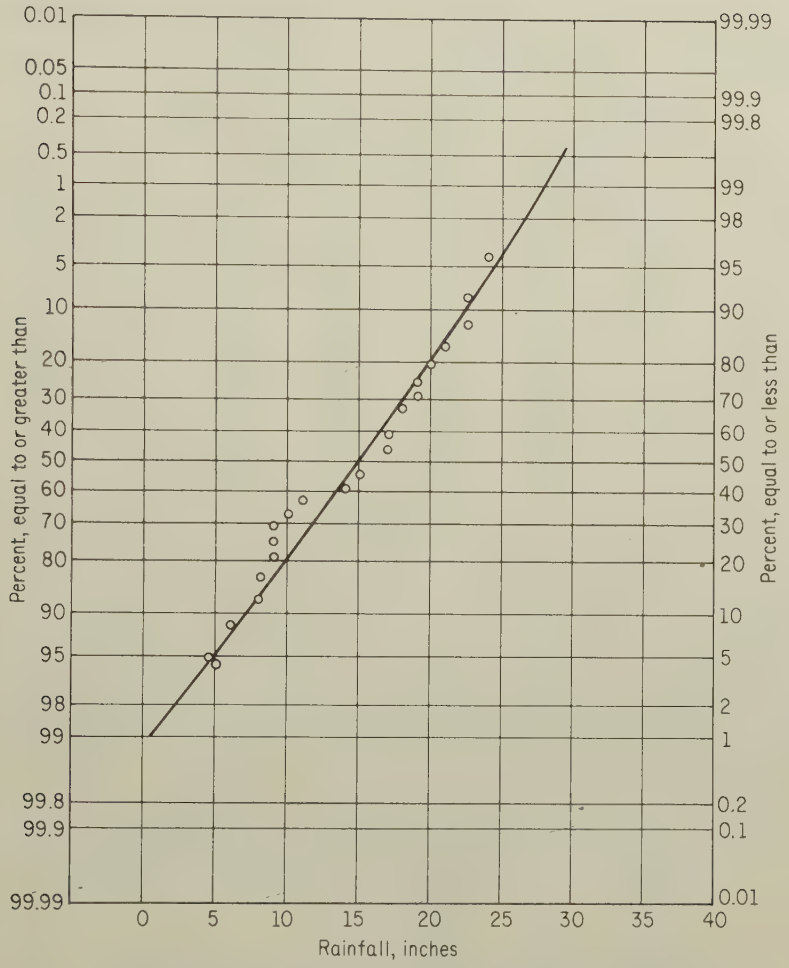


FIG. 4-8. Annual rainfall for Los Angeles, California, 1906–1928.

Gumbel's Extreme Value

Solving the extreme value distribution for the variate yields

$$x = u - \frac{1}{\alpha} \ln [\ln T - \ln (T - 1)] \tag{4-48}$$

and substituting u and α in the general frequency equation (Eq. 4-46) gives for the frequency factor

$$K = -\frac{\sqrt{6}}{\pi} \left[0.5772 + \ln \ln \frac{T}{T - 1} \right] \tag{4-49}$$

The frequency factor is sensitive also to length of record as shown in Table 4-5.[24] K vs. T curves have also been developed.[1]

TABLE 4-5 Extreme-Value Frequency Factors

Sample size	Recurrence interval						
	10	20	25	50	75	100	1000
15	1.703	2.410	2.632	3.321	3.721	4.005	6.265
20	1.625	2.302	2.517	3.179	3.563	3.836	6.006
25	1.575	2.235	2.444	3.088	3.463	3.729	5.842
30	1.541	2.188	2.393	3.026	3.393	3.653	5.727
40	1.495	2.126	2.326	2.943	3.301	3.554	5.476
50	1.466	2.086	2.283	2.889	3.241	3.491	5.478
60	1.446	2.059	2.253	2.852	3.200	3.446	
70	1.430	2.038	2.230	2.824	3.169	3.413	5.359
75	1.423	2.029	2.220	2.812	3.155	3.400	
100	1.401	1.998	2.187	2.770	3.109	3.349	5.261

Example 4-4 The mean of the annual maximum discharges for a streamflow site with 25 yr of record is 1,000 cfs. The standard deviation is 400 cfs. Estimate the magnitude of the 50-yr flood.

Solution:

From Table 4-5, $K = 3.088$; $x = \bar{x} + Ks = 1{,}000 + 3.088(400) = 2235$ cfs.

Example 4-5 The following annual maximum-discharge data have been obtained from the U.S. Geological Survey Water Resources Division for a small stream in Missouri. Rank the data and plot on extreme value probability paper.

Water year	Annual maximum discharge, cfs	Rank	$\frac{n+1}{m}$
1967	2,510	8	1.375
1966	4,150	1	11.0
1965	2,990	5	2.2
1964	2,120	10	1.1
1963	3,555	2	5.5
1962	2,380	9	1.22
1961	2,550	7	1.57
1960	2,800	6	1.83
1959	3,300	3	3.67
1958	3,150	4	2.75

The data are plotted on Fig. 4-9.

4-10 Reliability of Statistical and Frequency Studies

A truly remarkable development of theoretical statistics is the *central limit theorem*. As a consequence of the law of large numbers, the central limit theorem states that for a population with finite variance σ^2 and a mean μ, the distribution of sample means—that is, a number of equally

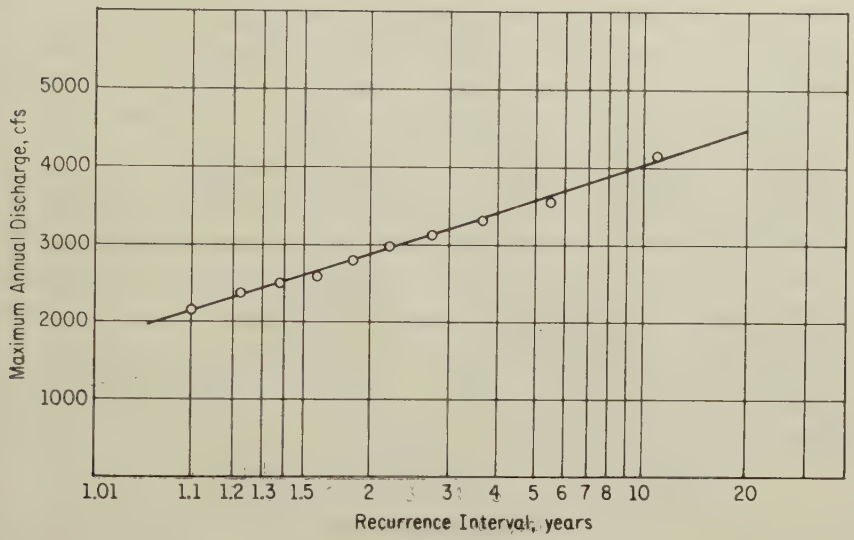


FIG. 4-9. Annual floods on small Missouri stream.

good means from repeated samples—will be distributed themselves as a normal distribution with mean μ and a variance equal to σ^2/n, where σ is the population standard deviation. This theorem does not limit the type of underlying population distribution but says that the distribution of sample means will approach a normal distribution as the sample size increases. The statistic $\sigma/\sqrt{n}$ is the standard deviation of the distribution of means and is called the *standard error* of the mean. Listed below are several parameters of distributions and their standard errors. It is apparent that standard errors, and therefore reliability, are almost completely a function of the sample size.

Measure	Standard error
Mean	$\sigma/\sqrt{n}$
Standard deviation	$\sigma/\sqrt{2n}$
Coefficient of variation	$C_v\sqrt{1 + 2C_v^2}/\sqrt{2n}$
Coefficient of skewness	$\sqrt{6n(n - 1)/(n + 1)(n - 2)(n + 3)}$

Confidence Limits

It is possible to place confidence limits on the measurement of a sample mean based upon the normal distribution of all means and regardless of the underlying population. Hence, as mentioned earlier, approximately two-thirds of the observations of a normal variate should fall between the limits of plus and minus one standard deviation. Therefore, two-thirds

of all sample means should occur between the limits plus and minus $\sigma/\sqrt{n}$. The 95 percent confidence limits for the mean are approximately $\mu \pm 2\sigma/\sqrt{n}$. Note that this statement requires knowledge of the underlying population variance. However, usually only s^2 instead of σ^2 is known and a slightly different technique is needed to estimate confidence limits for a sample mean with $\bar{x}$ and the standard deviation s. Theoretical statistics require Student's t distribution which converges to the normal as n grows large. It is one of a whole class of sampling distributions that are beyond the scope of this text. For more information in the field of inferential statistics—in particular, hypothesis testing and statistical decision theory—the reader must turn to basic sources.

Approximate error limits or control curves can be placed on frequency curves. A method proposed by Beard[9] involves placing lines above and below the fitted curve to form a reliability band. Table 4-6 shows the

TABLE 4-6 Error Limits for Flood Frequency Curves

Years of record (n)	Exceedence frequency, percent, @ 5% level						
	99.9	99	90	50	10	1	0.1
5	1.22	1.00	0.76	0.95	2.12	3.41	4.41
10	0.94	0.76	0.57	0.58	1.07	1.65	2.11
15	0.80	0.65	0.48	0.46	0.79	1.19	1.52
20	0.71	0.58	0.42	0.39	0.64	0.97	1.23
30	0.60	0.49	0.35	0.31	0.50	0.74	0.93
40	0.53	0.43	0.31	0.27	0.42	0.61	0.77
50	0.49	0.39	0.28	0.24	0.36	0.54	0.67
70	0.42	0.34	0.24	0.20	0.30	0.44	0.55
100	0.37	0.29	0.21	0.17	0.25	0.36	0.45
	0.1	1	10	50	90	99	99.9
	Exceedence frequency, percent, @ 95% level						

Note: Tabular values are multiples of the standard deviation of the variate. Five percent error limits are added to the flood value on the curve at the same exceedence frequency and the sum plotted. Ninety-five percent limits are subtracted from the frequency curve flow at the same exceedence frequency. Log values are added or subtracted before antilogging and plotting.

factors by which the standard deviations of the variate must be multiplied to mark off a 90 percent reliability band above and below the frequency curve. The 5 percent level, for example, means that only 5 percent of future values should fall higher than the limit, and, similarly, only 5 percent should fall under the 95 percent limit. Nine of ten should fall within the band.

Example 4-6 The following are maximum annual instantaneous flows from the Maury River near Lexington, Virginia, for a 26-yr period:

Water, yr	Discharge, cfs	Water, yr	Discharge, cfs	Water, yr	Discharge, cfs
1926	6,730	1935	13,800	1944	6,680
1927	9,150	1936	40,000	1945	6,540
1928	6,310	1937	10,200	1946	5,560
1929	10,000	1938	13,400	1947	7,700
1930	15,000	1939	8,950	1948	8,630
1931	2,950	1940	11,900	1949	14,500
1932	8,650	1941	5,840	1950	23,700
1933	11,100	1942	20,700	1951	15,100
1934	6,360	1943	12,300		

Plot the Log Pearson III curve of best fit and determine the magnitude of the flood to be equaled or exceeded once in 5, 10, 50, and 100 yr. Using Table 4-6 also plot the upper and lower confidence limits.

Solution:

1. The basic statistical calculations are summarized as follows:

	Arithmetic	Log
No. of observations: n	26	26
Mean: $\bar{x}$	11,606	4.001
Variance: s^2	53.87×10^6	.0516
Skew coefficient: C_s	2.4	.38

2. After forming an array and computing plotting positions, the data were plotted on Fig. 4-10.
3. Plotting data for Log Pearson III are developed from Table II, Appendix C:

Percent chance	$(C_s = .38)$ K	$(\bar{y} = 4.001)$ $(s = .227)$ $\bar{y} + Ks = \log Q$	Q
99	−2.050	3.535	3,424
95	−1.532	3.653	4,496
90	−1.241	3.719	5,235
80	−0.855	3.760	5,752
50	−0.062	3.987	9,700
20	0.818	4.187	15,370
10	1.315	4.300	19,930
4	1.872	4.426	26,670
2	2.248	4.511	32,470
1	2.597	4.590	38,950
.5	2.924	4.665	46,290

Confidence limits are plotted in Fig. 4-10 using Table 4-6. Note the position of the outlier and its adjusted position H using the Hazen method from Table 4-4. Additional information on outliers is given by Neville and Kennedy.[25]

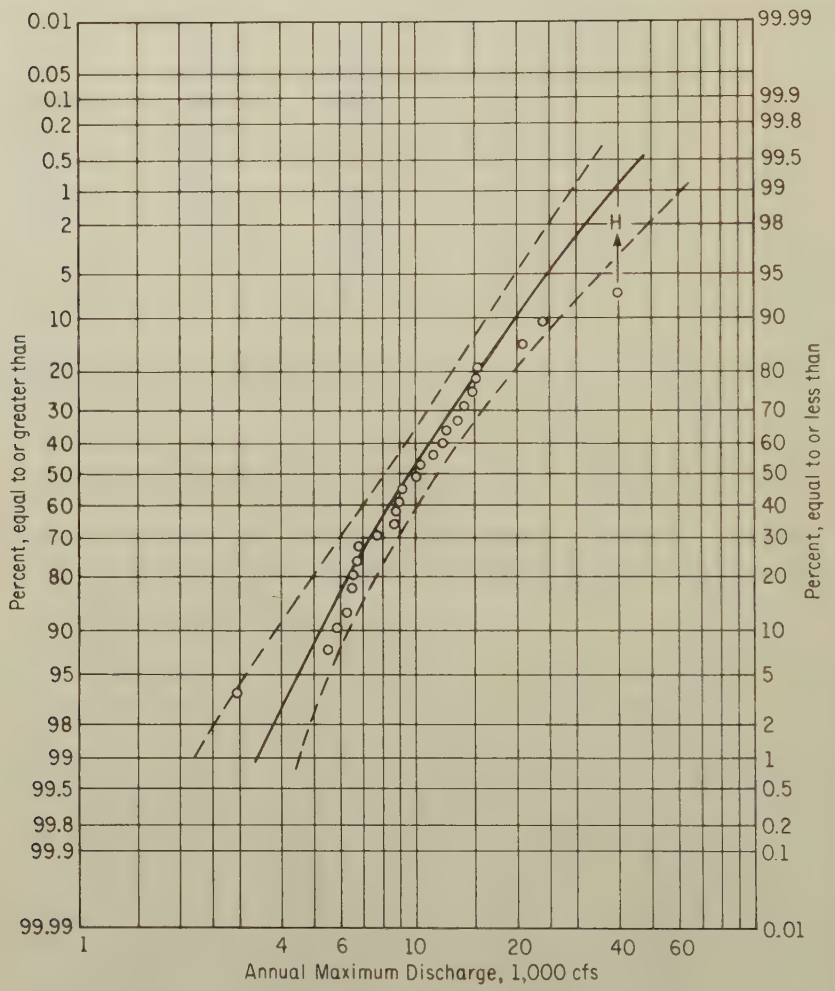


FIG. 4-10. Maximum instantaneous annual flows, Maury River, Lexington, Virginia.

4-11 Additional Frequency Analyses

In earlier examples of frequency analysis, only the series of annual maximum or minimum occurrences in the hydrologic record have been described. These extremes constitute an *annual series* which is consistent with frequency analysis and the manipulation of annual probabilities of occurrence. All of the observed data—say all floods, or all the daily streamflows—would constitute a *complete series*. Any subset of the complete series is a *partial series*. The partial series examined most commonly is a subset other than the annual series. In selecting the maximum annual

events from a record, it often happens that the second greatest event in one year exceeds the annual maximum in some other year. Analysis of the annual series neglects such events. Although they generally contain the same number of events, the extreme values analyzed without regard for the period (i.e., year) of occurrence, is usually termed the *partial-duration series*.

In Table 4-7 the maximum rainfall depths that occurred for any 30-min period during excessive rainfalls at Baltimore, Maryland, 1945–1954, are shown in the order of occurrence. The 65 observations represent a *complete series*. The 11 maximum annual events are underlined and represent the *annual series*. The greatest 11 events throughout the record are identified by an asterisk and represent the *partial-duration series*.

The higher values occur in both series, and hence recurrence intervals for the less frequent events are the same. The theoretical differences in recurrence intervals based on annual- and partial-duration series of the same length are shown in Table 4-8. The difference for intervals greater than 10 yr is negligible. The following example is illustrative.

Example 4-7 Perform a frequency analysis of the 30-min Baltimore rainfall data in Table 4-7 as an annual and a partial-duration series and plot the results.

Solution:

Order	Depth, in.		Recurrence interval
	Annual series	Partial series	$(n + 1)/m$
1	2.50	2.50	12
2	2.04	2.04	6
3	1.53	1.53	4
4	1.33	1.33	3
5	1.30	1.30	2.4
6	1.27	1.27	2
7	1.02	1.20	1.7
8	0.97	1.20	1.5
9	0.85	1.15	1.3
10	0.76	1.10	1.2
11	0.52	1.07	1.1

The data are plotted on Fig. 4-11.

The preceding example leads to consideration of the frequency analysis of rainfall depth or intensity for various durations of rainfall. Design problems often require the estimation of expected intensities for a critical time period. Frequency analysis of the rainfall record for periods other than the 30-min duration—for example the maximum 5-, 10-, 20-, and 60-min occurrences, would yield a family of curves similar to those of Fig. 4-11. The usual method of presenting these data is to convert depth in inches to an intensity in inches per hour and to summarize the data in

TABLE 4-7 Maximum 30-Minute Rainfall Depths, Baltimore, Maryland, 1945–1954

Year	Storm no.	RF depth, in.	Year	Storm no.	RF depth, in.	Year	Storm no.	RF depth, in.
1945	1	0.38	1948	1	1.33*	1953	1	0.40
	2	0.47		2	0.65		2	0.45
	3	0.39		3	0.47		3	0.53
	4	0.76		4	0.84		4	2.50*
	5	0.56		5	0.68		5	1.03
	6	0.35		6	0.63		6	0.75
	7	0.43		7	0.47		7	0.70
	8	0.40					8	1.00*
	9	0.36	1949	1	0.52			
				2	0.49			
1946	1	0.62				1954	1	0.42
	2	0.55					2	0.70
	3	0.88	1950	1	0.55		3	0.85
	4	0.47		2	0.63		4	0.60
	5	0.36		3	0.69			
	6	1.15*		4	1.27*	1955	1	0.70
	7	0.75		5	1.10*		2	0.95
	8	1.53*					3	1.02
	9	0.51	1951	1	0.88		4	0.50
				2	0.97		5	0.65
1947	1	0.88		3	0.59		6	0.55
	2	2.04*		4	0.46		7	0.52
	3	0.76		5	0.50		8	0.45
	4	0.97		6	0.55		9	0.54
	5	0.71					10	0.60
	6	1.07*	1952	1	0.47		11	0.80
	7	0.94		2	1.20*		12	0.95
	8	1.20*		3	0.93			
				4	0.70			
				5	0.57			
				6	0.46			
				7	0.48			
				8	1.30*			

Note: Underlined items are the annual series. Asterisks identify the partial-duration series.

TABLE 4-8 Relationship between the Partial-Duration Series and the Annual Series

Recurrence interval, years	
Partial-duration series	Annual series
0.5	1.2
1.0	1.6
1.5	2.0
2.0	2.5
5.0	5.5
10.0	10.5

intensity-duration-frequency curves as shown in Fig. 4-12. These curves are typical of the point analysis of rainfall data. It should be emphasized that the frequency curves join occurrences that are not necessarily from the same storm—that is, they do not represent a sequence of intensities during a single storm but only the average intensity expected for a specific duration. Fitting the curves and consideration of regional effects is covered in Chapter 5.

Figures 4-13 through 4-16 illustrate further applications of frequency analysis. Figures 4-13 and 4-14 represent standard point frequency analyses of the annual series of high and low flows for different durations. Figure 4-15 is a low-flow-duration-frequency curve based on the same data as Fig. 4-14. Figure 4-16 is based on an analysis of the *complete* series of daily flows, although all observed values are not plotted. Pre-

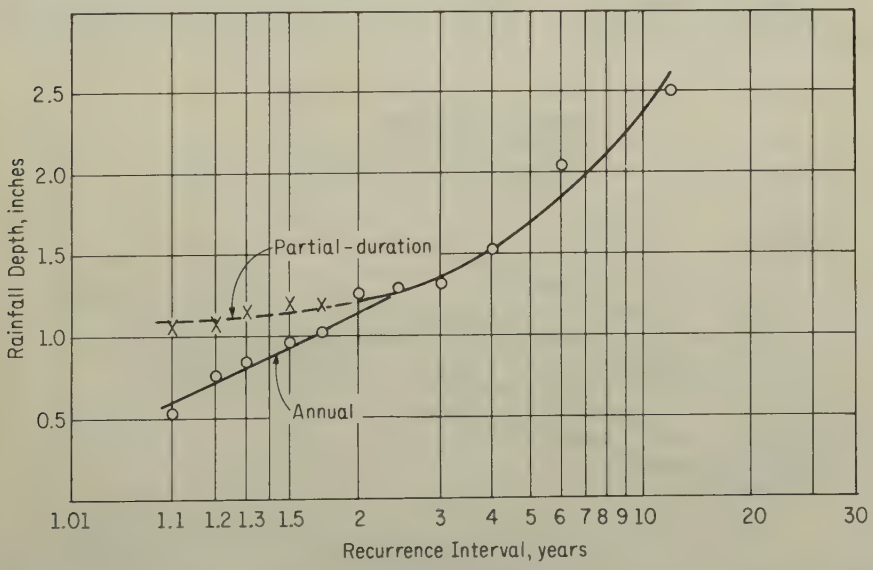


FIG. 4-11. Difference in annual and partial-duration series 11-year record of maximum 30-min durations, Baltimore, Maryland.

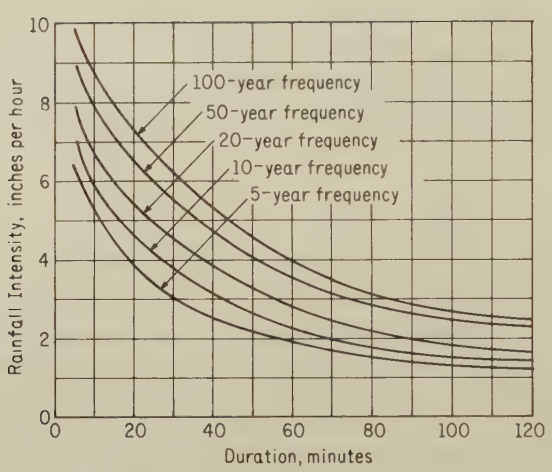


FIG. 4-12. Typical intensity-duration-frequency curves.

sented in this form such a curve usually is called a *duration curve*. Note that the probability scale must be labeled "percent of time," since the annual series was not used. Duration curves are useful in predicting the availability and variability of sustained flows, but, again they do not represent the actual sequence of flows.

Concluding Comment

Statistics is not an end in itself, and the treatment in this chapter has been nothing more than an introduction to statistics. Serious students

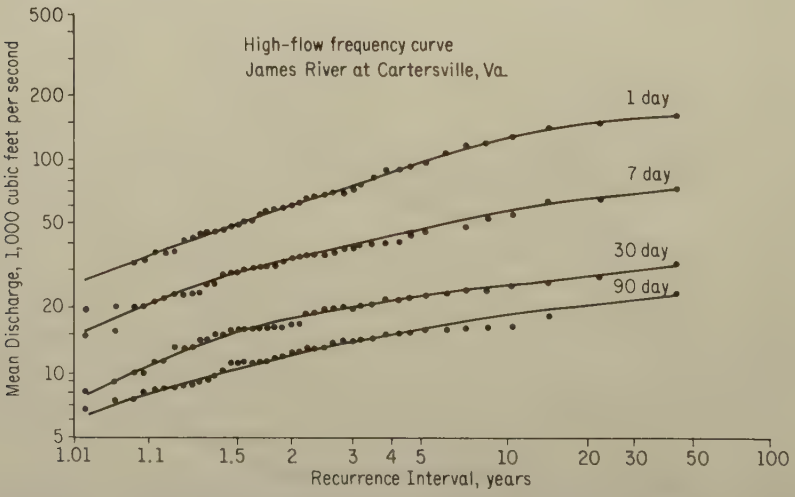


FIG. 4-13. High-flow frequency curves. (*Source:* Virginia Division of Water Resources.)

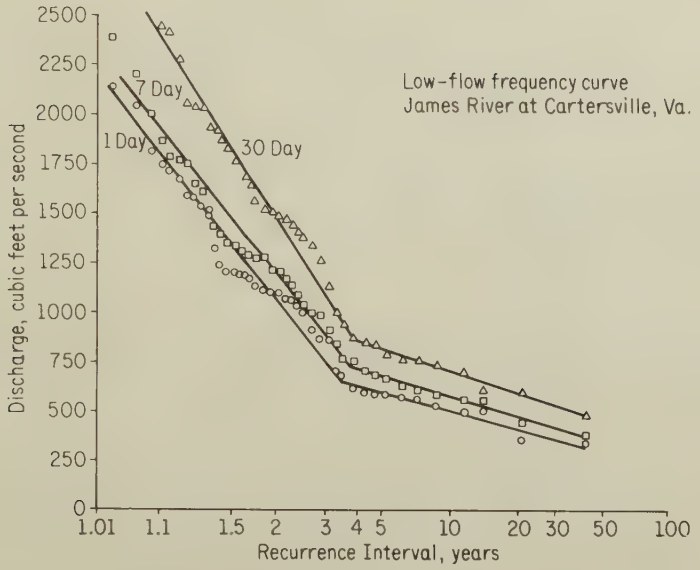


FIG. 4-14. Low-flow frequency curves. (*Source:* Virginia Division of Water Resources.)

and practitioners must return again and again to the theory in standard works. They will find that evaluating new developments and techniques must claim a large share of their time. Only certain aspects of statistical hydrology have been presented, principally the common distributions and

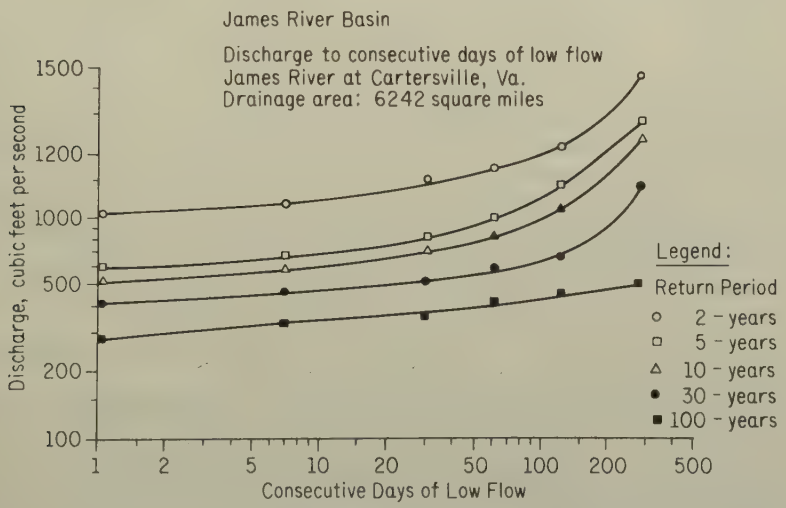


FIG. 4-15. Low-flow-duration-frequency curves. (*Source:* Virginia Division of Water Resources.)

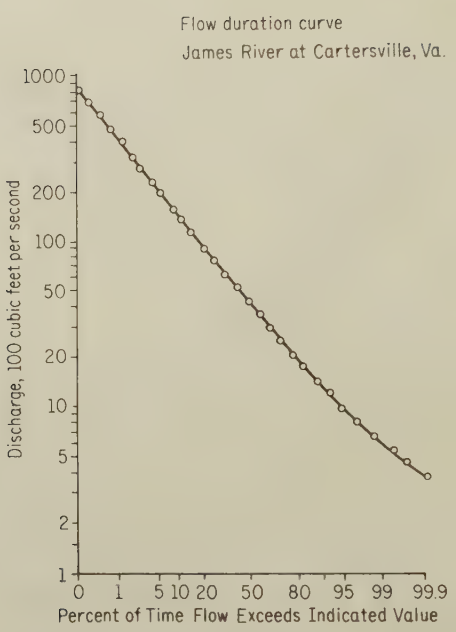


FIG. 4-16. Flow-duration curve. (*Source:* Virginia Division of Water Resources.)

the methods for analyzing frequency of events observed at a single point. In the next chapter this information will be extended to joint variables and time series.

References

1. Chow, Ven T., "Statistical and Probability Analysis of Hydrologic Data," Section 8-I, in V. T. Chow (ed.), *Handbook of Applied Hydrology*. New York: McGraw-Hill Book Company, 1964.

2. Fiering, M. B., "Information Analysis," in G. M. Fair, J. C. Geyer, and D. A. Okun, *Water Supply and Waste Water Removal*. New York: John Wiley & Sons, Inc., 1966, Chap. 4.

3. Benjamin, J. R., and C. Cornell, *Probability, Statistics and Decision For Civil Engineers*. New York: McGraw-Hill Book Company, 1969.

4. Mood, A. M., and F. A. Graybill, *Introduction to the Theory of Statistics*, 2nd ed. New York: McGraw-Hill Book Company, 1963.

5. Duncan, A. J., *Quality Control and Statistics*. Homewood, Ill.: Richard D. Irwin, 1959.

6. Hoel, P. G., *Introduction to Mathematical Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962.

7. Aitchison, J., and J. A. C. Brown, *The Log-Normal Distribution*. New York: Cambridge University Press, 1957.

8. Foster, H. A., "Theoretical Frequency Curves," *Trans. A.S.C.E.*, Vol. 87 (1924), pp. 142–203.

9. Beard, L. R., *Statistical Methods In Hydrology*, Civil Works Investigations; Sacramento District, U. S. Army Corps of Engineers, 1962.

10. "A Uniform Technique For Determining Flood Flow Frequencies," Bulletin No. 15, Water Resources Council, 1967.

11. "New Tables of Percentage Points of the Pearson Type III Distribution," Technical Release No. 38, Central Technical Unit. Washington, D.C.: U. S. Department of Agriculture, 1968.

12. Fiering, M. B., *Streamflow Synthesis*. Cambridge, Mass.: Harvard University Press, 1967.

13. Perkins, F. E., Simulation Lecture Notes, Summer Institute, "Applied Mathematical Programming In Water Resources," University of Nebraska, 1970.

14. Gumbel, E. J., "The Return Period of Flood Flows," *Ann. Math. Statist.*, Vol. 12, No. 2 (June 1941), pp. 163–190.

15. Chow, Ven T., "The Log-Probability and Its Engineering Application," *Proc. A. S. C. E.*, Vol. 80, Paper no. 536 (November 1954), pp. 1–25.

16. "Probability Tables and Other Analysis of Extreme Value Data," Series 22, National Bureau of Standards Applied Mathematics, 1953.

17. Gumbel, E. J., *Statistics of Extremes*. New York: Columbia University Press, 1958.

18. Gumbel, E. J., "Statistical Theory of Extreme Values for Some Practical Application," National Bureau of Standards, Applied Mathematical Series 33, 1954.

19. Benson, M. A., "Plotting Positions and Economics of Engineering Planning," *A.S.C.E., J. Hydr. Div.*, (November 1962), pp. 57–71.

20. Gringorten, I. I., "A Plotting Rule for Extreme Probability Paper," *J. Geophys. Res.*, Vol. 68, No. 3 (February 1963), pp. 813–814.

21. Chow, Ven T., A General Formula for Hydrologic Frequency Analysis, *Trans. Am. Geophys. Union*, Vol. 32 (1951), pp. 231–237.

22. Hazen, A., *Flood Flows*. New York: John Wiley & Sons, Inc., 1930.

23. Fair, G. M., J. C. Geyer, and D. A. Okun, *Water and Waste Water Engineering*. New York: John Wiley & Sons, Inc., 1966.

24. Kendall, G. R., "Statistical Analysis of Extreme Values," First Canadian Hydrology Symposium, National Research Council of Canada, Nov. 4 and 5, 1959.

25. Neville, A. M., and J. B. Kennedy, *Basic Statistical Methods for Engineers and Scientists*. Scranton, Pa.: Intext Educational Publishers, 1964.

Problems

4-1. The probabilities of events E_1 and E_2 are each .3. What is the probability of E_1 or E_2 occurring when (a) the events are independent but not mutually exclusive, and (b) when the probability of E_1, given E_2, is .1?

4-2. A temporary cofferdam is to be built to protect the 5-yr construction activity for a major cross-valley dam. If the cofferdam is designed to withstand the 20-yr flood, what is the risk that the structure will be overtopped (a) in the first

year, (b) in the third year exactly, (c) at least once in the 5-yr construction period, and (d) not at all during the 5-yr period?

4-3. The distribution of mean annual rainfall at 35 stations in the James River Basin, Virginia, is given in the following summary:

Interval (2 in. groupings)	36 or 37 in.	38 or 39 in.	40 or 41 in.	42 or 43 in.
Number of observations	2	4	7	9
Interval (2 in. groupings)	44 or 45 in.	46 or 47 in.	48 or 49 in.	50 or 51 in.
Number of observations	5	4	2	2

Compute relative frequencies and plot the frequency distribution and the cumulative distribution. Estimate the probability that mean annual rainfall will (a) exceed 40 in., (b) will exceed 50 in., and (c) will be between these values.

4-4. Show that Eq. 4-30 can be expressed in the following form for ease in computation

$$s^2 = \frac{1}{n-1}\left\{\sum_i x_i^2 - \left(\sum_i x_i\right)^2/n\right\}$$

Develop a similar form for Eq. 4-32.

4-5. Visit your computer center and inquire about the library programs available for statistical analysis. List the programs, the computations each will perform, and the input information required. Write a simple program to READ in N data points and compute mean, standard deviation, and skewness coefficient.

4-6. Complete the following mathematical statements about the properties of a PDF by inserting in the boxes on the left the correct item number from the right. Assume X is a series of annual occurrences from a normal distribution.

(a) $\int_{-\infty}^{\infty} f(x)\,dx = \square$ — 1. Zero

(b) $\int_{-\infty}^{m_1} f(x)\,dx = \square$ — 2. Unity

(c) $\int_{\mu}^{\mu+\square} f(x)\,dx = 0.34$ — 3. Value with 5 percent chance of exceedance each year

(d) $\int_{m_1}^{m_2} f(x)\,dx = \square$ — 4. 0.68

(e) $\int_{-\infty}^{\square} f(x)\,dx = 0.5$ — 5. Value expected every 50 yr on the average

(f) $\int_{\mu-\sigma}^{\mu+\sigma} f(x)\,dx = \square$ — 6. $P(X \leq m_1)$

(g) $\int_{\square}^{\infty} f(x)\,dx = 0.02$

(h) $\int_{\mu}^{\mu} f(x)\,dx = \square$

(i) $1 - \int_{m_1}^{m_2} f(x)dx = \square$

(j) $\int_{-\infty}^{\square} f(x)\,dx = 0.95$

7. $P(m_1 \leq X \leq m_2)$

8. $P(m_1 \geq X \geq m_2)$

9. Median

10. Standard deviation

4-7. The mean monthly temperature for September at the Tech first-order weather station is found to be normally distributed. The mean is 65.5°F, the variance is 39.3°F^2, and the record is complete for 63 yr. With the aid of Table I, Appendix C, find: (a) The midrange within which two-thirds of all future mean monthly values is expected to fall. (b) The midrange within which 95 percent of all future values is expected. (c) The limit below which 80 percent of all future values is expected. (d) The highest and lowest values which are expected to be exceeded with a frequency of once in 10 yr and once in 100 yr. Verify the results by plotting the cumulative distribution on normal probability paper.

4-8. The total annual pumpage (in acre-ft) over the last 30 yr from a fully developed irrigation well field was observed as follows:

2,450	3,300	3,400	3,650	3,800
2,650	3,150	3,100	3,500	2,850
3,050	4,300	3,300	3,300	3,150
2,100	3,300	3,650	3,150	3,550
2,900	3,250	3,000	3,400	3,750
3,900	3,600	3,150	3,600	3,000

Form an array of the data in preparation for plotting on normal probability paper. Plot only the highest and lowest values in the array and every third point in between. Compute and plot the mean of the data and draw the line of best fit through the mean and as close as possible to the other points by eye. Estimate from the line the standard deviation and the highest value expected to be equaled or exceeded once in 50 yr.

4-9. The mean and standard deviation of floods on a small eastern river are known to be 1,200 cfs and 750 cfs respectively. Assuming the floods to be log-normally distributed: (a) find the mean and standard deviation of the logs using Eqs. 4-36 and 4-37; (b) using Table II, Appendix C, plot on log-normal probability paper; and (c) estimate the 100-yr event.

4-10. Compare the log-normal estimate of the 100-yr event in Prob. 4-9c with the value predicted by Gumbel's distribution (Eq. 4-41 and Table 4-3).

4-11. The 20-yr record of annual flood peaks on Furr's Run are:

Year	Q, cfs	Year	Q, cfs
1951	1,060	1961	1,350
1952	2,820	1962	1,140
1953	1,970	1963	2,100
1954	1,760	1964	1,090
1955	1,650	1965	2,890
1956	1,140	1966	1,100
1957	1,020	1967	1,840
1958	2,260	1968	1,710
1959	1,650	1969	1,630
1960	870	1970	1,260

Log the values of peak discharge and compute mean, standard deviation, and skewness coefficient. Plot the data on log-normal probability paper. With the aid of Table 2, Appendix C, fit both the log-normal and Log-Pearson III distributions. Compare estimates of the 50- and 100-yr events.

4-12. Compare the results of Prob. 4-11 by plotting on extreme-value paper. Compute the mean and variance of the discharges, and, using frequency factors from Table 4-4, find the Gumbel estimates of the 50- and 100-yr events.

4-13. Expand the computer program of Prob. 4-5 to include the computation of mean, standard deviation, and skewness coefficient of the logarithms of input data. Also, include a routine to sort the data by placing them in descending order and compute the corresponding plotting positions. Verify, using the data in Prob. 4-11.

4-14. Perform a complete frequency analysis on one of the three 33-yr records given below. Fit a Pearson Type III or Log-Pearson III and compare with the normal or log-normal of best fit. Plot the data and place control curves around the theoretical curve of best fit using Table 4-6.

Year	Trempeuleau River Dodge, Wis. (DA = 643 sq mi) Q_{peak}, cfs	Bow River Banff, Alberta (DA = 858 sq mi) Q_{peak}, cfs	James River Scottsville, Va. (DA = 4,570 sq mi) Q_{peak}, cfs
1928	3,700	10,200	75,600
1929	1,700	7,590	44,700
1930	3,360	9,280	45,800
1931	1,650	6,610	21,100
1932	3,600	9,850	31,400
1933	11,000	11,000	59,500
1934	2,570	9,490	38,800
1935	4,490	6,940	93,400
1936	7,180	7,720	126,000
1937	1,780	5,210	62,200
1938	3,170	7,770	87,400
1939	6,400	6,270	68,400
1940	3,120	7,220	130,000
1941	2,890	4,450	27,100
1942	5,680	5,850	80,600
1943	5,060	7,380	95,200
1944	2,040	5,590	133,000
1945	8,120	4,450	57,000

(*continued on next page*)

Year	Trempeuleau River Dodge, Wis. (DA = 643 sq mi) Q_{peak}, cfs	Bow River Banff, Alberta (DA = 858 sq mi) Q_{peak}, cfs	James River Scottsville, Va. (DA = 4,570 sq mi) Q_{peak}, cfs
1946	4,570	7,210	41,200
1947	5,410	5,880	33,200
1948	4,840	10,320	59,600
1949	1,920	4,290	94,200
1950	3,600	10,080	73,300
1951	4,840	8,570	64,900
1952	6,950	5,460	54,500
1953	4,040	9,180	67,000
1954	5,710	10,120	62,900
1955	10,400	8,680	70,000
1956	17,400	9,060	20,400
1957	713	5,360	64,200
1958	1,140	6,730	44,500
1959	8,000	7,480	29,300
1960	1,480	6,440	64,200

4-15. Compare results of Prob. 4-14 with estimates by Gumbel's extreme-value distribution for the 50- and 100-yr events.

4-16. Listed below are the pan-evaporation data for the month of July at a site in Missouri. Determine the mean, standard deviation, and coefficient of variation. What are the standard errors of these statistics? Establish the approximate 95 percent confidence limits of the mean.

Evaporation, in.	9.7,	11.7,	11.2,	11.3,	11.5,
	11.2,	8.8,	11.4,	11.8,	8.9,
	9.3,	9.2,	9.3,	9.3,	10.4,
	9.8,	8.7,	11.5,	10.9,	10.2

4-17. A 40-yr record of rainfall indicates mean monthly precipitation during April is 3.85 in. with a standard deviation of .92. The distribution is normal. With 95 percent confidence, estimate the limits within which (a) next April's precipitation is expected to fall and (b) the mean April precipitation for the next 40 yr?

4-18. From the data given in the accompanying table of low flows, prepare a set of low flow frequency curves for the daily, weekly, and monthly durations.

Lowest Mean Discharge, cfs, for the Following Number of Consecutive Days, Maury River near Buena Vista, Va.

Year	1-day	7-day	30-day
1939	100.0	103.0	125.0
1940	167.0	171.0	194.0
1941	22.0	59.4	69.1
1942	101.0	127.0	173.0
1943	86.0	93.9	103.0
1944	62.0	65.9	77.4
1945	78.0	80.7	90.3

(*continued on next page*)

Year	1-day	7-day	30-day
1946	76.0	78.6	87.1
1947	97.0	102.0	123.0
1948	154.0	176.0	215.0
1949	136.0	138.0	163.0
1950	113.0	125.0	139.0
1951	95.0	95.3	101.0
1952	115.0	116.0	120.0
1953	85.0	86.1	90.8
1954	68.0	70.0	81.7
1955	83.0	96.1	99.9
1956	64.0	66.3	71.7
1957	62.0	64.1	75.8
1958	88.0	92.6	107.0
1959	76.0	80.9	117.0
1960	83.0	91.7	103.0
1961	99.0	103.0	152.0
1962	90.0	95.0	105.0
1963	60.0	60.6	70.8
1964	51.0	54.1	62.0
1965	64.0	68.7	76.2

4-19. For the 7-day low flows at Buena Vista given in Prob. 4-18, attempt to fit a straight-line frequency curve on log-normal or extreme-value probability paper, proceeding as follows: From the original plot of the data, estimate the lowest flow (say, q) at high recurrence intervals; subtract this flow from all observed flows ($Q - q = Q^1$); and replot, Q^1 vs. original recurrence intervals. Repeat if necessary. The best fitting curve will be a three-parameter frequency distribution.

4-20. The following table summarizes the number of occurrences of intensities of various durations for a 34-yr record of rainfall. Maximum intensities for the given durations were determined for all excessive storms and a count made of the exceedences. Interpolate for the average number of exceedences expected on a 5-yr frequency, and plot the 5-yr intensity-duration-frequency curve.

Duration, min	Number of times stated intensities were equaled or exceeded						
	Intensity, in./hr						
	1.0	2.0	3.0	4.0	5.0	6.0	7.0
5			73	48	21	9	2
10		68	51	26	11	3	1
15	72	35	23	11	5	1	
30	29	17	7	3	1		
60	15	6	2	1			
120	8	1					

5

REGIONAL ANALYSIS: JOINT DISTRIBUTIONS AND TIME SERIES

In Chapter 4 the probability and statistical properties of random variables were introduced, but applications were restricted to a single variable and the analysis of point-frequency distributions. Complex hydrologic processes often require knowledge of the joint distribution of several random variables and consideration of the correlation between them. Dependence of sequential events in a time series is also an important concept in hydrology. Methods of analysis can be extended to the study of regional relationships, over both space and time, and techniques can be developed to produce synthetic sequences of variables which are artificial traces of the possible occurrence of hydrologic events that preserve the characteristics of the historical record. With the development of electronic computers, these methods and techniques have become a valuable element in planning and design.

In the present chapter an extension of probability theory leads to consideration of regression and correlation analysis, Markov and other time-dependent series, and the basis for generation of synthetic hydrologic variables (covered in Chapter 10).

JOINT DISTRIBUTIONS AND CORRELATION

5-1 Additional Probability Concepts

Many hydrologic events must be studied as the joint occurrence of two or more random variables, and the probability or frequency analysis concerns their joint probability distribution. Joint probabilities and con-

ditional probabilities were introduced in Chapter 4, but were not treated as complete distribution functions.

Joint and Marginal Distributions

If an outcome results from the simultaneous occurrence of two discrete random variables x and y, their joint probability function is $P(X = x_i \cap Y = y_j)$; or in shorter form, $P(x_i, y_j)$. For $x_i (i = 1, 2, \ldots, n)$ and $y_j (j = 1, 2, \ldots, m)$, the familiar rule for summing to unity holds:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) = 1 \tag{5-1}$$

The summation can be carried to limiting values of X and Y, say x_k and y_l, to find the joint cumulative probability:

$$F(x_k, y_l) = \sum_{i=1}^{k} \sum_{j=1}^{l} P(x_i, y_j) \tag{5-2}$$

The concept of *marginal distributions* is defined by summing $P(x_i, y_j)$ over all values of one variable. The resulting marginal distribution is the probability function of the other variable without regard for the first. In other words, for two jointly distributed random variables, X and Y, the distribution of X is defined by finding the marginal distribution of the joint probability function. Thus

$$P(x_i) = \sum_{j=1}^{m} P(x_i, y_j) \tag{5-3}$$

and similarly

$$P(y_j) = \sum_{i=1}^{n} P(x_i, y_j) \tag{5-4}$$

For the marginal CDF of X,

$$F(x_k) = \sum_{i=1}^{k} \sum_{j=1}^{m} P(x_i, y_j) \tag{5-5}$$

For continuous variables the analogous relationships follow directly. For the joint PDF $f(x, y)$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1 \tag{5-6}$$

and the cumulative function $F(x, y)$

$$F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u, v)\, dv\, du \tag{5-7}$$

while the marginal distribution of, say, X is

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy \tag{5-8}$$

and the marginal CDF of X is

$$F(x) = \int_{-\infty}^{x} f(u)\, du = \int_{-\infty}^{x} \int_{-\infty}^{\infty} f(u, y)\, dy\, du \tag{5-9}$$

The marginal density functions are particularly helpful in finding conditional probabilities, $P(x \mid y)$ or $f(x \mid y)$. Recall

$$P(x \mid y) = \frac{P(X = x \cap Y = y)}{P(y)} \tag{5-10}$$

and also

$$f(x \mid y) = \frac{f(x, y)}{f(y)} \tag{5-11}$$

In both cases the joint probability is the product of the conditional and the marginal probability functions.

Moments

Moments again provide the best description of the distribution. The joint moment of order r and s is defined as

$$\mu'_{r,s} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^r y^s f(x, y)\, dy\, dx \tag{5-12}$$

When $r = 1$ and $s = 0$,

$$\mu'_{1,0} = \int_{-\infty}^{\infty} x \left[\int_{-\infty}^{\infty} f(x, y)\, dy \right] dx \tag{5-13}$$

and the expression in brackets is simply $f(x)$, the marginal PDF of x, as shown. Hence the moment is the mean of x:

$$\mu'_{1,0} = \mu_x = \int_{-\infty}^{\infty} x f(x)\, dx \tag{5-14}$$

Similar results hold for y.

When the moments are written about the means,

$$\mu_{r,s} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mu_x)^r (y - \mu_y)^s f(x, y)\, dy\, dx \tag{5-15}$$

With $r = 2$, $s = 0$, the result is σ_x^2, and for $r = 0$, $s = 2$, it is σ_y^2. Both are familiar quantities. But a new concept evolves from the third type of second-order moment, that is, $r = 1$, $s = 1$. This moment is called the *covariance*, cov (x, y), or $\sigma_{x,y}$

$$\sigma_{x,y} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mu_x)(y - \mu_y) f(x, y)\, dy\, dx \tag{5-16}$$

The correlation coefficient, usually identified as ρ, is a standardized and dimensionless form of the covariance as follows:

$$\rho_{x,y} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y} \tag{5-17}$$

Because the correlation coefficient varies between ± 1, it is widely referred to as a measure of dependence or association between two variables. It actually indicates the degree of linear dependence. Thus a high value would indicate, for positive correlation, that large values of x are associated with large values of y and similarly for small values. A low value of the coefficient may indicate lack of association, but not conclusively.[1] If x and y are independent, then $f(x, y) = f(x) \times f(y)$, and Eq. 5-16 gives a covariance of zero.

Example 5-1 For the joint discrete distribution of quarterly flows on the two rivers shown in Fig. 5-1, find the marginal distributions of X and Y, and compute the means, variances, standard deviations, covariance, and correlation coefficient.

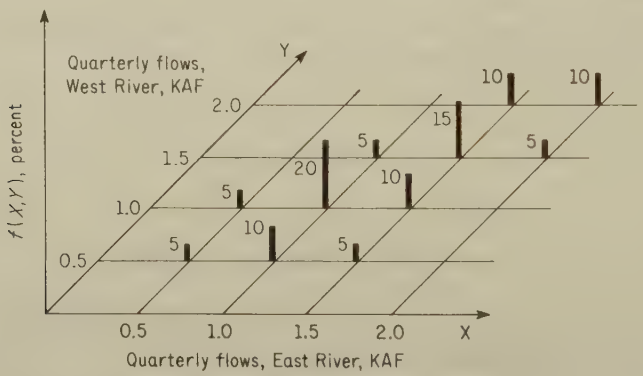


FIG. 5-1. Joint discrete distribution of quarterly flows on east and west rivers.

Solution:

1. Summing over Y gives the marginal distribution of X. Thus

x:	0.5	1.0	1.5	2.0
$f(x)$:	0.1	0.35	0.40	0.15

2. Summing over X gives the marginal distribution of Y. Thus

y:	0.5	1.0	1.5	2.0
$f(y)$:	0.2	0.35	0.25	0.2

3. $\bar{x} = \Sigma x f(x) = 0.5(0.1) + 1.0(0.35) + 1.5(0.40) + 2.0(0.15) = 1.3$ KAF (thousands of acre-ft).

4. $s_x^2 = \Sigma(x - \bar{x})f(x) = (0.5 - 1.3)^2(0.1) + (1.0 - 1.3)^2(0.35) + (1.5 - 1.3)^2(0.40) + (2.0 - 1.3)^2(0.15) = 0.459\ \text{KAF}^2$.
5. $s_x = 0.677$ KAF.
6. Similarly for Y, $\bar{y} = 1.23$ KAF; $s_y^2 = 0.262\ \text{KAF}^2$; $s_y = 0.512$ KAF.
7. $\text{cov}(x, y) = \Sigma(x - \bar{x})(y - \bar{y})f(x, y) = (0.5 - 1.3)(0.5 - 1.23)(0.05) + (0.5 - 1.3)(1.0 - 1.23)(0.05) + \cdots + (2.0 - 1.3)(2.0 - 1.23)(0.10) = 0.145$.
8. $\rho_{x,y} = \text{cov}(x, y)/(s_x)(s_y) = (0.145)/(0.677)(0.512) = 0.418$.

5-2 Multivariate Distributions

Joint-density functions can be either discrete or continuous. A multivariate discrete distribution that is analogous to the binomial (Table 4-3) is termed the *multinomial*. Like the binomial, it also is essentially a counting technique and is treated in most standard texts on probability and statistics. Of more direct interest are the continuous multivariate distributions. The bivariate normal is used as an example. Figure 5-2

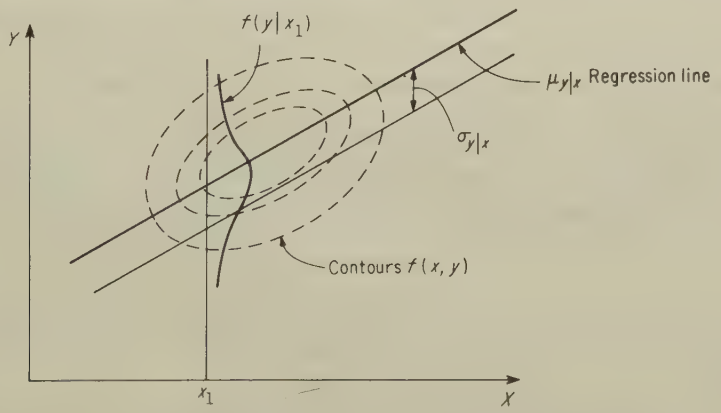


FIG. 5-2. Bivariate density with conditional probability function.

shows the density of $f(x, y)$. The PDF has the form

$$f(x, y) = K \exp\{M\} \tag{5-18}$$

where

$$K = 1/[2\pi\sigma_x\sigma_y(1 - \rho^2)^{1/2}] \tag{5-19}$$

$$M = \left\{-\left[\frac{1}{2(1 - \rho^2)}\right]\left[\left(\frac{x - \mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x - \mu_x}{\sigma_x}\right)\left(\frac{y - \mu_y}{\sigma_y}\right) + \left(\frac{y - \mu_y}{\sigma_y}\right)^2\right]\right\} \tag{5-20}$$

It is a five-parameter distribution, as follows: μ_x, μ_y, σ_x, σ_y, and ρ. The marginal distributions of $f(x, y)$ are univariate normal distributions and

the conditional distribution is defined as usual, for example, $f(y \mid x) = f(x, y) \div f(x)$, which is also a univariate normal.

The conditional distribution has the form

$$f(y \mid x) = K' \exp\{M'\} \tag{5-21}$$

where

$$K' = 1/[\sqrt{2\pi}\,\sigma_y\sqrt{1 - \rho^2}] \tag{5-22}$$

and

$$M' = \left\{-\left[\frac{1}{2\sigma_y^2(1 - \rho^2)}\right]\left[(y - \mu_y) - \rho\,\frac{\sigma_y}{\sigma_x}(x - \mu_x)\right]^2\right\} \tag{5-23}$$

This distribution is normal with mean

$$\mu_{y \mid x} = \mu_y + \rho\,\frac{\sigma_y}{\sigma_x}(x - \mu_x) \tag{5-24}$$

and variance

$$\sigma_{y \mid x}^2 = \sigma_y^2(1 - \rho^2) \tag{5-25}$$

Both of these last two equations have great usefulness in regression and correlation studies. Equation 5-24 is linear and expresses the linear dependence between y and x as shown in Fig. 5-2. The mean value of y can be computed for fixed values of x. And, if the correlation between them is significant, we can predict values of y with less error than from the marginal distribution of y alone. In fact, from Eq. 5-25, the fraction of the original variance explained or removed by the regression is

$$\rho^2 = 1 - \frac{\sigma_{y \mid x}^2}{\sigma_y^2} \tag{5-26}$$

It can be seen also from Eq. 5-24 that the slope of the regression line is

$$\rho\,\frac{\sigma_y}{\sigma_x} = \frac{\mu_{y \mid x} - \mu_y}{x - \mu_x} \tag{5-27}$$

or, if x and y are standardized, then ρ itself is the slope;

$$\rho = \frac{(\mu_{y \mid x} - \mu_y)/\sigma_y}{(x - \mu_x)/\sigma_x} \tag{5-28}$$

The bivariate case can be expanded generally to cover higher-order, multivariate distributions.

5-3 Regression Theory

Regression lines as expressed by Eq. 5-24 and shown in Fig. 5-2 are useful in explaining linear dependence and, where significant correlation exists, in making predictions. For the bivariate case, in general, the procedure is to fit a linear model to a sample of random variables observed in

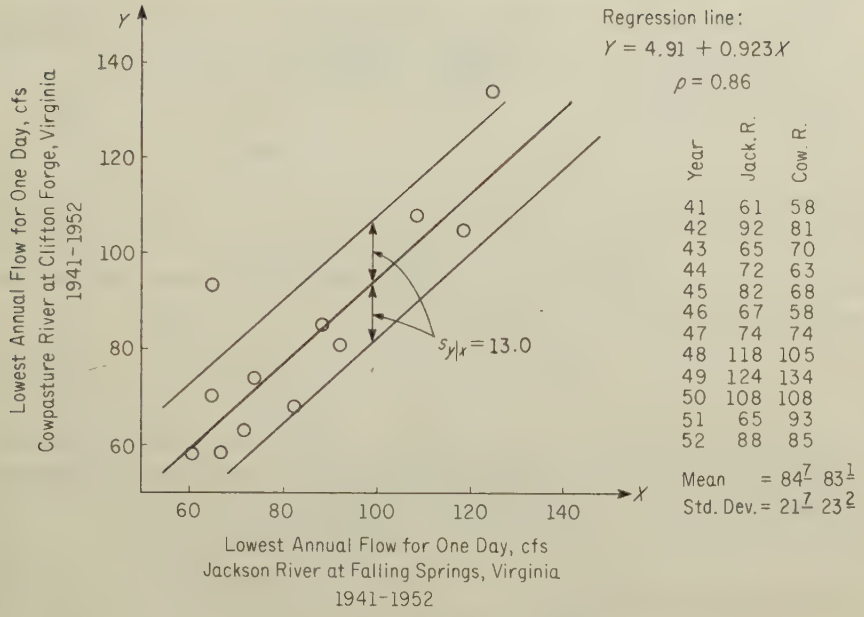


FIG. 5-3. Cross correlation of low flows.

pairs (see Fig. 5-3). The fitting technique is the method of least squares which minimizes the sum of the residuals squared. Residuals as shown in the figure are the difference, vertically in this instance, from the mean value of y predicted by the line and the y value observed for the same corresponding value of x. The line to be fitted is

$$\bar{y}_i = \alpha + \beta x_i \tag{5-29}$$

The best estimates of α and β are sought. Thus to minimize

$$\Sigma(y_i - \bar{y}_i)^2 = \Sigma[y_i - (\alpha + \beta x_i)]^2 \tag{5-30}$$

take partial derivatives as follows

$$\partial/\partial\alpha\{\Sigma[y_i - (\alpha + \beta x_i)]^2\} \tag{5-31}$$

$$\partial/\partial\beta\{\Sigma[y_i - (\alpha + \beta x_i)]^2\} \tag{5-32}$$

After carrying out the differentiations and summations, two equations result in α and β called *normal equations*.

$$\Sigma y_i - n\alpha - \beta\Sigma x_i = 0 \tag{5-33}$$

$$\Sigma x_i y_i - \alpha\Sigma x_i - \beta\Sigma x_i^2 = 0 \tag{5-34}$$

Solving Eqs. 5-33 and 5-34 simultaneously yields

$$\alpha = \Sigma y_i/n - \beta\Sigma x_i/n \tag{5-35}$$

$$\beta = \frac{\Sigma x_i y_i - \Sigma x_i \Sigma y_i / n}{\Sigma x_i^2 - (\Sigma x_i)^2 / n} \tag{5-36}$$

Recall that the slope is

$$\beta = \rho \frac{\sigma_y}{\sigma_x} \tag{5-37}$$

Also

$$\sigma_{y|x}^2 = \sigma_y^2(1 - \rho^2) \tag{5-38}$$

the square root of which is called the *standard error of estimate* and can be estimated from

$$s_{y|x}^2 = \frac{n}{n-2} s_y^2(1 - \rho^2) \tag{5-39}$$

The bivariate example can be extended to multiple linear regressions. For example, the linear model in three variables, with y the dependent variable and x_1 and x_2 the independent variables, has the form

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 \tag{5-40}$$

The normal equations are

$$\Sigma y = \alpha n + \beta_1 x_1 + \beta_2 x_2 \tag{5-41}$$

$$\Sigma y x_1 = \alpha \Sigma x_1 + \beta_2 \Sigma x_1^2 + \beta_2 \Sigma x_1 x_2 \tag{5-42}$$

$$\Sigma y x_2 = \alpha \Sigma x_2 + \beta_1 \Sigma x_1 x_2 + \beta_2 \Sigma x_2^2 \tag{5-43}$$

The standard error of estimate is

$$s_{y|x_1,x_2}^2 = \frac{1}{n-3} \Sigma (y_i - \bar{y}_i)^2 \tag{5-44}$$

where y_i are the observed values and $\bar{y}_i$ are predicted by Eq. 5-40. The *multiple correlation coefficient* is

$$R = \left(1 - \frac{s_{y|x_1,x_2}^2}{s_y^2}\right)^{1/2} \tag{5-45}$$

It should be noted again that a small correlation coefficient, or one of zero, signifies lack of linear dependence, but it is possible there is nonlinear dependence. Furthermore, transformations of an assumed model can be made functionally linear in many ways, the most familiar of which is reduction of a multiplicative relationship by logarithms, for example,

$$y = \alpha x_1^{\beta_1} x_2^{\beta_2} \tag{5-46}$$

which becomes linear when

$$\log y = \log \alpha + \beta_1 \log x_1 + \beta_2 \log x_2 \tag{5-47}$$

Example 5-2 Lowest annual flows for a 12-yr period on the Jackson and

Cowpasture Rivers are tabulated on Fig. 5-3. The stations are upstream of the confluence of the two rivers which form the James River. Find the correlation between low flows and the regression equation.

Solution:

1. The basic statistics are: $\Sigma x = 1016$; $\Sigma y = 997$; $\Sigma x^2 = 91{,}216$; $\Sigma xy = 89{,}209$.
2. For the two-variable regression, α and β are found from Eqs. 5-35 and 5-36:

$$\beta = \frac{[(89{,}209) - (1016)(997)/(12)]}{(91{,}216) - (1016)^2/(12)} = 0.923$$

$$\alpha = (997)/(12) - (0.941)(1016)/(12) = 4.91$$

   The regression is $y = 4.91 + 0.923x$.
3. The correlation coefficient from Eq. 5-37 is

$$\rho = (0.923)(21.7)/(23.2) = 0.86$$

4. From Eq. 5-39 the standard error of estimates, $s_{y|x}$, is 13.0 which is plotted as limits around the regression line on Fig. 5-3.

5-4 Stochastic Time Series

Markov Processes

When dependence exists between sequential events in a random process, and the dependence is such that the next step depends only on the present state and not on other past states, the process is described as Markovian. In general, a Markov process describes only step-by-step dependence, called a *first-order process*, or exhibiting lag-one serial correlation. The theory can be extended conceptually, however, to multiple-lag correlations which then are referred to as *higher-order Markov processes*. In all cases, the process is defined in terms of discrete probabilities.

The probability relationships for a Markov process must provide for the conditional probabilities of the process moving from any state at period t to any subsequent state at period $t + 1$. Thus

$$P(X_{t+1} = j \mid X_t = i) \tag{5-48}$$

expresses the conditional probability of "transitioning" from state i at period t to state j at $t + 1$. Two conditions must be defined to describe completely the properties of the process.[2] One is the initial state—that is, the value of the variable at the beginning. The other is the complete matrix of transition probabilities. A third and resultant property is a statement, or matrix, of steady-state probabilities, which occur in all Markov processes after sufficient transitioning from state to state. The steady-state probabilities are independent of the beginning state and the transition probabilities. The following example developed by Loucks[3] illustrates the principles and application.

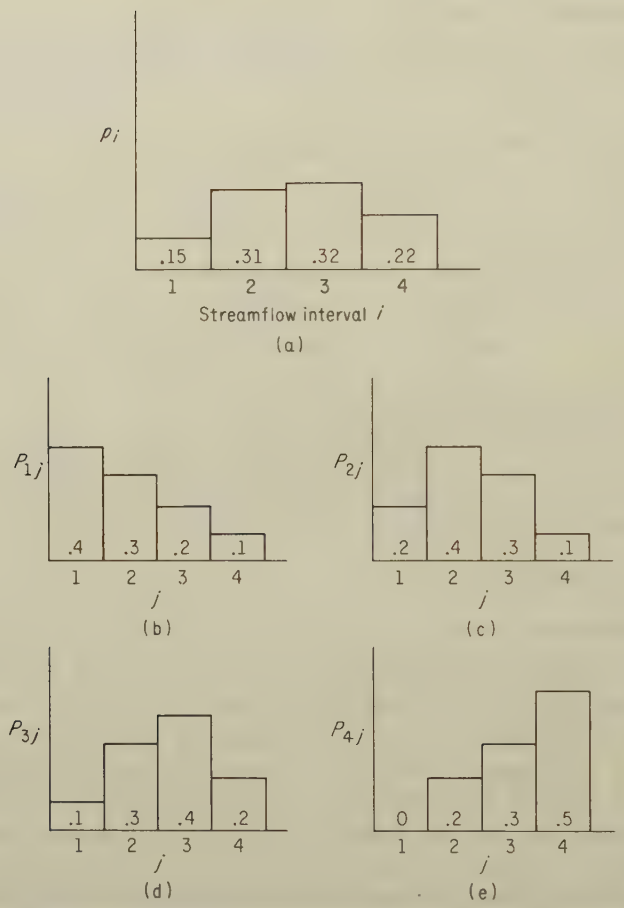


FIG. 5-4. Discrete probability distributions of annual streamflows: (a) unconditional distribution. Conditional distributions: (b) state 1 to state j; (c) state 2 to state j; (d) state 3 to state j; and (e) state 4 to state j.

Example 5-3 Figure 5-4 summarizes in discrete form the probability distribution of annual streamflows which vary from 60 to 100 KAF. Figure 5-4a shows the complete, unconditional probabilities of the historical record after determining the percent of past flows that fall into four discrete intervals. On analyzing the annual flow record further, it is found that serial correlation exists and can be summarized in the four conditional probability distributions shown in Figs. 5-4b through e. Develop the matrix of transition probabilities, and beginning in state 2 (flow interval, 70 to 80 KAF) show that the steady state probabilities are as given in Fig. 5-4a.

Solution:

1. The steady state probabilities from Fig. 5-4a are P_i = (0.15, 0.31, 0.32, 0.22), a probability vector. *Note:* $\Sigma p_i = 1$.

2. The transition probabilities, p_{ij}, expressing the probability of being in state i and transitioning to state j in the next year, are taken from Figs. 5-4b through e. Table 5-1 is the transition probability matrix, which can be expressed simply as P_{ij}. Note that the rows must sum to one.

TABLE 5-1 Transition Probability Matrix

		Flow state j in year $y+1$			
		1	2	3	4
Flow state i in year y	1	.4	.3	.2	.1
	2	.2	.4	.3	.1
	3	.1	.3	.4	.2
	4	.0	.2	.3	.5

3. Starting in state 2 for year y, the probability state vector is $P_i^{(y)} = (0, 1, 0, 0)$. For transition to year, $y + 1$, solve $P_i^{(y+1)} = P_i^{(y)} P_{ij}$. For example,

$$P_i^{(y+1)} = (0 \quad 1 \quad 0 \quad 0)\begin{pmatrix} .4 & .3 & .2 & .1 \\ .2 & .4 & .3 & .1 \\ .1 & .3 & .4 & .2 \\ 0 & .2 & .3 & .5 \end{pmatrix} = (.2 \quad .4 \quad .3 \quad .1)$$

Similarly, for $P_i^{(y+2)}$, the next transition

$$P_i^{(y+2)} = (.2 \quad .4 \quad .3 \quad .1)\, P_{ij} = (.19 \quad .33 \quad .31 \quad .17)$$

4. After nine transitions the steady-state vector is reproduced as shown in Table 5-2.

TABLE 5-2 Successive Streamflow Probability Vectors (After Loucks, Ref. 3)

Year y	Streamflow state probabilities			
	$P_1^{(y)}$	$P_2^{(y)}$	$P_3^{(y)}$	$P_4^{(y)}$
y	.000	1.000	.000	.000
$y + 1$	.200	.400	.300	.100
$y + 2$	.190	.330	.310	.170
$y + 3$	.173	.316	.312	.199
$y + 4$	.164	.311	.314	.211
$y + 5$	.159	.310	.315	.216
$y + 6$	.157	.309	.316	.218
$y + 7$	.156	.309	.316	.219
$y + 8$	.154	.309	.317	.220
$y + 9$	.15	.31	.32	.22
$y + 10$	.15	.31	.32	.22

Serial Correlation (Continuous Variables)

Sequential events in a time series represent a special case of joint distributions. They are not a true joint density because x_t and x_{t-1} are

actually from the same distribution, X. But the density $f(x_t, x_{t-1})$ has meaning and usefulness in hydrology because of serial correlation. Some natural events exhibit a time dependent trend, or persistence between successive occurrences, and their historical traces preserve the tendency. The analogy between joint and marginal distributions holds, however, because one variable can be "summed out."

$$f(x_t) = \int_{-\infty}^{+\infty} f(x_t, x_{t-1})\, dx_{t-1} \tag{5-49}$$

Moments of the synthetic joint distribution are, by the principle of stationarity, independent of time no matter what the averaging period is. The mean of x_t or x_{t-1} is μ_x. In defining second moments, a covariance function appears that is a function of the time interval t. Thus

$$\phi(t) = \int_{-\infty}^{\infty} (x_t - \mu)(x_{t-1} - \mu) f(x_t)\, dx_t \tag{5-50}$$

which for a stochastic time series is called the *autocovariance function*. When it is normalized by forming the ratio $\phi(t)/\phi(0) = \rho(t)$, where $\phi(0)$ is actually the variance of X, we have the definition for a parameter called the *autocorrelation function*. Autocorrelation is evident when $\rho(t)$ is plotted for all increments of t. Periodicity or lags at various time intervals are immediately apparent.

The typical model for an autoregressive process is an annual streamflow model of the first order, i.e., considering only lag-one serial correlation. If autocorrelation is significant, a first-order model is usually sufficient. Consider

$$(x_{i+1} - \mu) = c_1(x_i - \mu) + c_2 t \tag{5-51}$$

a transformation from year i to year $i + 1$. The mean is constant, and t is a standardized random variate. If lag-one autocorrelation did not exist, c_1, obviously would be zero, and c_2 would have to be σ of the X_{i+1} and, by stationarity, the whole process. The model would be simply $x = \bar{x} + Ks$, with $K = t$. But, if correlation exists, it must follow the joint considerations outlined. It can be shown that

$$c_1 = \rho(1) \tag{5-52}$$

and

$$c_2 = \sigma[1 - \rho^2(1)]^{1/2} \tag{5-53}$$

Thus the model is

$$(x_{i+1} - \mu) = \rho(1)[(x_i - \mu)] + \sigma t[(1 - \rho^2(1)]^{1/2} \tag{5-54}$$

Such a model is like a Markov process because there is a transition from one state to the succeeding state without concern for lags greater than one. Variations of this model appear in Chapter 10 on generating synthetic sequences.

REGIONAL FREQUENCY ANALYSIS

In dealing with single historical sequences of hydrologic variables, the predictive value of fitted distributions is limited because the records are generally short and the sampling errors correspondingly large.[4,6] Additional and more reliable information often can be obtained within a homogeneous region by correlating dependent hydrologic variables with other causative or physically related factors. In such ways, hydrologic characteristics within the region can be summarized, and estimates of statistical parameters can be derived from general regional relationships. Typical examples are (1) the prediction of rainfall depths and frequencies in ungauged or incompletely gauged areas from characteristics at well-gauged sites in the same area; (2) the prediction of peak flood flows from the correlation of observed flows on measurable quantities such as drainage area size, precipitation, and stream slope; and (3) the prediction of the standard deviation of annual streamflows throughout a large basin based on a host of physical and climatic factors. Recourse to multiple regression techniques is apparent in such cases.

Definition of the regional boundaries depends on the parameters or variables to be estimated. In some cases, significant generalizations can be made over large physiographic regions (mean annual precipitation in the United States, for example); in other situations the region might have to be limited to drainage basins of certain sizes within smaller physiographic zones (for example, the peak rates of runoff for areas up to one square mile in the Piedmont Plateau). Homogeneity tests have been proposed for testing the significance of regional delineations.[7,8]

5-5 Generalized Hydrologic Characteristics

The rainfall atlases published by the U.S. Weather Bureau represent a convenient method for summarizing mean annual precipitation and storm rainfall depths expected with various frequencies. The controlling features of physiographic regions and climatic patterns are immediately apparent in such summaries, and estimates of mean values are easily interpolated between the isohyetals. Similar summaries can be made for other hydrologic variables—for example, mean annual evapotranspiration or runoff. The longer the records, the more reliable are the data.

Less widely tabulated are measures of variability: the variance or coefficient of variation, for example, of rainfall and runoff. But regional distinctions are still clearly established. Fair et al.[9] report that C_v for mean annual rainfall varies from 0.1 in well-watered regions to 0.5 in arid regions, whereas C_v for mean annual runoff ranges from 0.15 to 0.75. Since runoff is usually considerably less than total rainfall in any region, the

greater variability in runoff indicates the effect of other factors and presents difficulty in determining rainfall-runoff relationships. Maps showing isolines of the standard deviation of regional flood flows have been prepared by the U. S. Army Corps of Engineers.[4]

As mentioned in Chapter 4, the large sampling errors in computing skewness from short records of hydrologic sequences suggest the use of regional values. Beard[4] proposed the following figures for the logarithms of flood flows in the absence of regional studies.

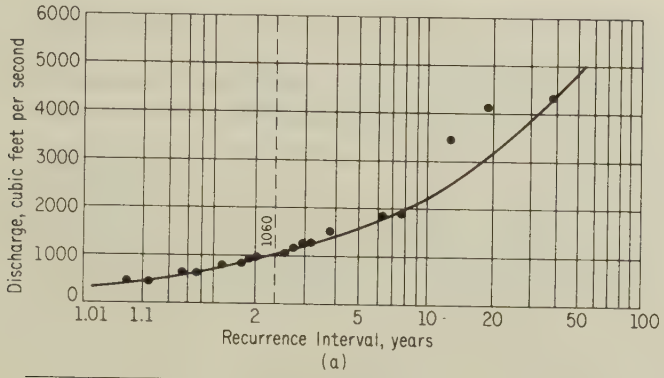
Duration	Skew coefficient
Instantaneous	0.00
1 day	−0.04
3 day	−0.12
10 day	−0.23
30 day	−0.32
90 day	−0.37
1 year	−0.40

Methods of "smoothing" and averaging regional values of skewness have been proposed also.[5,10]

Many techniques used in the past for generalizing regional characteristics did not rely on statistical considerations. The so-called "station-year" method of extending rainfall records has proved helpful but has questionable statistical validity, especially if applied to dependent series or to stations in nonhomogeneous areas. The method has been used to combine, say, two 25-yr records to obtain a single 50-yr sequence. In practice, the analyst may have to use imagination and ingenuity to summarize regional characteristics, while remaining aware of actual and theoretical considerations.

The *index-flood* method proposed by the U. S. Geological Survey is an example of summarizing regional characteristics successfully.[8,11] The method uses statistical data but combines them in graphical summaries. It can be supplemented and generally improved by using statistical methods —employing, for example, the regression techniques explained in the next article. The index method, as illustrated in Fig. 5-5, can be outlined as follows.

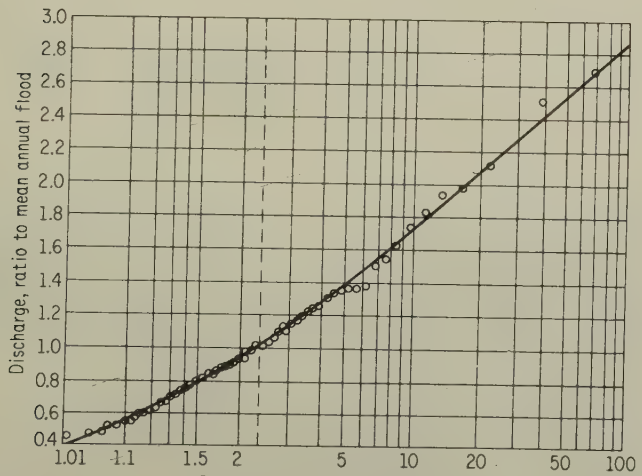
1. Prepare single-station, flood-frequency curves for each station within the homogeneous region (Fig. 5-5a).
2. Compute the ratio of flood discharges taken from the curves at various frequencies to the mean annual flood for the same station. (*Note:* The mean annual flood as defined by the USGS has a recurrence interval of 2.33 yr; in Eq. 4-49, for a frequency factor $K = 0$, $T = 2.33$ yr.)



(a)

Station	Recurrence intervals, in years					
	1.1	1.5	5	10	20	50
1	0.49	0.75	1.46	1.93	2.55	3.03
2	.57	.78	1.36	1.74	2.18	2.73
3	.54	.79	1.32	1.55	1.79	2.09
4	.57	.80	1.33	1.62	1.94	2.48
5	.56	.79	1.34	1.63	1.97	2.52
6	.49	.78	1.37	1.65	1.93	2.28
7	.50	.75	1.34	1.70	2.08	2.57
8	.45	.76	1.39	1.76	2.21	3.02
9	.69	.83	1.37	1.80	2.38	3.00
10	.62	.81	1.39	1.77	2.28	2.92
11	.58	.78	1.32	1.67	2.10	2.90
12	.63	.80	1.36	1.80	2.39	2.91
13	.47	.74	1.47	2.13	3.02	4.57
14	.46	.76	1.35	1.64	1.91	2.25
15	.54	.75	1.35	1.73	2.22	2.80
Median	0.54	0.78	1.36	1.73	2.18	2.80

(b)



(c)

FIG. 5-5. Index-flood method of regional flood-frequency analysis: (a) single-station flood-frequency curve, (b) ratios Q_t to $Q_{2.33}$, for 15 stations, and (c) regional flood-frequency curve.

3. Compile ratios for all stations and find the median ratio for each frequency (Fig. 5-5b).
4. Plot the median ratios against recurrence interval to produce a regional frequency curve (Fig. 5-5c).

Two statistical considerations involved are (1) a homogeneity test to justify definition of a region and (2) a method for extending short records to place all stations on the same base period. A somewhat similar technique has been developed by Potter for the Bureau of Public Roads.[12] It relies on graphical correlation of floods with physical and climatic variables, and is thus a technique that refers in part to the discussion in the next Article.

Regional Rainfall Characteristics

The variation of rainfall frequencies with duration was introduced in Chapter 4. Regression analysis can be used to fit intensity-duration-frequency curves similar to those shown in Fig. 4-12, and the constants interpreted as regional characteristics. Many formulas have been used in the past to fit these curves, but most of them are in a form with intensity (i) inversely proportional to duration (t). Steel[13] has used a model of the form $i = A/(t + B)$ to fit rainfall data throughout the United States. The constants A and B therefore serve as characteristic features of both the rainfall region and the frequency of occurrence in each area.

Example 5-4 Fit the following rainfall data to determine the 10-yr intensity-duration-frequency curve.

t = Duration, min:	5	10	15	30	60	120
i = Intensity, in./hr:	7.1	5.9	5.1	3.8	2.3	1.4
$1/i$	.14	.17	.20	.26	.43	.71

Solution:

1. A model of the form $i = A/(t + B)$ can be expressed in linear form as $(1/i) = (t/A) + (B/A)$.
2. The regression of $(1/i)$ vs. t yields $(1/i) = 0.005t + 0.12$, from which $A = 200$ and $B = 24$.
3. Thus the rainfall formula is $i = 200/(t + 24)$. The correlation coefficient is 0.997 and the fit when plotted on the 10-year curve of Fig. 4-12 is seen to be very good.

Maximum average rainfall depths have been published by the U. S. Weather Bureau[14] for durations between 30 min and 24 hr and for frequencies between 1 yr and 100 yr. Depth-duration-frequency curves can be constructed for any location by plotting successive values from the various rainfall maps, preferably on logarithmic paper to facilitate fitting flatter curves. Correction factors are given to permit estimates

of depths for durations less than 30 min. Special attention must be given to the extrapolation of point rainfall data to account for spatial variations. Duration-frequency analyses based on maximum point gauge data exhibit the average area-depth relationship shown in Fig. 5-6.

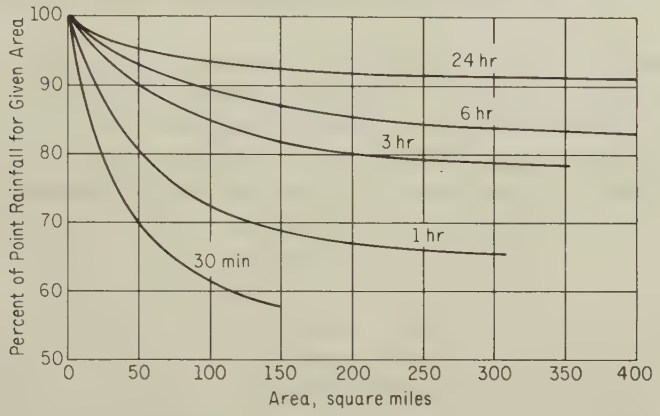


FIG. 5-6. Area-depth curves for use with duration-frequency values. (U. S. Weather Bureau.)

The accuracy of area rainfall data depends heavily upon the density and location of gauges throughout the area considered. Simple averaging of the accumulation in all gauges gives no consideration to the effective area around each gauge or to the storm pattern. Two methods are available in calculating the weighted average of gauge records, the Thiessen polygon method and the isohyetal method. The Thiessen method assumes a linear variation of rainfall between each pair of gauges. Perpendicular

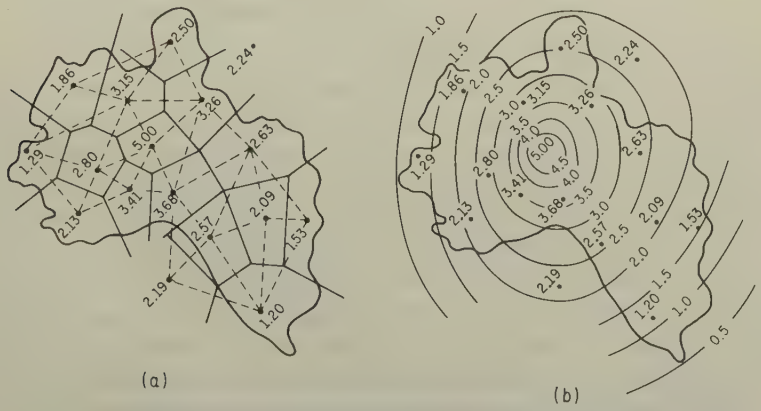


FIG. 5-7. Methods of determining rainfall averages: (a) Thiessen network (24-hr total; average basin precipitation = 2.54 in.); and (b) isohyetal map (24-hr total; average basin precipitation = 2.50 in.).

bisectors of the connecting lines form polygons around each gauge (or partial polygons within the area boundary). If a sufficient number of gauges are available to construct contours of rainfall depth (isohyets), the weighting process can be carried out by using the average depth between isohyets and the area included between the isohyets and the area boundaries. Figure 5-7 shows both schemes.

An example of the effect of gauge location and density is shown in Figs. 5-8a and b. Figure 5-8a shows the increase in variability between Thiessen-weighted storm rainfalls and rainfall at a single gauge as the distance of single gauges from the watershed center increases. Figure 5-8b shows the effect of gauge density and total area on the standard error of the mean. Complete studies of precipitation patterns over large areas

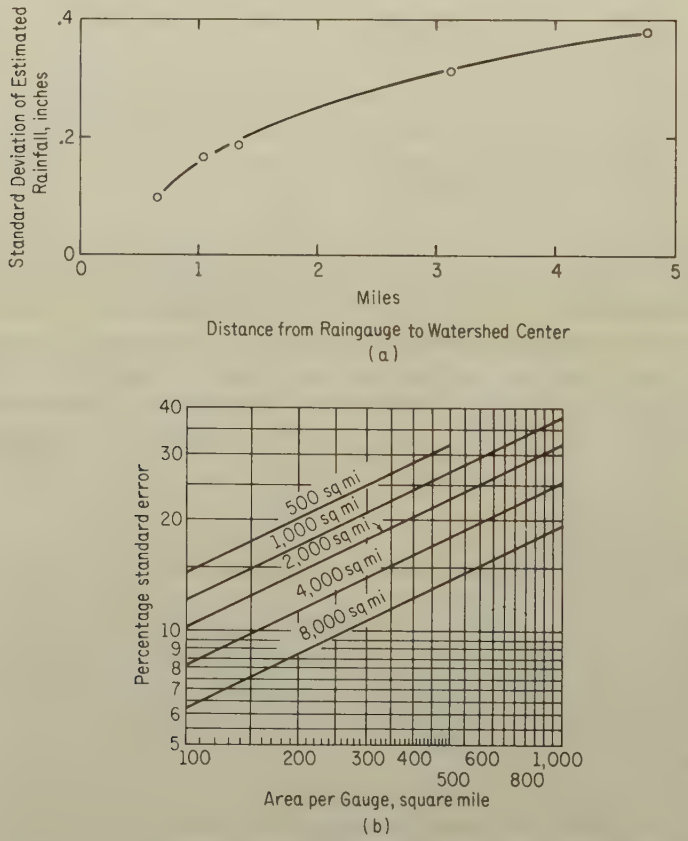


FIG. 5-8. (a) Relation between standard deviation of watershed and point rainfall and distance of point from center of watershed. (After Riesel, Texas, Ref. 16.) (b) Standard error of average precipitation as a function of network density and drainage area for the Muskingum Basin. (U.S. Weather Bureau.)

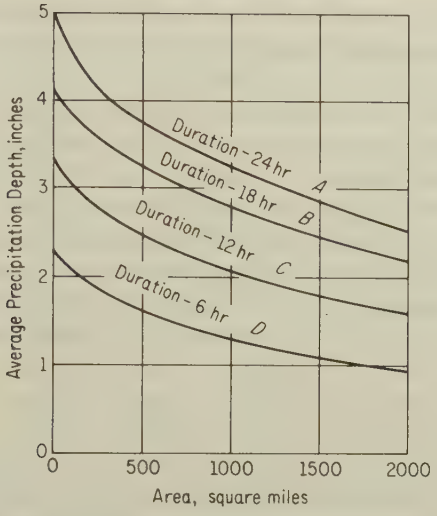


FIG. 5-9. Depth-area-duration curves for 24-hr storm of Fig. 5-7. (Ref. 15.)

require detailed analysis of depth-area-duration (DAD) data which depend upon the mass curves of accumulation from a network of gauges. The method is described in detail in other references.[15,17] Figure 5-9 depicts the depth-area relationship for the 24-hr storm shown in Fig. 5-7. It also required observations taken at various durations and the successive determination of average depths by the isohyetal method.

5-6 Multiple-Regression Techniques

The regression techniques developed in Sec. 5-3 provide a powerful means for performing regional frequency analyses. These techniques are used to identify the mathematical dependence between observed values of physically related variables and thus can account for the additional information contained in correlated sequences of events. Sampling errors are reduced and the reliability of estimates is improved. In addition to predicting the mean or expected value of a hydrologic variable such as rainfall, runoff, or peak flows, the technique can be used to predict the expected value of other statistical parameters—for example, standard deviation, skewness, or autocorrelation. The correlation is determined between the desired statistical parameter as dependent variable, and the appropriate physical and climatic variables within the basin or region as the independent variables. The procedures are significantly better than using relatively short historical sequences and point-frequency analysis.[4,5,11]

Not only does the method reduce the inherently large sampling errors, but it furnishes a means to estimate parameters at ungauged locations.

There are limitations to the techniques of Sec. 5-3. First, the analyst assumes the form of the model which can express only linear, or logarithmically linear, dependence. Second, he selects the independent variables to be included in the regression analysis. And, third, the theory assumes that the independent variables are indeed independent and are observed or determined without error. Advanced statistical methods which are beyond the scope of this text offer means to overcome some of these limitations, but in practice it may be impossible to satisfy any of them. Therefore, care must be exercised in selecting the model and in interpreting results.

Accidental or casual correlation may exist between variables that are not functionally correlated. For this reason, correlation should be determined between hydrologic variables only when a physical relationship can be presumed. Because of the natural dependence between many factors treated as independent variables in hydrologic studies, the correlation between the dependent variable and each of the independent variable is different from the relative effect of the same independent variables when analyzed together in a multivariate model. One way to guard against this effect is by screening the variables initially by graphical methods. Another is to examine the results of the final regression equation to determine physical relevance.

Alternatively, regression techniques themselves aid in screening significant variables. When electronic computation is available, a procedure can be followed in which successive independent variables are added to the multiple-regression model, and the relative effect of each is judged by the increase in the multiple-correlation coefficient. Although statistical tests can be employed to judge significance, it is useful otherwise to specify that any variable remain in the regression equation if it contributes or explains, say, 1 percent or 5 percent of the total variance, or R^2. A frequently used method is to compute the *partial-correlation coefficients* for each variable. This statistic represents the relative decrease in the variance remaining $(1 - R^2)$ by addition of the variable in question. If the variance remaining with the variable included in the regression is $(1 - R^2) = D^2$ and the variance remaining after removal is $(1 - R'^2) = D'^2$, then the partial-regression correlation is $(D'^2 - D^2)/D'^2$.

At any modern electronic computer facility, a multiple-regression program is normally included in the standard library routines. Most are extremely flexible, requiring minimal instructions and input data other than raw data. Special manipulations can effect an interchange of dependent and independent variables, bring one variable at a time into the regression equation, rearrange the independent variables in order of significance, and perform various statistical tests.

Extending Hydrologic Records

Regression frequently extends short records based on significant correlation between the station of short record and a nearby station with a longer record. In Example 5-2, if the Jackson River records were complete from 1941 to date but the Cowpasture records were incomplete after 1952, the cross-correlation could be used to estimate the missing years by solving the regression equation for Y from 1953 on using the X flows as observed. The reliability of such methods depends on the correlation coefficient and the length of the concurrent records. If the concurrent record is too short or the correlation weak, the standard error of the parameter to be estimated can be increased and nothing is gained. The limiting value of cross-correlation for estimating means is approximately $\rho = 1/\sqrt{n}$, where n is length of concurrent record.[13] Thus any correlation above 0.3 would improve the Cowpasture records. Estimates of other parameters with larger standard errors require higher cross-correlation for significant improvement. Extending or filling in deficient records often is necessary for regional studies in which every record used should be adjusted to the same length. Synthetic, stochastic methods of augmenting records are discussed in Chapter 10.

Predicting Regionalized Hydrologic Variables

Cruff and Rantz[11] studied various methods of regional flood analysis and found the multiple-regression technique a better predicter than either the index-flood method or the fitting of theoretical frequency distributions to individual historical records. They first used regression techniques to extend all records to a base length. Next they extrapolated by various methods to estimate the 50- and 100-yr flood events and with multiple correlation examined several dependent variables including: drainage area size A; basin-shape factor (ratio of diameter of a circle size A to length of basin measured parallel to the main channel) S_h; channel shape S; annual precipitation P; and others. They found only A and S_h to be significant, which resulted in prediction equations of the form $Q_t = cA^a S_h^b$. These equations were superior to those of the other techniques. The multiple-correlation coefficient was as high as 0.954. It is interesting that regression techniques were employed in still a third way—that is, to estimate regional values of the mean and standard deviation after adjusting record length. Example 5-4 illustrates application of regression analysis to regionalize the standard deviation of annual maximum flow logarithms as a function of the drainage area size and the number of rainy days each year.

Example 5-5 In the following exhibit given by Beard,[4] the regional correlation is sought of the standard deviation of flow logarithms with the logarithms

TABLE 5-3 Logarithmic Data for 50 Gauging Stations

$X_1 = 1 + \log s$		$X_2 = \log$ D.A.		$X_3 = \log$ no. rainy days per year			
Sta. no. (1)	X_2 (2)	X_3 (3)	X_1 (4)	Sta. no. (5)	X_2 (6)	X_3 (7)	X_1 (8)
1	1.61	2.11	0.29	33	1.94	1.87	0.20
2	2.89	2.12	0.18	34	2.73	1.36	0.58
3	4.38	2.11	0.17	35	3.63	1.81	0.64
4	3.20	2.04	0.44	36	1.91	1.58	0.37
5	3.92	2.07	0.38	37	2.26	1.48	0.27
6	1.61	2.04	0.37	38	2.97	1.89	0.54
7	3.21	2.09	0.30	39	0.70	1.32	0.63
8	3.65	1.99	0.35	40	0.30	1.54	0.78
9	3.23	2.15	0.16	41	3.38	1.62	0.46
10	4.33	2.08	0.11	42	2.87	2.03	0.44
11	1.60	2.09	0.32	43	2.42	2.26	0.24
12	2.82	2.00	0.34	44	4.53	1.93	−0.03
13	2.40	2.00	0.25	45	3.04	1.78	0.30
14	3.69	2.09	0.43	46	4.13	2.00	0.17
15	2.18	2.19	0.27	47	1.49	2.01	0.14
16	2.09	2.17	0.25	48	5.37	1.95	0.10
17	4.48	1.91	0.52	49	1.36	2.11	0.27
18	4.95	1.95	0.18	50	2.31	2.23	0.18
19	2.21	1.97	0.39				
20	3.41	2.08	0.40	ΣX	147.55	96.24	17.89
21	4.82	1.88	0.25	$\bar{X}$	2.951	1.925	0.358
22	1.78	1.93	0.23	$\Sigma X X_2$	503.7779	285.5627	51.1527
23	4.39	1.74	0.54	$\Sigma X \Sigma X_2 / n$	435.4200	284.0042	52.7934
24	3.23	2.01	0.51	$\Sigma x x_2$	68.3579	1.5585	−1.6407
25	3.58	2.04	0.45				
26	1.64	1.78	0.63	$\Sigma X X_3$		187.5912	33.2598
27	4.58	1.76	0.45	$\Sigma X \Sigma X_3 / n$		185.2428	34.4347
28	3.26	1.93	0.59	$\Sigma x x_3$	1.5585	2.3484	−1.1749
29	4.29	1.81	0.46				
30	1.23	1.89	0.32	$\Sigma X X_1$			8.1635
31	3.44	1.48	0.96	$\Sigma X \Sigma X_1 / n$			6.4010
32	2.11	1.97	0.12	$\Sigma x x_1$	−1.6407	−1.1749	1.7625

Note: $x = (X - \bar{X})$.

of the drainage area size and the number of rainy days per year. X_1 is set equal to $(1 + \log s)$ to avoid negative values. Find the regression equation and the multiple correlation coefficient.

Solution:

1. From Eqs. 5-41, 5-42, and 5-43, the parameters are

$$\alpha = 1.34; \quad \beta_1 = -0.013; \quad \beta_2 = -0.49$$

and the regression equation is

$$X_1 = 1.34 - 0.013X_2 - 0.49X_3$$

or

$$\log S = 0.34 - 0.013 \log (\text{DA}) - 0.49 \log (\text{days})$$

2. The multiple correlation coefficient from Eqs. 5-44 and 5-45 is $R = 0.56$.

References

1. Benjamin, J. R., and C. Cornell, *Probability, Statistics and Decision For Civil Engineers*. New York: McGraw-Hill Book Company, 1959.

2. Feller, W., *An Introduction to Probability Theory*. New York: John Wiley & Sons, Vol. I, 1957, and Vol. II, 1966.

3. Loucks, D. P., "Stochastic Methods For Analyzing River Basin Systems," Tech. Report No. 16, Cornell University Resources and Marine Sciences Center, 1969.

4. Beard, L. R., *Statistical Methods In Hydrology*, Civil Works Investigations; Sacramento District, U. S. Army Corps of Engineers, 1962.

5. Benson, M. A., and N. C. Matalas, "Synthetic Hydrology Based on Regional Statistical Parameters," *Water Resources Research*, Vol. 3, No. 4 (1967).

6. Matalas, M. C., "Mathematical Assessment of Synthetic Hydrology," *Water Resources Research*, Vol. 3, No. 4 (1967).

7. Chow, Ven T., "Statistical and Probability Analysis of Hydrologic Data," Section 8-I, V. T. Chow (ed.), *Handbook of Applied Hydrology*. New York: McGraw-Hill Book Company, 1964.

8. Dalrymple, T., "Flood-Frequency Analysis," *Manual of Hydrology*, Part 3, U. S. Geological Survey Water-Supply Paper 1543-A. Washington, D. C.: Government Printing Office, 1960.

9. Fair, G. M., J. C. Geyer, and D. A. Okun, *Water and Waste Water Engineering*. New York: John Wiley & Sons, Inc., 1966.

10. "Monthly Stream Simulation," Hydrologic Engineering Center, Computer Program 23-C-L267, Sacramento District, Corps of Engineers, July 1967.

11. Cruff, R. W., and S. E. Rantz, "A Comparison of Methods Used In Flood-Frequency Studies For Coastal Basins in California," *Flood Hydrology*, U. S. Geological Survey Water-Supply Paper 1580. Washington, D. C.: Government Printing Office, 1965.

12. Potter, W. D., "Peak Rates of Runoff from Small Watersheds," Hydraulic Design Series No. 2, Bureau of Public Roads. Washington, D. C.: Government Printing Office, April 1961.

13. Steel, E. W., *Water Supply and Sewerage*, 4th Ed. New York: McGraw-Hill Book Company, 1960.

14. Hershfield, D. M., Rainfall Frequency Atlas of the United States, Technical Paper No. 40, U. S. Weather Bureau, 1961.

15. *Hydrology Handbook*, A. S. C. E. Manual of Practice, No. 28, 1949.

16. Knisel, W. G., Jr., and R. W. Baird, in *ARS Precipitation Facilities and Related Studies*. Washington, D. C.: U. S. Department of Agriculture, Agricultural Research Service, 1971, Chapter 14.

17. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, *Applied Hydrology*. New York: McGraw-Hill Book Company, 1949.

18. Fiering, M. D., "Information Analysis," in G. M. Fair, J. C. Geyer, and D. A. Okun, *Water Supply and Waste Water Removal*. New York: John Wiley & Sons, Inc., 1966, Chapter 14.

Problems

5-1. Observations for the past 10 yr of withdrawals and estimated recharge of an artesian aquifer are given below. Find the means, variances, standard deviations, covariance, and correlation coefficient.

Measured discharge, 1,000 acre-ft	Estimated recharge, 1,000 acre-ft
12.2	12.0
10.4	9.8
10.6	11.0
12.6	13.2
14.2	14.6
13.0	14.0
14.0	14.0
12.0	12.4
10.4	10.4
11.4	11.6

5-2. Fit a regression equation to the data in Prob. 5-1, treating discharge as the dependent variable. Compute the standard error of estimate. Estimate the expected discharge when recharge is 13 KAF. What would be the estimate of discharge if no information were available on recharge? What is the relative improvement provided by the regression estimate?

5-3. Prepare a computer program for simple, two-variable, linear regression. The program should read in N pairs of observations, Y and X; compute the means, variances, and standard deviations of both Y and X; find the regression constants, the standard error of estimate, and the correlation coefficient. Verify with the data in Prob. 5-1.

5-4. From the following observations of variation of mean annual rainfall with altitude of the gauge, determine a linear prediction equation for the catchment. What is the correlation between rainfall and altitude?

Gauge no.	Mean annual rainfall, in.	Altitude of gauge, 1,000 ft.
1	22	4.2
2	28	4.4
3	25	4.5
4	31	5.4
5	32	5.6
6	37	5.6
7	36	5.8
8	35	6.0
9	36	6.6
10	46	6.6
11	41	6.8
12	41	7.0

5-5. Estimate the expected rainfall in Prob. 5-4 for a gauge to be installed at

an altitude of 5,500 ft. What are the approximate 95 percent confidence limits of the estimate?

5-6. In Example 5-3, verify the eventual generation of the steady-state probabilities by beginning in state 3 and transitioning to succeeding states according to the given transition probability matrix.

5-7. Using the information in Example 5-3 and Prob. 5-6, compile a probability matrix which shows the conditional probability after five transitions, i.e., the probabilities of being in state i at period y and state j at period $y + 5$.

5-8. Given the following transition probability matrix, find the steady-state probability vector:

		State j in period $y + 1$		
		1	2	3
State i in period y	1	0.5	0.3	0.2
	2	0.3	0.6	0.1
	3	0.1	0.4	0.5

5-9. From the partial record of streamflows in Plum Creek near Meadville, Neb., find the monthly serial correlation coefficient between October and November. Repeat for April and May. Comment on the hydrologic reasons for any difference in the correlation between the two pairs of data.

Streamflow Data, Plum Creek at Meadville, Neb.
(100 acre-ft)

Year	Oct.	Nov.	Apr.	May	Annual total
1949	49.9	47.5	97.5	63.7	710.5
1950	48.5	47.9	76.8	106.5	702.8
1951	55.8	52.2	74.1	88.1	897.9
1952	72.2	73.2	116.4	163.7	1012.0
1953	55.0	52.0	71.0	115.2	810.6
1954	71.1	55.4	67.9	92.7	795.7
1955	56.3	58.4	62.8	61.9	730.0
1956	56.1	52.4	59.6	64.0	664.8
1957	55.7	58.6	72.2	102.3	761.6
1958	57.8	59.6	161.7	77.0	849.3
1959	60.2	57.8	69.9	63.6	707.5
1960	54.7	56.8	89.0	101.5	781.9
1961	57.9	56.1	55.7	67.6	689.2
1962	55.5	54.1	59.3	101.0	1010.0
1963	62.3	61.3	74.8	76.3	882.7
1964	60.5	57.9	89.5	90.0	792.1
1965	52.3	55.2	62.1	63.9	705.6
1966	58.8	54.4	71.1	63.8	731.9
1967	58.0	55.3	54.7	64.0	788.3

5-10. For the total annual flow given in Prob. 5-9, determine the serial correlation between successive years. Explain how you would use this information (a) to predict the expected flow in any future year when the preceding year's flow is 75,000 acre-ft, and (b) to add a random component to the estimate in (a) for a simulation study.

5-11. The results of a multiple regression analysis of over 200 flood records in Virginia led to the following regional flood-frequency equations:

$$
\begin{aligned}
Q_{1.2\text{-yr}} &= 9.13 \quad A^{.909} \quad S^{.293} \\
Q_{2.33\text{-yr}} &= 20.8 \quad A^{.861} \quad S^{.309} \\
Q_{5\text{-yr}} &= 38.1 \quad A^{.830} \quad S^{.300} \\
Q_{10\text{-yr}} &= 63.0 \quad A^{.802} \quad S^{.283} \\
Q_{25\text{-yr}} &= 104 \quad A^{.779} \quad S^{.266} \\
Q_{50\text{-yr}} &= 118 \quad A^{.795} \quad S^{.279}
\end{aligned}
$$

where flood discharge for the given frequency is in cfs, A is the drainage area in square miles, and S is the channel slope in feet per mile (measured between the points that are 10 and 85 percent of total river miles upstream of the gauging station to the drainage divide). Devise a satisfactory method for graphically portraying these regional flood-frequency relations (*Note:* There are four factors, Q, T, A, and S.)

5-12. Using the regression equations in Prob. 5-11, find the predicted floods for the North Fork, Shenandoah River at Cootes Store. Drainage area = 215 sq mi and channel slope = 44.3 ft per mi.

5-13. Compare the predictions from the regression equations in Prob. 5-11 with the values estimated by the frequency analysis in Example 4-6. Drainage area = 487 sq mi and channel slope = 21.1 ft per mi.

5-14. Referring to Fig. 2-8b, compare the average storm rainfall over the city of Baltimore on Sept. 10, 1957, computed by the isohyetal method with the simple average of total accumulation at the rain gauges within the city. Neglect the area to the south of the 1.0 in. isohyet.

5-15. Fit the formula $i = A/(t + B)$ to the data derived in Prob. 4-20 for the 5-yr intensity-duration-frequency curve.

6

HYDROLOGIC AND HYDRAULIC ROUTING

Flood forecasting, reservoir design, watershed analysis, and comprehensive water-resources projects generally utilize some form of routing technique. Routing can be used to predict the temporal and spatial variations of a flood wave as it traverses a river system, or can be employed to predict the outflow hydrograph from a watershed subjected to a known amount of precipitation. Routing is defined as "a technique used to compute the effect of system storage and system dynamics on the shape and movement of the flood wave." The techniques may be classified into two catagories—*hydrologic routing* and *hydraulic routing.*

Hydrologic routing employs the equation of continuity and a relationship describing storage in the system. Additionally, an assumption regarding functional relationship of the discharge must also be made when used in reservoir analysis. Hydraulic routing on the other hand uses both the equation of continuity and the equation of motion, customarily the momentum equation. This particular form of routing utilizes presumptions normally resorted to in deriving the unsteady flow equations. It more adequately describes the dynamics of flow than does the hydrologic-routing technique, and is treated in the latter portion of this chapter.

Applications of hydrologic-routing skills to problems of flood prediction or effect of urbanization are numerous. Most flood-warning systems instituted by NOAA and the U.S. Army Corps of Engineers incorporate some form of this technique to predict flood stages in advance of a severe storm. It is the method most frequently used to size spillways for small, intermediate, and large dams. Additionally, synthesis of runoff hydrographs from gauged and ungauged watersheds is possible by use of basic assumptions inherent in this approach. Three major areas of pertinence will be described in detail in the following material.

6-1 Hydrologic River Routing

First reference to real application of routing a flood hydrograph from one station to another was by Graeff in 1833.[1] The technique employed by Graeff was based on use of a rating curve and celerity of the flood wave. Existing hydrologic river routing techniques, however, generally utilize some form of the Muskingum Method.[2] All hydrologic-routing methods, however, are founded upon the equation of continuity

$$I - O = \frac{dS}{dt} \tag{6-1}$$

where I = inflow to reach, cfs
O = outflow from reach, cfs
dS/dt = rate of change of storage within reach

Muskingum Method

Storage in a stable river reach can be expected to depend primarily on the discharge into and out of a reach, and upon hydraulic characteristics of the channel section. The storage within the reach at a given time can be expressed as[3]

$$S = \frac{b}{a}[xI^{m/n} + (1 - x)O^{m/n}] \tag{6-2}$$

Constants a and n reflect the stage discharge characteristics of control sections at each end of the reach, and b and m mirror stage volume characteristics of the section. Factor x defines the relative weights given to inflow and outflow within the reach.

The Muskingum method assumes $m/n = 1$ and lets $b/a = K$, resulting in

$$S = K[xI + (1 - x)O] \tag{6-3}$$

where K is referred to as the storage time constant for the reach and x is the weighting factor which varies between 0 and 1.0 for a given river section. Application of this equation has shown that K is usually reasonably close to the travel time within the reach and x will average about 0.2.

Behavior of the flood wave due to changes in value of the weighting factor x is readily apparent from examination of Fig. 6-1. The resulting downstream flood wave is commonly described by the amount of translation—that is, time lag—and by the amount of attenuation, or reduction in peak discharge. As can be noted from Fig. 6-1, the value $x = 1.0$ results in a pure translation of the flood wave.

Application of Eq. 6-3 to a river reach is a straightforward procedure. Necessary information required to define K and x for a given river section are the inflow and outflow hydrographs to the section of the river under

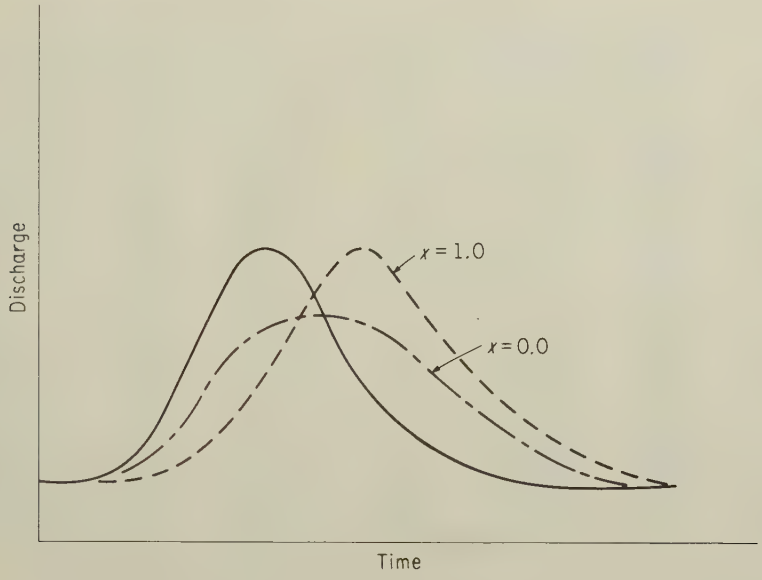


FIG. 6-1. Effect of weighting factor.

investigation. The volume of storage for each given time interval is simply the difference between cumulative inflow and outflow. This quantity is plotted versus the computed storage from Eq. 6-3 for various assumed values of x. The x factor which produces the narrowest loop is chosen as a representative figure for the reach in question. The value for K equals the reciprocal of the slope of the straight line through the narrowest loop, since

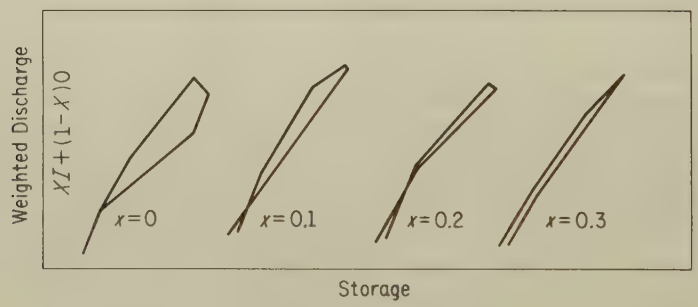
$$K = \frac{S}{xI + (1 - x)O} \tag{6-4}$$

Example 6-1 Given inflow and outflow hydrographs on the Muckwamp River, determine K and x for the river reach.

Date	Total inflow, cfs	Total outflow, cfs	Storage volume, 1,000 cfs-days	Weighted discharge $x = 0.1$	$x = 0.2$	$x = 0.3$
3-16	5,870	4,180	1.7	4,350	4,520	4,690
17	9,310	6,970	4.0	7,200	7,440	7,670
18	12,900	7,560	9.4	8,090	8,630	9,160
19	20,500	14,200	15.7	14,800	15,500	16,100
20	21,000	18,300	18.4	18,600	18,800	19,100
21	23,400	18,500	23.3	19,000	19,500	20,000
22	32,500	21,300	34.5	22,400	23,500	24,700
23	55,400	29,300	60.6	31,900	34,500	37,100

(*Continued on next page*)

Date	Total inflow, cfs	Total outflow, cfs	Storage volume, 1,000 cfs-days	Weighted discharge $x = 0.1$	$x = 0.2$	$x = 0.3$
24	62,700	39,700	83.6	42,000	44,300	46,600
25	52,600	48,700	97.5	49,100	49,500	50,000
3-26	43,200	53,300	87.4	52,300	51,300	50,300
27	25,200	48,700	73.9	46,400	44,000	41,700
28	22,800	37,100	59.6	35,700	34,200	32,800
29	41,200	35,800	65.0	36,300	36,900	37,400
30	50,400	35,800	79.6	37,300	38,700	40,200
31	45,300	35,800	89.1	36,800	37,700	38,600
4- 1	38,800	42,700	85.2	42,300	41,900	41,500
2	27,000	44,100	68.0	42,400	40,800	39,000
3	16,200	35,400	48.9	33,500	31,600	29,600
4	12,400	25,200	36.1	23,900	22,600	21,400
5	10,200	16,400	29.9	15,800	15,200	14,500
6	8,080	11,500	26.5	11,200	10,800	10,500
7	6,010	9,380	23.1	9,040	8,710	8,370
8	5,050	7,860	20.3	7,300	7,300	7,020



Solution:

$$x = 0.3; \quad K = 20/10 = 2.0 \text{ days}$$

The computations shown in Example 6-1 demonstrate an application of the Muskingum River Routing technique. Once K and x have been ascertained for a reach, the problem of future flood forecasting for the particular section becomes merely a matter of calculation, since values of outflow can readily be computed from a synthesized inflow hydrograph and values of K and x for the reach.

The computational procedure is shown by expressing Eq. 6-1 in finite-difference form:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t} \tag{6-5}$$

where Δt = time interval for routing

1, 2 = subscripts denoting beginning and ending of time interval

Storage change in the river reach over the routing time period is

$$S_2 - S_1 = K[x(I_2 - I_1) + (1 - x)(O_2 - O_1)] \tag{6-6}$$

and combining with Eq. 6-3 results in

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \tag{6-7}$$

in which

$$C_0 = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

Inherent in this procedure is the postulation that the water surface in the reach is a uniform unbroken surface profile between upstream and downstream ends of the section. Additionally, it is presupposed that K and x are constant throughout the range in stage of investigation. If significant departures from these restrictions are present, it may be necessary to work with shorter reaches of the river, or to employ a more sophisticated approach to the problem. A demonstration of this method is given in Example 6-2.

Example 6-2 Perform the flood routing for a reach of river given $x = 0.2$ and $K = 2$ days. The inflow hydrograph is shown in col. 1.

Date	Inflow (1)	$C_0 I_2$ (2)	$C_1 I_1$ (3)	$C_2 O_1$ (4)	Computed outflow (5)
16	4,260	—	—	—	4,260
17	7,646	364	1,823	2,232	4,419
18	11,167	532	3,272	2,315	6,119
19	16,730	798	4,779	3,206	8,783
20	21,590	1,029	7,160	4,602	12,791
21	20,950	999	9,240	6,702	16,941
22	26,570	1,267	8,966	8,877	19,110
23	46,000	2,194	11,371	10,013	23,578
24	59,960	2,860	19,688	12,355	34,903
25	57,740	2,754	25,662	18,289	46,705
26	47,890	2,284	24,712	24,473	51,469
27	34,460	1,643	20,496	26,970	49,109
28	21,660	1,033	14,748	25,733	41,514
29	34,680	1,654	9,270	21,753	32,677
30	45,180	2,155	14,843	17,122	34,120
31	49,140	2,343	19,337	17,879	39,559
1	41,290	1,969	21,031	20,729	43,729
2	33,830	1,613	17,672	22,914	42,199
3	20,510	978	14,479	22,112	37,569
4	14,720	702	8,778	19,686	29,166

(*Continued on next page*)

Date	Inflow (1)	C_0I_2 (2)	C_1I_1 (3)	C_2O_1 (4)	Computed outflow (5)
5	11,436	545	6,300	15,283	22,128
6	9,294	443	4,894	11,595	16,932
7	7,831	373	3,977	8,872	13,222
8	6,228	297	3,351	6,928	10,576
9	6,083	290	2,665	5,542	8,497

$x = 0.2$, $K = 2$ days, $C_0 = 0.0477$, $C_1 = 0.428$, $C_2 = 0.524$.

Graphical Integration Method

Many different graphical methods are used to solve the equation of continuity. A method shown here is rapid and produces a satisfactory outflow discharge hydrograph.[4] It deals with two factors K and L: L represents lag time and K is a coefficient which when multiplied by the change in outflow with respect to time gives the difference in storage with respect to time (or the storage time constant of Eq. 6-3 when $x = 0$).

$$K \frac{dO}{dt} = \frac{dS}{dt} \tag{6-8}$$

or, substituting Eq. 6-1 results in

$$\frac{I - O}{K} = \frac{dO}{dt} \tag{6-9}$$

where the difference between inflow and the outflow divided by K represents the change in outflow with respect to time. Graphical determination of K and L values for a particular river reach permits subsequent construction of outflow hydrographs from given or synthesized inflow hydrographs.

L, the time difference between points A and B, is obtained from a previous flood wave of record as shown in Fig. 6-2a. The procedure to get K for a known flood wave is as follows.

1. Lag the inflow hydrograph L hours as in Fig. 6-2b.
2. Select point A on the rising side of the inflow hydrograph Fig. 6-2c.
3. Drop a vertical line downward through point A to a point B on the outflow hydrograph.
4. Draw a horizontal line through point A to the right until it intersects the outflow hydrograph.
5. From point B construct a line tangent to the outflow hydrograph upward until it intersects the horizontal line at a point C.
6. The distance in hours from A to C is K for the value of outflow at point B.

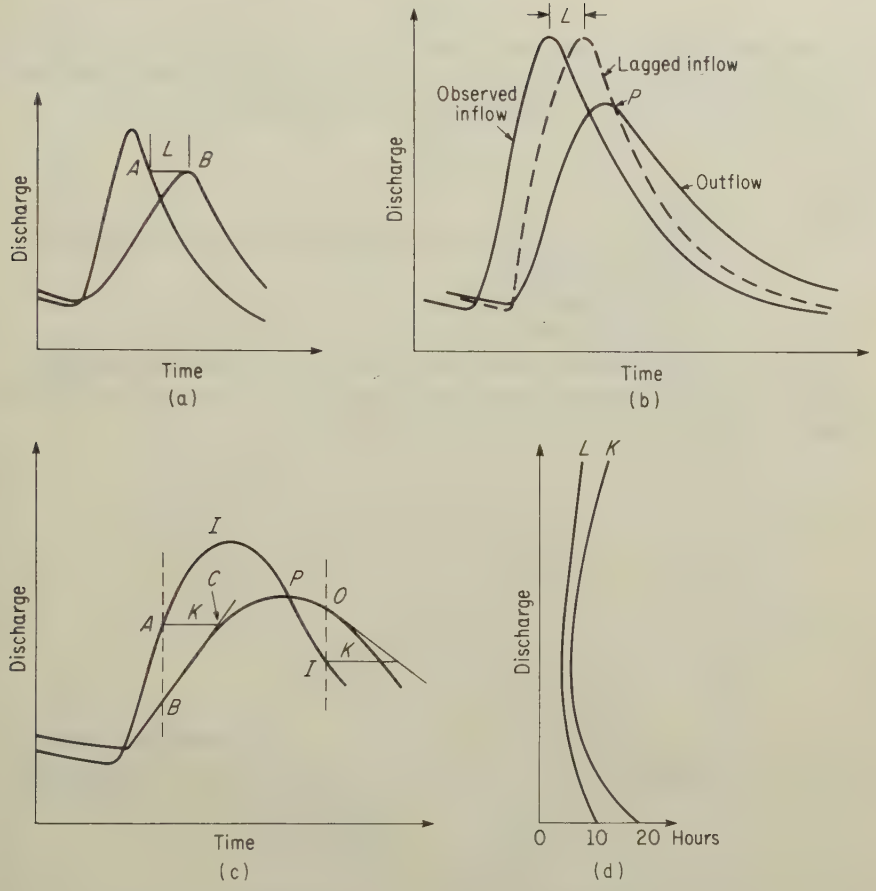


FIG. 6-2. Determining K and L. (U. S. Weather Bureau, NOAA.)

7. Compute K for several points along the outflow hydrograph and plot as shown in Fig. 6-2d.

Application of known K and L values to an inflow hydrograph can now be used to forecast an outflow hydrograph. Steps in the procedure are:

1. Lag the inflow L hours, Fig. 6-3a.
2. Plot a point K hours from I_2 with the same discharge value.
3. A straightedge from this point to O_1 gives the slope at O_2.
4. Draw a short segment of the outflow hydrograph at time O_2.
5. The lagged inflow and graphically derived outflow for an entire storm are shown in Fig. 6-3b.

6-2 Hydrologic Reservoir Routing

Reservoir routing can be considered a special case of the Muskingum method. When $x = 0$, Eq. 6-3 simplifies to

$$S = KO \tag{6-10}$$

Thus storage behind a level pool reservoir can be considered to depend only upon outflow from the reservoir spillway. For an uncontrolled outlet, that is, without gates, discharge from a spillway is generally in the form

$$O = CLH^x \tag{6-11}$$

where O = outflow, cfs
C = coefficient of discharge for weir or section used for spillway
L = length of spillway, ft
H = head over spillway crest, ft
x = exponent, generally $3/2$

With a given inflow hydrograph, the equation of continuity, discharge relationship, and a storage elevation curve for the reservoir, a routing of the inflow hydrograph can be readily obtained.

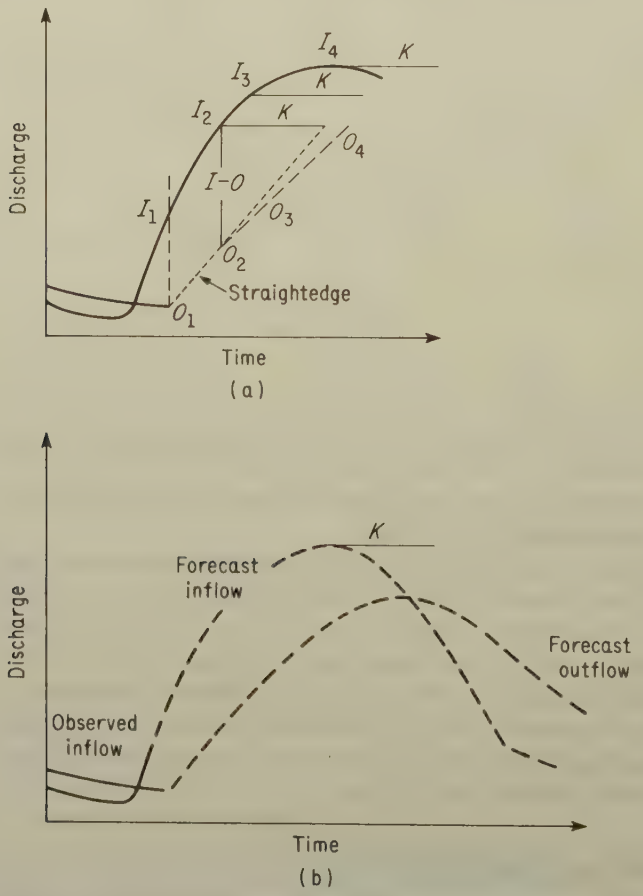


FIG. 6-3. *K* and *L* routing. (U. S. Weather Bureau, NOAA.)

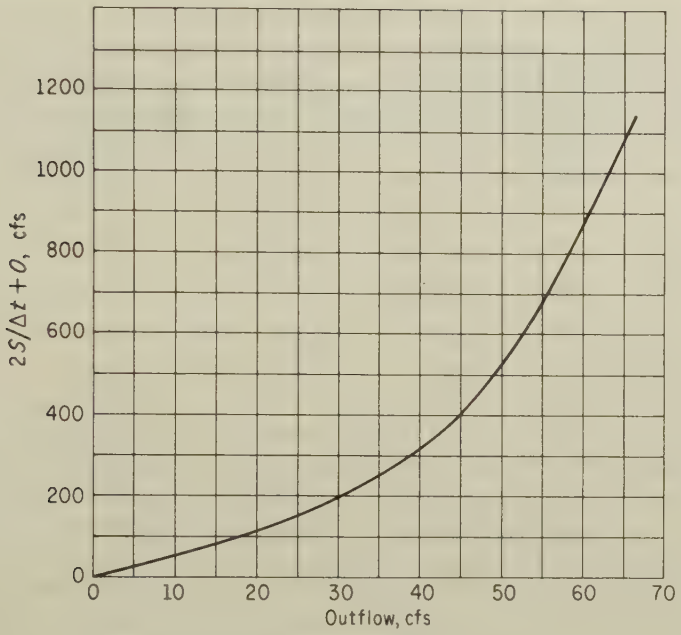


FIG. 6-4. Curve of $2S/\Delta t + O$ vs. O.

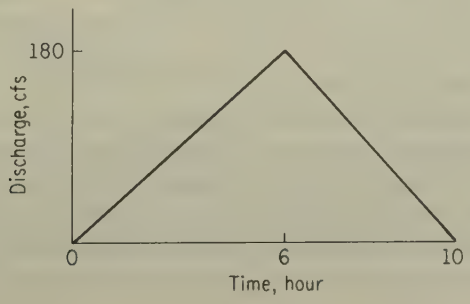
Numerical Integration

Equation 6-5 can be rewritten in the form

$$I_n + I_{n+1} + \left(\frac{2S_n}{\Delta t} - O_n\right) = \frac{2S_{n+1}}{\Delta t} + O_{n+1} \tag{6-12}$$

in which the only unknowns are successive values of S_{n+1} and O_{n+1}.

Example 6-3 Given the triangular-shaped inflow hydrograph and the $2s/\Delta t + O$ curve of Fig. 6-4 find the outflow hydrograph for the reservoir assuming it completely full at the beginning of the storm.



Routing Table

(1) Time, hr	(2) n	(3) I_n, cfs	(4) $I_n + I_{n+1}$, cfs	(5) $\frac{2S_n}{\Delta t} - O_n$, cfs	(6) $\frac{2S_{n+1}}{\Delta t} + O_{n+1}$ cfs	(7) O_{n+1} cfs
0	1	0	30	0		0
1	2	30	90	20	30	5
2	3	60	150	74	110	18
3	4	90	210	160	224	32
4	5	120	270	284	370	43
5	6	150	330	450	554	52
6	7	180	315	664	780	58
7	8	135	225	853	979	63
8	9	90	135	948	1078	65
9	10	45	45	953	1085	65
10	11	0	0	870	998	64
11	12	0	0	746	870	62
12	13	0	0	630	746	58

The common parameter between storage and elevation and between outflow and elevation can be utilized to prepare a curve relating both storage and outflow at a common height. This curve then allows a stepwise solution to Eq. 6-12 by the procedure demonstrated in Example 6-3 for an assumed triangular inflow hydrograph into a reservoir with known storage-elevation and outflow-elevation relationship. In selecting a routing period Δt, generally at least five points on the rising limb of the inflow hydrograph are employed in the calculations. An increased number of points on the rising limb, i.e., a small Δt, improves accuracy, since as $\Delta t \rightarrow 0$ the numerical integration approaches the true limit of the function being integrated, in this case dS/dt.

The stepwise solution of Eq. 6-12 depends upon the relationship between storage behind the reservoir at a given elevation and the discharge over the spillway at a given height. Calculation of the term $2S/\Delta t + O$ can be made for values of S and O at several common levels for plotting vs. the outflow at those elevations. This procedure results in a curve similar to Fig. 6-4. The computations proceed stepwise in time for $1 < n < 13$.

As can be seen from Example 6-3, tabulation of col. 3 comes from the given inflow hydrograph, col. 4 is simply the addition of $I_n + I_{n+1}$, and cols. 5 and 7 are initially zero, since in this problem no outflow occurs and the reservoir is assumed full at commencement of inflow. Therefore there is no available storage.

The starting value for col. 6 may be readily computed as 30 cfs from Eq. 6-12, since

$$I_1 + I_2 + \frac{2S_1}{\Delta t} - O_1 = \frac{2S_2}{\Delta t} + O_2$$

$$30 + 0 = \frac{2S_2}{\Delta t} + O_2$$

The answer $O_2 = 5$ cfs corresponding to $2S_2/\Delta t + O_2 = 30$ cfs is taken from Fig. 6-4. An insert in col. 5 for $n = 2$ can then be obtained, since O_2 is now known: subtract two times O_2 from $2S_2/\Delta t + O_2$, resulting in 20 cfs for the solution of $2S_2/\Delta t - O_2$ shown in col. 5.

The stepwise procedure used to get outflow figures for all n can be summarized as:

1. Entries in cols. 1 and 3 are known from the given inflow hydrograph.
2. Entries in col. 4 are the additions of $I_n + I_{n+1}$ in col. 3.
3. The $2S/\Delta t + O$ vs. O plot is entered with known values of $2S/\Delta t + O$ to find values of O.
4. From the tabulations in col. 6 subtract twice the number in col. 7 to enter a $2S/\Delta t - O$ in col. 5.
5. Add the value in col. 5 to the advanced sum in col. 4 and enter result in col. 6 for the new period under consideration.
6. The new outflow is again found from the relation of $2S/\Delta t + O$ as in Fig. 6-3.
7. Steps 3 through 6 are repeated until the entire outflow hydrograph is generated.

6-3 Hydrologic Watershed Routing

Watershed runoff can be thought of as being modified by two types of storage, channel and reservoir. This concept allows the construction of conceptual models to describe the resulting runoff hydrographs. Simulation of watershed characteristics by a series of linear reservoirs was first proposed by Nash[5] although Zoch[6] utilized the concept with a single routing in 1934.

Linear Reservoirs

Following the conceptual development of Nash, the watershed can be considered as n serially arranged reservoirs each having a relationship between storage and outflow satisfying Eq. 6-8. Assuming the first reservoir is instantaneously full and discharges into the second, the second into the third, etc., combination of Eqs. 6-1 and 6-8 results in

$$I - O = K \frac{dO}{dt} \tag{6-13}$$

Since for first reservoir I = zero,

$$-O_1 = K \frac{dO_1}{dt} \tag{6-14}$$

Solution of this differential equation is

$$O_1 = \frac{1}{K} e^{-t/K} \tag{6-15}$$

which represents outflow from the first reservoir.

By definition of the mathematical model, outflow from the first reservoir is the inflow to the second, resulting in

$$O_1 - O_2 = K \frac{dO_2}{dt} \tag{6-16}$$

and solving,

$$O_2 = \frac{1}{K} \left(\frac{t}{K}\right) e^{-t/K} \tag{6-17}$$

For the third reservoir the outflow is

$$O_3 = \frac{1}{2K} \left(\frac{t}{K}\right)^2 e^{-t/K} \tag{6-18}$$

and from the nth it is

$$O_n = \frac{1}{K\Gamma(n)} \left(\frac{t}{K}\right)^{n-1} e^{-t/K} \tag{6-19}$$

where $\Gamma(n)$ is the gamma function of n. Equation 6-19 describes the outflow from routing through n linear reservoirs as an instantaneous unit hydrograph. For application of this technique the reader is referred to Gray's method described in Chapter 3.

Muskingum Method

The Muskingum method can also be used to derive an outflow hydrograph from a watershed. A procedure was utilized by Overton[7] to analyze runoff from small ARS watersheds assuming an isosceles triangular inflow such as shown in Fig. 6-5. Combining Eqs. 6-1 and 6-3 results in

$$I - O = Kx \frac{dI}{dt} + K(1 - x) \frac{dO}{dt} \tag{6-20}$$

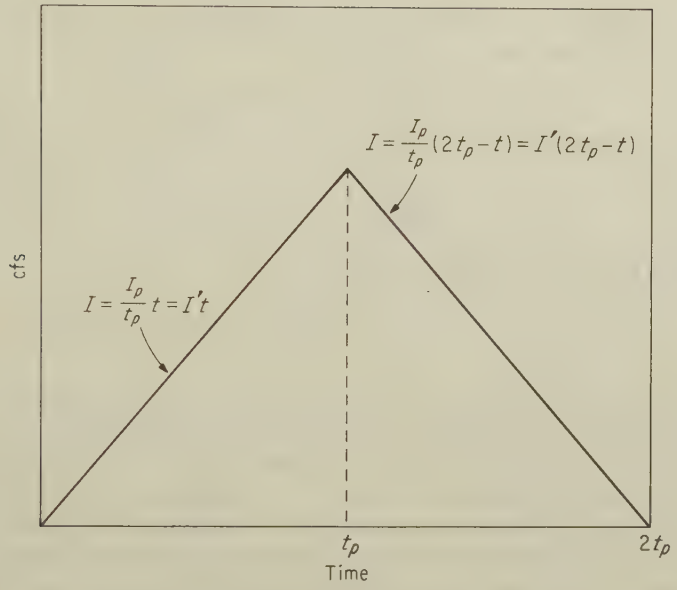


FIG. 6-5. Triangular inflow for Muskingum method.

or

$$\frac{dO}{dt} + \frac{O}{K(1 - x)} = \frac{I}{K(1 - x)} - \left(\frac{x}{1 - x}\right)\frac{dI}{dt} \qquad (6\text{-}21)$$

The solution of this equation for the assumed inflow of Fig. 6-5 is

$$O = I'\{t - K(1 - \exp(-t/K(1 - x))\} \qquad \text{for } 0 < t < t_p \qquad (6\text{-}22)$$

where

$$I' = \frac{I_p}{t_p}$$

and

$$O = I'\{2t_p - t + K - K(2 - \exp(-t_p/K(1 - x)) \cdot \exp(-(t - t_p)/K(1 - x))\} \qquad \text{for } t_p < t < 2t_p \qquad (6\text{-}23)$$

Peak outflow O_p and time to its peak T_p can be computed by setting the first derivative of Eq. 6-23 equal to zero, solving for T_p and combining with Eq. 6-23 for the value of O_p. This results in

$$T_p = t_p - K(1 - x)\ln_e\left\{\frac{1 - x}{2 - \exp(-t_p/K(1 - x))}\right\} \qquad (6\text{-}24)$$

and

$$O_p = I'\{t_p - (T_p - t_p) + Kx\} \qquad (6\text{-}25)$$

Simplification of Eq. 6-24 can be made for $K < t_p$ by using an average

value of the term $(1 - x) \ln \{(1 - K)/2\}$ equal to -0.71 for x in the range $0 \leq x \leq 0.5$. Equation 6-24 reduces to

$$T_p = t_p + 0.71K \tag{6-26}$$

and K is described by

$$K = \frac{T_p - t_p}{0.71} = 1.41(T_p - t_p) \tag{6-27}$$

The result for x is obtained from Eq. 6-25 as

$$x = 0.71 - \frac{t_p}{K}\left\{\frac{I_p - O_p}{I_p}\right\} \tag{6-28}$$

Values for K and x can now be determined by comparing the peak, and time to peak, of the inflow and outflow to a watershed. Application of this method is similar to the Muskingum method for river reaches since it is necessary only to calculate C_0, C_1, and C_2 and apply the results to a known inflow to generate a given outflow. The need to fill in missing peak records from a gauged watershed or to generate additional runoff records from prior rainfall records can be undertaken by the procedure described.

Time-Area Curves

The concept of routing a time-area curve can also be used to describe the discharge hydrograph from a watershed. This method requires construction of a curve representing the area of the watershed contributing to the outflow discharge as a function of time. Formation of this time-area curve is accomplished by drawing lines of equal travel time "isochrones" from the watershed outlet. The area contributing to the outlet at equal time intervals is then plotted versus time as a time-area curve.

Diversified rainfall intensity patterns or spatial variations for the selected time interval can be applied to the time-area curve prior to routing it through storage. Time area-curves are lagged or summed, depending upon the nature of the rainfall differences required, and then routed through either reservoir storage or by means of the Muskingum routing equation.

An application of time-area curves in urban drainage design is called the Road Research Laboratory Method.[8] It employs the time-area curve but only contributions to runoff from the impervious tracts in the urban location are considered. The basin is first divided into a number of time zones, Fig. 6-6a, and the connecting contributing area within each time zone determined. The area in each isochrone is then plotted versus time as shown in Fig. 6-6b.

The inflow hydrograph ordinates to the routing procedure are determined by applying a selected design hyetograph, Fig. 6-7 to the time-area

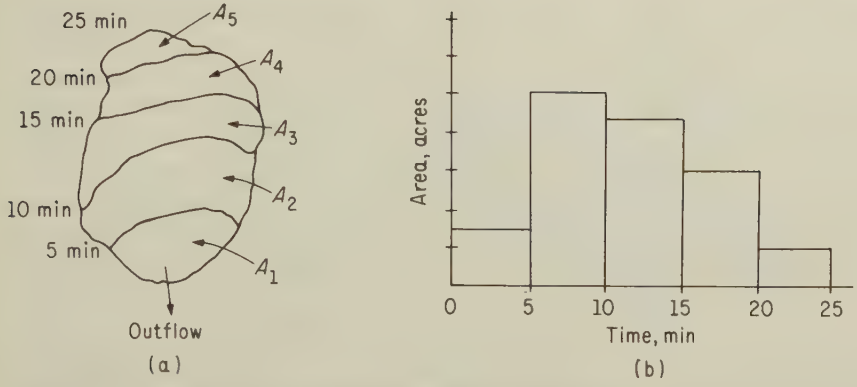


FIG. 6-6. Time-area construction: (a) basin isochrones; and (b) time-area histogram.

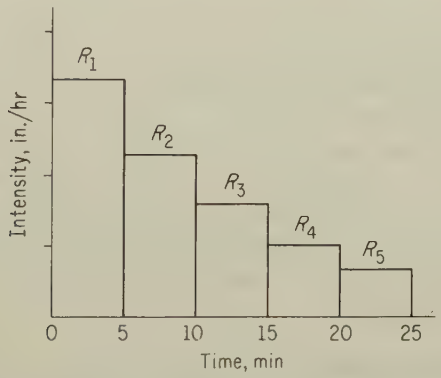


FIG. 6-7. Design-storm hyetograph.

curve according to the formula

$$I_i = R_1 A_i + R_2 A_{i-1} + \cdots + R_i A_1 \tag{6-29}$$

where I_i = inflow hydrograph ordinates, cfs
R_i = rainfall hyetograph ordinates, in./hr
A_i = time-area histogram ordinates, acres

This results in the inflow hydrograph shown in Fig. 6-8 whose routing requires a discharge-storage relationship such as given in Fig. 6-4. This coupling can readily be developed by calculating the total storage available in the urban storm drain system. Depth of flow throughout the system can be assumed proportional to flow at the outlet. Discharge and depth of flow at the outlet can be presumed related by the equation

$$O = AV = \frac{1.49}{n} AR^{2/3} S^{1/2} \tag{6-30}$$

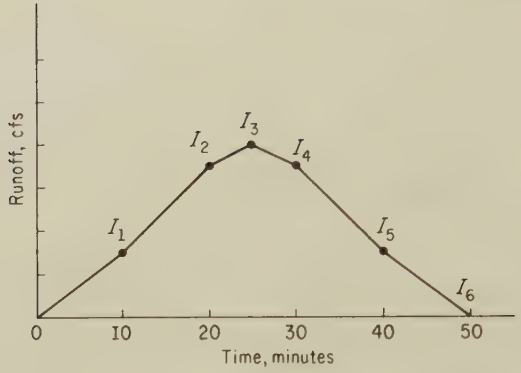


FIG. 6-8. Inflow hydrograph.

where O = outflow discharge, cfs
A = area of storm drain, sq ft
V = average velocity of flow, ft/sec
R = hydraulic radius, ft
S = slope of energy gradient, ft/ft

Construction of the discharge-storage relation involves selecting various flow depths, calculating the discharge, the storage, and $2S/\Delta t + O$ values needed to draw a curve similar to that of Fig. 6-4. The inflow hydrograph of Fig. 6-8 is then routed by use of Eq. 6-12.

6-4 Hydraulic River Routing

As noted previously, hydraulic routing employs both the equation of continuity and the equation of motion. Closed-form solutions to the complete hydraulic routing equations do not exist. Thus application of these techniques demand computer operations. Numerous approaches are available for numerical integration of the equations. Two basic systems will be used in the following discussion. The first is termed the *explicit method* because values of velocity and depth or discharge are explicitly calculated based on previously known information on the river, watershed or reservoir. The second system is the *characteristic method*. Its name is derived from the fact that computations are performed along the so-called characteristic curves well known in the study of water waves and gas dynamics.

Hydraulic routing techniques are helpful in solving river routing problems, overland flow or sheet flow, and in systems simulation of a composite watershed.[9] The main disadvantage of the hydraulic routing approach is the need for a high-speed digital computer. Generally this drawback is not severe, since the majority of sophisticated routing needs

are undertaken in conjunction with other studies requiring use of a computer.

Hydraulic routing proceeds from simultaneous solution of expressions of continuity and momentum. The general form of the statements is called the *spatially varied unsteady flow equation.*

These equations also apply to sheet flow or overland flow and include terms for laterally incoming rainfall. They can be simplified and used to resolve river-routing problems. For completeness of presentation, a general form of the spatially varied unsteady flow equations will be presented.

Equation of Continuity

The equation of continuity states simply that inflow minus outflow equals the rate of change of storage. To relate this concept to a river section under a condition of rainfall, consider an element of length Δx and unit width as shown in Fig. 6-9, where ρ = density of water, V = average velocity, y = depth, A = cross-sectional area, i = lateral inflow per elemental, Δx, and S = slope of the river bottom.

Total inflow is

$$\rho\left(V - \frac{\partial V}{\partial x}\frac{\Delta x}{2}\right)\left(A - \frac{\partial A}{\partial x}\frac{\Delta x}{2}\right)\Delta t + \rho \int_x^{x+\Delta x} \int_t^{t+\Delta t} i(x,t)\,dt\,dx \quad (6\text{-}31)$$

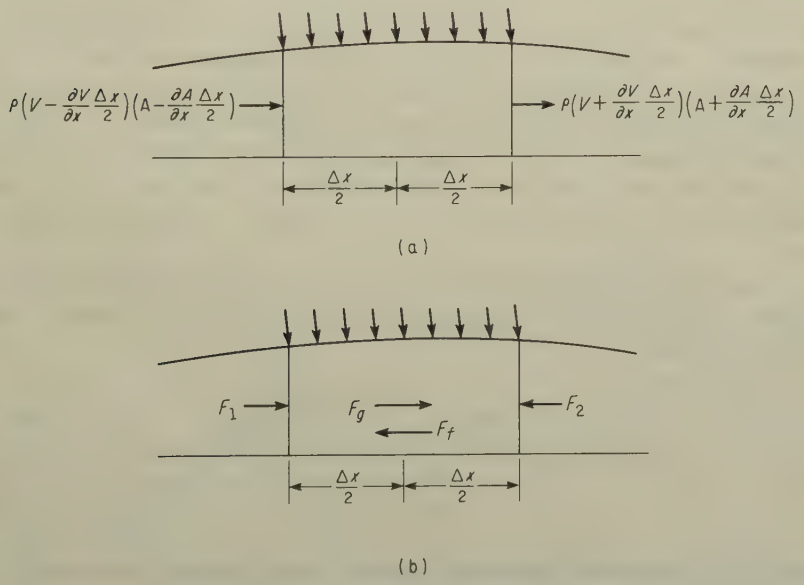


FIG. 6-9. Continuity and momentum elements.

Total outflow is

$$\rho\left(V + \frac{\partial V}{\partial x}\frac{\Delta x}{2}\right)\left(A + \frac{\partial A}{\partial x}\frac{\Delta x}{2}\right)\Delta t \tag{6-32}$$

Rate of change in storage is

$$\rho\frac{\partial A}{\partial t}\Delta x\,\Delta t \tag{6-33}$$

Consequently, continuity gives:

$$-\rho\left(A\frac{\partial V}{\partial x}\Delta x + V\frac{\partial A}{\partial x}\Delta x\right)\Delta t + \rho\bar{i}\,\Delta x\,\Delta t - \rho\frac{\partial A}{\partial t}\Delta x\,\Delta t = 0 \tag{6-34}$$

where $\bar{i}$ = average lateral inflow resulting from rainfall over Δx and Δt.

The continuity equation of unsteady flow with lateral inflow is obtained by simplifying Eq. 6-34.

$$A\frac{\partial V}{\partial x} + V\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} = \bar{i} \tag{6-35}$$

For a unit width, $A = y$ and Eq. 6-35 takes the form

$$H_2 = y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} - \bar{i} = 0 \tag{6-36}$$

Momentum Equation

In accordance with Newton's second law of motion, the change of momentum per unit of time on a body is equal to the resultant of all external forces acting on the body. The following derivation of the momentum equation of spatially varied unsteady flow is presented subject to the following assumptions: (1) the flow is unidirectional and velocity uniform across the flow section; (2) pressure is hydrostatic; (3) slope of the river bottom is relatively small; (4) the Manning formula may be used to evaluate the friction loss due to shear at the channel wall; (5) lateral inflow enters the stream with no velocity component in the direction of flow; and (6) the value of $\bar{i}$ represents the spatial and time variations of lateral inflow.

Forces acting on an element of length Δx and unit width are shown in Fig. 6-9b. Forces F_1 and F_2 represent hydrostatic forces on the element and are expressed as

$$F_1 = \gamma\left\{\bar{y}A - \frac{\partial(\bar{y}A)}{\partial x}\frac{\Delta x}{2}\right\} \tag{6-37}$$

$$F_2 = \gamma\left\{\bar{y}A + \frac{\partial(\bar{y}A)}{\partial x}\frac{\Delta x}{2}\right\} \tag{6-38}$$

where $\bar{y}$ is the distance from water surface to centroid of the area. The resultant hydrostatic force if $F_1 \neq F_2$, or

$$F_p = -\gamma \frac{\partial(\bar{y}A)}{\partial x} \Delta x \tag{6-39}$$

By assuming a small slope for the river bottom, the gravitational force is given by

$$F_g = \gamma A S \Delta x \tag{6-40}$$

Frictional force along the bottom is equal to the friction slope S_f multiplied by the weight of water in an element Δx.

$$F_f = \gamma A S_f \Delta x \tag{6-41}$$

Rate of change of momentum in the length Δx may be expressed as

$$\frac{d(mV)}{dt} = m \frac{dV}{dt} + V \frac{dm}{dt} \tag{6-42}$$

in which m is the mass of fluid.

If it is assumed that incoming lateral inflow enters the moving fluid with no velocity component in the direction of flow and $\bar{i}$ represents the spatial and time variations of the lateral inflow, rate of change of momentum for the element can be expressed as

$$\frac{d(mV)}{dt} = \rho A \Delta x \frac{dV}{dt} + \rho V \bar{i} \Delta x \tag{6-43}$$

where dV/dt represents

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} \tag{6-44}$$

The rate of change of momentum is therefore

$$\rho A \Delta x \left\{ \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} \right\} + \rho V \bar{i} \Delta x \tag{6-45}$$

Equating rate of change of momentum to all external forces acting on the element results in

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{g}{A} \frac{\partial(\bar{y}A)}{\partial x} + \frac{V\bar{i}}{A} = g(S - S_f) \tag{6-46}$$

Now for a unit-width element, the relationship simplifies to

$$H_1 = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} + \frac{V}{y} \bar{i} - g(S - S_f) = 0 \tag{6-47}$$

Equations 6-36 and 6-47 form a set of simultaneous expressions which can be solved for V and y subject to the appropriate boundary conditions. Unraveling these equations is extremely difficult without the aid of a high-speed digital computer.

Explicit Method

Using an explicit method of fixed time intervals requires rewriting Eqs. 6-36 and 6-47 in finite-difference form so that values of velocity and depth

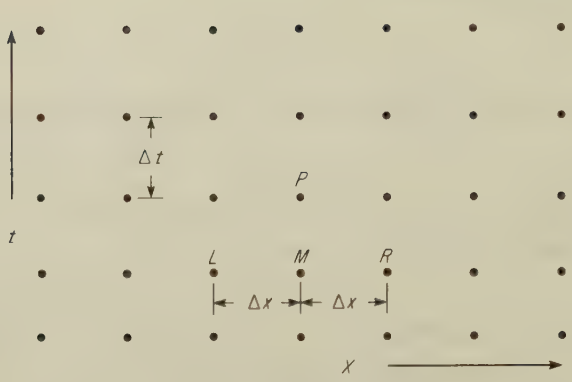


FIG. 6-10. Finite-difference network.

for each succeeding time interval can be calculated, based upon previously computed figures for velocity and depth. A scheme employed for this purpose is indicated in Fig. 6-10.

Finite-difference approximations for the terms in Eqs. 6-36 and 6-47 appear as

$$\left(\frac{\partial V}{\partial x}\right)_M = \frac{V_R - V_L}{2\Delta x} \tag{6-48}$$

$$\left(\frac{\partial V}{\partial t}\right)_P = \frac{V_P - V_M}{\Delta t} \tag{6-49}$$

$$\left(\frac{\partial y}{\partial x}\right)_M = \frac{Y_R - Y_L}{2\Delta x} \tag{6-50}$$

$$\left(\frac{\partial y}{\partial t}\right)_P = \frac{Y_P - Y_M}{\Delta t} \tag{6-51}$$

Substitution of these values in the equation of continuity with lateral inflow, Eq. 6-36 results in

$$Y_P = Y_M + \frac{\Delta t}{2\Delta x}[V_M(Y_L - Y_R) + Y_M(V_L - V_R)] + \bar{i}\Delta t \tag{6-52}$$

and similarly, the spatially varied unsteady flow equation, Eq. 6-47, appears as

$$\frac{V_P - V_M}{\Delta t} + V_M\frac{(V_R - V_L)}{2\Delta x} + g\frac{(Y_R - Y_L)}{2\Delta x} + \frac{V_P}{Y_P}\bar{i} = g(S - S_f) \tag{6-53}$$

The only existing unknown quantity at this phase in development of the equations is the term S_f, or the friction slope. If it is assumed that unsteady effects can be ignored in this term, the Chezy-Manning relationship

$$S_f = \frac{V\,|\,V\,|\,n^2}{2.2082\,Y^{4/3}} \tag{6-54}$$

can solve the frictional slope; thus, in finite difference form, evaluated at point P, the friction slope is

$$S_f = \frac{V_P \mid V_P \mid n^2}{2.2082\, Y_p^{4/3}} \tag{6-55}$$

Substituting this relationship in Eq. 6-53, and simplifying results,

$$V_P^2 + \frac{2.2082}{n^2 g \Delta t} Y_P^{4/3} \left(1 + \bar{i}\frac{\Delta t}{Y_P}\right) V_P - \frac{2.2082}{n^2 g \Delta t} Y_P^{4/3} \left\{V_M + \Delta t\left(V_M \frac{(V_L - V_R)}{2\Delta x} + g\frac{(Y_L - Y_R)}{2\Delta x} + gS\right)\right\} = 0 \tag{6-56}$$

This equation is quadratic in V_P; however an answer for V_P may be obtained from the general form for quadratic formula, resulting in

$$V_P = \frac{-2.2082}{2n^2 g \Delta t} Y_P^{4/3} \left(1 + \bar{i}\frac{\Delta t}{Y_P}\right) + \left\{\frac{2.2082}{n^2 g \Delta t} Y_P^{4/3}\left(\frac{2.2082}{4n^2 g \Delta t} Y_P^{4/3}\left(1 + \bar{i}\frac{\Delta t}{Y_P}\right)^2 + V_M\left[1.0 + \frac{\Delta t}{2\Delta x}(V_L - V_R)\right] + g\Delta t S + g\frac{\Delta t}{2\Delta x}(Y_L - Y_R)\right)\right\}^{1/2} \tag{6-57}$$

Solution of the finite difference Eqs. 6-52 and 6-57 proceeds by finding a value for each Y_P in the forward time line by Eq. 6-52, substituting this figure in Eq. 6-57 and computing each value of V_P in the time line. The discharge for a 1-ft width at the end of the overland flow plane is simply the product $V_P Y_P$ for each time line. A discharge hydrograph is prepared by storing the product Q_P for selected times and plotting the resulting curve. In prismatic channels the lateral inflow term may be omitted if it is not significant. The discharge would simply be a multiple of the width for a rectangular channel. Routing the flood wave entering the upper end of the channel is contingent upon proper description of the boundary conditions as will be described shortly.

Characteristic Method

Before dealing with boundary conditions or the inherent problems of stability encountered in the explicit method it is well to look at the characteristic solution to spatially varied unsteady flow equations. Equations 6-36 and 6-47 for H_1 and H_2 can be employed to form a linear combination with an unknown multiplier λ as follows.

$$H = H_1 + \lambda H_2 \tag{6-58}$$

It is obvious that any two real distinct values of λ will produce two equations in V and y retaining all the attributes of the original two equations H_1 and H_2. Often use of this technique simplifies solution for V and y.

Combining Eqs. 6-36 and 6-47 according to Eq. 6-58 and grouping terms results in

$$H = \left\{\frac{\partial V}{\partial x}(V + \lambda y) + \frac{\partial V}{\partial t}\right\} + \lambda\left\{\frac{\partial y}{\partial x}\left(V + \frac{g}{\lambda}\right) + \frac{\partial y}{\partial t}\right\} - \bar{i}\lambda - g(S - S_f) + \frac{V}{y}\bar{i} \qquad (6\text{-}59)$$

The first and second terms of Eq. 6-59 correspond to the total derivatives of dy/dt and dV/dt which are

$$\frac{dy}{dt} = \frac{\partial y}{\partial x}\frac{dx}{dt} + \frac{\partial y}{\partial t} \qquad (6\text{-}60)$$

and

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t} \qquad (6\text{-}61)$$

Thus H of Eq. 6-59 can now be rewritten as

$$H = \frac{dV}{dt} + \lambda\frac{dy}{dt} - \lambda\bar{i} - g(S - S_f) + \frac{V}{y}\bar{i} \qquad (6\text{-}62)$$

Equating the results of dx/dt of Eqs. 6-60 and 6-61 gives

$$\frac{dx}{dt} = V + \lambda y \qquad (6\text{-}63)$$

and

$$\frac{dx}{dt} = V + \frac{g}{\lambda} \qquad (6\text{-}64)$$

Solving these two equations for λ produces

$$\lambda = \pm\sqrt{g/y} \qquad (6\text{-}65)$$

The two real distinct roots for λ obtained will convert the two partial differential equations into a pair of ordinary differential equations in y and V subject to the restraint of Eqs. 6-63 and 6-64. Substituting Eq. 6-65 into Eqs. 6-62 produces

$$dV + \sqrt{g/y}\,dy + dt\left\{g(S_f - S) + \frac{V}{y}\bar{i} - \sqrt{g/y}\,\bar{i}\right\} = 0 \qquad (6\text{-}66)$$

$$dx = (V + \sqrt{gy})\,dt \qquad (6\text{-}67)$$

$$dV - \sqrt{g/y}\,dy + dt\left\{g(S_f - S) + \frac{V}{y}\bar{i} + \sqrt{g/y}\,\bar{i}\right\} = 0 \qquad (6\text{-}68)$$

$$dx = (V - \sqrt{gy})\,dt \qquad (6\text{-}69)$$

Solution of Eqs. 6-66 through 6-69 can be carried out numerically to completely describe the movement of a flood wave from one point in the river to another.

The curve described by Eq. 6-67 is the positive characteristic emanating from point L in Fig. 6-11; that for Eq. 6-69 is the negative characteristic

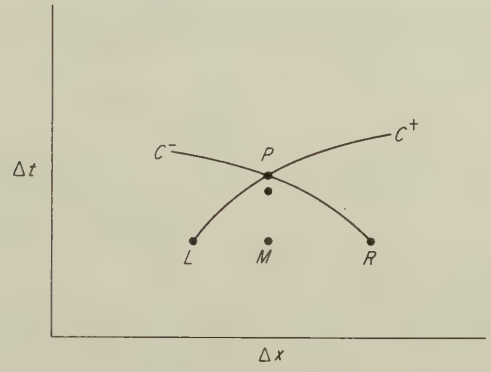


FIG. 6-11. Characteristic curves.

issuing from point R. Thus solving the equations for values of V and y at various points P throughout the x-t plane, gives a complete picture of the flood-wave movement.

Substitution of first-order finite-difference forms for V, y, and t into Eqs. 6-66 through 6-69 results in

$$\left.\begin{aligned} &V_P - V_L + \sqrt{\frac{g}{Y_L}}(Y_P - Y_L) \\ &+ (t_P - t_L)\left\{g(S_{fL} - S) + \frac{V_L}{Y_L}\bar{i} - \sqrt{\frac{g}{Y_L}}\bar{i}\right\} = 0 \end{aligned}\right\} C^+ \qquad (6\text{-}70)$$

$$x_P - x_L = (V_L + \sqrt{gY_L})(t_P - t_L) \qquad (6\text{-}71)$$

$$\left.\begin{aligned} &V_P - V_R - \sqrt{\frac{g}{Y_R}}(Y_P - Y_R) \\ &+ (t_P - t_R)\left\{g(S_{fR} - S) + \frac{V_R}{Y_R}\bar{i} + \sqrt{\frac{g}{Y_R}}\bar{i}\right\} = 0 \end{aligned}\right\} C^- \qquad (6\text{-}72)$$

$$x_P - x_R = (V_R - \sqrt{gY_R})(t_P - t_R) \qquad (6\text{-}73)$$

These equations allow computation of the time step along the characteristic by subtracting Eq. 6-73 from Eq. 6-71, yielding

$$t_P = \{x_L - x_R + t_R(V_R - \sqrt{gY_R}) - t_L(V_L + \sqrt{gY_L})\} \div (V_R - V_L - \sqrt{gY_L} - \sqrt{gY_R}) \qquad (6\text{-}74)$$

Distance along the X plane is obtained from Eq. 6-71:

$$x_P = x_L + (V_L + \sqrt{gY_L})(t_P - t_L) \tag{6-75}$$

Equation 6-72 can now be subtracted from Eq. 6-70 to get the following expression

$$\begin{aligned} Y_P = {} & V_L - V_R + (\sqrt{gY_L})(Y_L) + (\sqrt{g/Y_R})(Y_R) \\ & - (t_P - t_L)\left\{g(S_{f_L} - S) + \frac{V_L}{Y_L}\bar{i} - (\sqrt{g/Y_L})(\bar{i})\right\} \\ & + (t_P - t_R)\left\{g(S_{f_R} - S) + \frac{V_R}{Y_R}\bar{i} + (\sqrt{g/Y_R})(\bar{i})\right\} \\ & /(\sqrt{g/Y_L} + \sqrt{g/Y_R}) \end{aligned} \tag{6-76}$$

The value of V_P found by Eq. 6-70 is

$$\begin{aligned} V_P = {} & V_L - \sqrt{g/Y_L}(Y_P - Y_L) \\ & - (t_P - t_L)\left\{g(S_{f_L} - S + \frac{V_L}{Y_L}\bar{i} - (\sqrt{g/Y_L})(\bar{i})\right\} \end{aligned} \tag{6-77}$$

Initial and Boundary Conditions

The initial state of a river reach must be known or assumed before a hydraulic-routing technique is appropriate. Steady uniform flow and backwater conditions, control by weirs or dams, or a nonuniform lateral inflow all are initial and/or boundary effects which must be incorporated into the numerical solution. State conditions, i.e., subcritical or supercritical flow, dictate the type of finite-difference network to be employed in the explicit method and determine the direction of the curves in the method of characteristics. Simple subcritical steady uniform flow is treated here—more complex initial and boundary conditions are covered in texts and articles on the subject of hydraulic routing.

The distinction between initial and boundary conditions is essentially one of position on the x plane at commencement of the solution procedure. The term *initial condition* describes the flow depth, velocity or discharge at all points in the channel at time $t = 0$. *Boundary condition* refers to the depth, velocity or discharge at the upper and lower ends of the river reach at all times $t > 0$. For subcritical flow, boundary conditions are necessary at both ends of the river reach whereas only two upstream boundary conditions are necessary in supercritical flow, since downstream effects cannot be propagated backward—that is, $V \pm c$ is always positive.

The general case of river routing has an initial condition of uniform steady flow. Steady uniform flow in an open channel can be calculated by the Chézy equation

$$Q = CA\sqrt{RS} \tag{6-78}$$

where Q = discharge, cfs
C = factor of flow resistance, Chézy's C

A = cross-sectional area, ft^2
R = hydraulic radius, ft
S = slope of energy line

The Manning equation provides another method of calculating steady uniform flow.

$$Q = \frac{1.486}{n} AR^{2/3}S^{1/2} \tag{6-79}$$

where Q = discharge, cfs
n = factor of flow resistance, Manning's n
A = cross-sectional area, ft^2
R = hydraulic radius, ft
S = slope of energy line

Thus given the geometric properties of a channel, estimating the factor of flow resistance allows computation of the normal flow for a given depth of water. For initial conditions of flow in the channel it is necessary only to obtain the boundary conditions. Examples of boundary conditions for both the explicit and characteristics methods are shown in Examples 6-4 and 6-5.

Example 6-4 A 20-ft-wide rectangular channel 2 mi long having uniform flow of 6-ft depth is subjected to an upstream increase in flow to 2,000 cfs in a period of 20 min. This flow then decreases uniformly to the initial flow depth in an additional period of 40 min. The channel has a bottom slope of 0.0015 ft/ft and an estimated Manning's n of 0.02. Calculate the explicit solution of hydraulic routing for this situation.

Solution:

Since all parameters for calculating initial uniform flow are given, computation readily made by Eq. 6-79 gives values for depth and velocity at every point in the channel. The upstream boundary must then be ascertained by modifying Eq. 6-52 to read

$$Y_P = Y_M + \frac{\Delta t}{\Delta x}[V_M(Y_M - Y_R) + Y_M(V_M - V_R)]$$

and

$$V_P = \frac{Q}{BY_P}$$

since values of Y_L and V_L are not available to the left of the upstream boundary. The downstream boundary computation is similarly

$$Y_P = Y_M + \frac{\Delta t}{\Delta x}[V_M(Y_L - Y_M) + Y_M(V_L - V_M)]$$

and

$$V_P = C_M\left(\frac{BY_P}{B + 2Y_P}\right)^{2/3}$$

where $C_M = 1.486S^{1/2}/n$.

Interior points are found by Eqs. 6-52 and 6-57 modified to eliminate the lateral flow terms. A Fortran IV computer program is tabulated on pages 200–202 to demonstrate this example.

Note that a primary disadvantage of the explicit method is its stability problem. Unstable solutions ensue if selection of Δt is too large relative to Δx. The Courant condition $\Delta t \leq \Delta x/V + c$ provides a guide in selecting values for Δt. However, it is generally found for this type of explicit formulation that values of Δt should be approximately 20 percent of those given by the Courant condition. Another way to achieve more stable solutions is by a diffusing difference approximation—simply substituting the following expressions into Eqs. 6-36 and 6-47:

$$\left(\frac{\partial V}{\partial t}\right)_M = \frac{V_P - \frac{1}{2}(V_L + V_R)}{\Delta t} \qquad \left(\frac{\partial V}{\partial x}\right)_M = \frac{V_L - V_R}{2\Delta x}$$

$$\left(\frac{\partial y}{\partial t}\right)_M = \frac{Y_P - \frac{1}{2}(Y_L + Y_R)}{\Delta t} \qquad \left(\frac{\partial y}{\partial x}\right)_M = \frac{Y_L - Y_R}{2\Delta x}$$

$$S_f = \frac{S_{f_L} + S_{f_R}}{2}$$

This exchange allows significant increases in the Δt interval to be used; however, Δt is still bounded by the Courant condition as well as an additional limitation arising from influence of the term S_f on stability of an explicit numerical solution of the unsteady flow equations. The latter condition is known as the "friction criteria" and is given by

$$\Delta t \leq \frac{2.21\, R^{4/3}}{gn^2 \mid V \mid}$$

The lesser value of Δt, as computed from the Courant condition and the friction criteria, determined the maximum allowable Δt interval.

Example 6-5 Investigate the same channel geometry and imposed flow defined in Example 6-4 by the characteristic method of hydraulic routing.

Solution:

Initial conditions of flow depth and velocity are calculated using Eq. 6-79. The upstream boundary point is then found from Eq. 6-73:

$$x_P - x_R = F_2(t_P - t_R)$$

where

$$F_2 = V_R - \sqrt{gY_R}$$

Since $x_P = 0$ at the upstream boundary,

$$t_P = t_R - x_R/F_2$$

Now, applying Eq. 6-72,

$$V_P - V_R - F_6(Y_P - Y_R) + (t_P - t_R)g(S_{f_R} - S) = 0$$

in which $F_6 = \sqrt{g/Y_R}$.

Next, substituting $V_P = Q/BY_P$ and simplifying,

$$\frac{Q}{By_P} - F_6 Y_P + FU = 0$$

where

$$FU = -V_R + F_6 y_R + (t_P - t_R) g(S_{f_R} - S)$$

Rearranging to make the equation quadratic in Y_P results in

$$Y_P = \frac{FU}{2F_6} + \frac{1}{2}\sqrt{\frac{(FU)^2}{(F_6)^2} + \frac{4Q}{F_6 B}}$$

and computing the upstream velocity is

$$V_P = \frac{Q}{BY_P}$$

Downstream boundary values are worked out in a similar manner. From Eq. 6-71,

$$t_P = t_L + \frac{(L - x_L)}{F_1}$$

with L = length of reach;

$$F_1 = V_L + \sqrt{gY_L}$$

Then from Eq. 6-70,

$$V_P - V_L + F_5(Y_P - Y_L) + (t_P - t_L) g(S_{f_L} - S) = 0$$

or the reduced downstream boundary figures for velocity and depth follow the relation

$$V_P + F_5 Y_P + FD = 0$$

where

$$F_5 = \sqrt{g/Y_L}$$

$$\text{FD} = -V_L - F_5 Y_L + (t_P - t_L) g(S_{f_L} - S)$$

However, since Manning's equation can be utilized to find the velocity

$$V_P = \frac{1.486}{n} S^{1/2} \left(\frac{BY_P}{B + 2Y_P}\right)^{2/3} = C_M \left(\frac{BY_P}{B + 2Y_P}\right)^{2/3}$$

Rearranging and substituting in the downstream boundary equation yields the following expression for the depth

$$CM \left(\frac{BY_P}{B + 2Y_P}\right)^{2/3} + F_5 Y_P + FD = 0$$

This equation is nonlinear in Y_P and can quickly be evaluated by Newton's iteration technique.[10] The general form of this procedure is

$$Y_{P_{k+1}} = Y_{P_k} - f(Y_{P_k})/f'(Y_{P_k})$$

where the prime denotes the first derivative of the function of Y_P and the

```
C       EXPLICIT METHOD
C       RIVER ROUTING PORBLEM : RECTANGULAR CHANNEL WITH UNIFORM NORMAL
C         FLOW IS SUBJECTED TO A FLOOD WAVE AT THE UPSTREAM REACH OF
C         CHANNEL BEING CONSIDERED.     INITIAL UNIFORM DEPTH IS 6.0 FT.
C         THE FLOOD WAVE IS SUCH THAT THE DISCHARGE AT THE UPSTREAM POINT
C         INCREASES TO A MAXIMUN OF 2000 CFS IN A 20 MINUTE PERIOD AND
C         THEN DECREASES TO THE IINITIAL UNIFORM FLOW IN A PERIOD OF 40
C         MINUTES.
C       DIMENSIONS ARE AS FOLLOWS : TIME (SECONDS),LENGTH (FT.),
C         DEPTH (FT.),VELOCITY (FT. PER SEC),DISCHARGE (CFS)
C       B IS THE WIDTH OF THE CHANNEL
C       SO IS THE CHANNEL SLOPE
C       RN IS THE MANNING ROUGHNESS COEFICIENT
C       TTL IS THE LENGTH OF CHANNEL REACH BEING INVESTIGATED
C       G IS THE ACCELERATION OF GRAVITY,32.2 FT. PER SEC PER SEC
C       YO IS THE INITIAL UNIFORM NORMAL DEPTH
C       NM IS THE NUMBER OF INCREMENTAL REACHES ALONG THE LENGTH OF RIVER
C       ER IS THE ERROR TOLERANCE FOR ITERATIVE SOLUTIONS OF EQUATIONS
C       TTP IS THE TIME PERIOD BETWEEN QO AND QMAX
C       QMAX IS THE PEAK FLOOD DISCHARGE OF INFLOW HYDROGRAPH
C       TAP IS THE TIME PERIOD AFTER QMAX UNTIL QO
C       Q IS THE DISCHARGE AT THE UPSTREAM BOUNDARY POINT
C       QOUT IS THE DISCHARGE AT THE DOWNSTREAM BOUNDARY POINT
C       Y IS THE DEPTH OF FLOW
C       V IS THE VELOCITY OF FLOW
C       DT IS THE INCREMENTAL TIME STEP
C       I IS THE TIME ASSOCIATED WITH ANY VALUES OF DISCHARGE ,DEPTH,OR
C         VELOCITY CALCULATED
0001          DIMENSION Y(2,41),V(2,41),QQ(2,41)
0002          READ(1,950) RN,SO,TTL,B,ER,YO,G
0003          READ(1,951) TPRINT,DTP,TMAX,DT,NM
0004          READ(1,952) QMAX,TTP,TAP
0005          WRITE(3,955) RN,SO,TTL,B,ER,YO
0006          WRITE(3,956) TPRINT,DTP,TMAX,DT,NM
0007          WRITE(3,957) QMAX,TTP,TAP
0008          DX=TTL/NM
0009          N=NM
0010          M=N+1
0011          EFFF=1./3.
0012          EF=4./3.
0013          EFF=2./3.
0014          CN=RN*RN/2.2082
0015          CM=1.486/RN*SQRT(SO)
C       CALCULATE THE INITIAL UNIFORM NORMAL DISCHARGE AND VELOCITY
0016          QO=CM*B*YO*(B*YO/(B+2.*YO))**EFF
0017          VO=QO/(B*YO)
0018          WRITE(3,960) YO,VO,QO
0019          WRITE(3,963)
0020          WRITE(3,965)
C       SPECIFY THE INITIAL CONDITIONS
0021          DO 5 J=1,M
0022          Y(1,J)=YO
0023          V(1,J)=VO
0024        5 CONTINUE
0025          DTX=DT/DX
0026          DTX2=DTX/2.
0027          GDT=G*DT
0028          GTS=GDT*SO
```

```
0029              GTN=GDT*CN
0030              T=0.0
0031            9 T=T+DT
0032              TD=T-TTP
0033              IF(TD) 16,16,17
0034           16 Q=QO+(QMAX-QO)/TTP*T
0035              GO TO 20
0036           17 IF(TD-TAP) 18,18,19
0037           18 Q=QMAX-(QMAX-QO)/TAP*TD
0038              GO TO 20
0039           19 Q=QO
0040           20 CONTINUE
          C        CALCULATE THE UPSTREAM BOUNDARY POINT
0041              Y(2,1)=Y(1,1)+DTX*(V(1,1)*(Y(1,1)-Y(1,2))+Y(1,1)*(V(1,1)-V(1,2)))
0042              V(2,1)=Q/(B*Y(2,1))
          C        CALCULATE THE INTERIOR POINTS
0043              DO 100 J=2,N
0044              J1=J-1
0045              J2=J+1
0046              YD=Y(1,J1)-Y(1,J2)
0047              VD=V(1,J1)-V(1,J2)
0048              FI=-V(1,J)+DTX2*(-V(1,J)*VD-G*YD)-GTS
0049              Y(2,J)=Y(1,J)+DTX2*(V(1,J)*YD+Y(1,J)*VD)
0050              BYFP=(B*Y(2,J))/(B+2.*Y(2,J)))**EF
0051              V(2,J)=-BYFP/(2.*GTN)+0.5*SQRT(BYFP/GTN*BYFP/GTN-4.*FI*BYFP/GTN)
0052              QQ(2,J)=V(2,J)*Y(2,J)*B
0053          100 CONTINUE
          C        CALCULATE THE DOWNSTREAM BOUNDARY POINT
0054              Y(2,M)=Y(1,M)+DTX*(V(1,M)*(Y(1,N)-Y(1,M))+Y(1,M)*(V(1,N)-V(1,M)))
0055              V(2,M)=CM*(B*Y(2,M)/(B+2.*Y(2,M)))**EFF
0056              QOUT=Y(2,M)*V(2,M)*B
0057              DO 130 J=1,M
0058              Y(1,J)=Y(2,J)
0059          130 V(1,J)=V(2,J)
0060              IF(T-TPRINT+0.002) 140,135,135
0061          135 TPRINT=TPRINT+DTP
0062              TT=T/60.0
0063              WRITE(3,970)TT,Q,Y(2,1),V(2,1),QQ(2,11),Y(2,11),V(2,11),QOUT,
                 1Y(2,M),V(2,M)
0064          140 IF(T-TMAX) 9,299,299
0065          299 STOP
0066          950 FORMAT(7F10.6)
0067          951 FORMAT(4F10.4,I10)
0068          952 FORMAT(3F10.4)
0069          955 FORMAT(2X,'RN=',F10.4,5X,'SO=',F10.6,5X,'TTL=',F10.2,5X,'B=',
                 1F10.2,5X,'ER=',F10.6,5X,'YO=',F10.2)
0070          956 FORMAT(2X,'TPRINT=',F10.2,5X,'DTP=',F10.2,5X,'TMAX=',F10.2,5X,
                 1'DT=',F10.2,5X,'NM=',I5)
0071          957 FORMAT(2X,'QMAX=',F10.2,5X,'TTP=',F10.2,5X,'TAP=',F10.2)
0072          960 FORMAT(2X,'INITIAL UNIFORM DEPTH=',F10.4,5X,'INITIAL VELOCITY=',
                 1F10.4,5X,'INITIAL DISCHARGE=',F10.4)
0073          963 FORMAT(25X,'UPSTREAM',24X,'MIDPOINT',24X,'DOWNSTREAM')
0074          965 FORMAT(1X,'TIME(MIN)',7X,'DISCHARGE',4X,'DEPTH',3X,'VELOCITY',6X,
                 1'DISCHARGE',4X,'DEPTH',3X,'VELOCITY',6X,'DISCHARGE',4X,'DEPTH',
                 23X,'VELOCITY')
0075          970 FORMAT(2X,F10.2,5X,3F10.4,5X,3F10.4,5X,3F10.4)
0076              END
```

RN= 0.0200 SO= 0.001500 TTL= 10560.00 B= 20.00 ER= 0.001000 YO= 6.00
TPRINT= 60.00 DTP= 60.00 TMAX= 3600.00 DT= 2.00 NM= 20
QMAX= 2000.00 TTP= 1200.00 TAP= 2400.00
INITIAL UNIFORM DEPTH= 6.0000 INITIAL VELOCITY= 6.9458 INITIAL DISCHARGE= 833.4939

	UPSTREAM			MIDPOINT			DOWNSTREAM		
TIME(MIN)	DISCHARGE	DEPTH	VELOCITY	DISCHARGE	DEPTH	VELOCITY	DISCHARGE	DEPTH	VELOCITY
1.00	891.8191	6.1185	7.2879	833.3853	6.0000	6.9449	833.3877	5.9995	6.9455
2.00	950.1443	6.3249	7.5111	833.3853	6.0000	6.9449	833.3508	5.9993	6.9454
3.00	1008.4697	6.5306	7.7211	833.4058	6.0001	6.9449	833.3599	5.9993	6.9455
4.00	1066.7949	6.7535	7.8981	835.0549	6.0050	6.9530	833.3564	5.9993	6.9454
5.00	1125.1204	6.9840	8.0550	845.1335	6.0334	7.0038	833.3196	5.9991	6.9454
6.00	1183.4456	7.2185	8.1973	869.0608	6.1049	7.1178	833.2925	5.9990	6.9453
7.00	1241.7710	7.4587	8.3243	903.4121	6.2156	7.2673	833.3059	5.9990	6.9453
8.00	1300.0962	7.6999	8.4423	940.0215	6.3445	7.4082	833.5425	6.0002	6.9459
9.00	1358.4214	7.9439	8.5501	980.4680	6.4933	7.5498	835.5205	6.0103	6.9507
10.00	1416.7468	8.1878	8.6516	1027.1169	6.6669	7.7031	841.5051	6.0407	6.9653
11.00	1475.0720	8.4318	8.7471	1074.7683	6.8520	7.8428	853.0667	6.0992	6.9933
12.00	1533.3975	8.6755	8.8375	1125.8032	7.0518	7.9824	870.2615	6.1860	7.0342
13.00	1591.7227	8.9179	8.9243	1178.5686	7.2617	8.1150	892.3354	6.2968	7.0856
14.00	1650.0481	9.1601	9.0067	1232.4243	7.4783	8.2400	918.6567	6.4282	7.1455
15.00	1708.3733	9.4018	9.0853	1288.7878	7.7044	8.3640	949.8958	6.5832	7.2146
16.00	1766.6987	9.6430	9.1605	1344.8564	7.9330	8.4763	985.6157	6.7591	7.2911
17.00	1825.0239	9.8833	9.2328	1402.5435	8.1672	8.5864	1025.0811	6.9519	7.3727
18.00	1883.3491	10.1221	9.3032	1460.3350	8.4039	8.6885	1068.2063	7.1610	7.4585
19.00	1941.6746	10.3593	9.3716	1518.8965	8.6425	8.7874	1114.2644	7.3824	7.5467
20.00	1999.9998	10.5969	9.4367	1577.4963	8.8822	8.8801	1162.9456	7.6146	7.6363
21.00	1970.8372	10.6783	9.2282	1636.2183	9.1225	8.9680	1213.9810	7.8560	7.7264
22.00	1941.6746	10.6913	9.0807	1695.6743	9.3645	9.0537	1267.0659	8.1052	7.8164
23.00	1912.5120	10.6863	8.9484	1752.4111	9.6015	9.1257	1321.4702	8.3586	7.9049
24.00	1883.3494	10.6611	8.8328	1791.6033	9.8005	9.1404	1377.2195	8.6163	7.9919
25.00	1854.1865	10.6205	8.7293	1801.2517	9.9286	9.0710	1433.9558	8.8768	8.0770
26.00	1825.0239	10.5664	8.6360	1807.3760	10.0301	9.0098	1491.0176	9.1369	8.1593
27.00	1795.8613	10.5027	8.5495	1809.6597	10.1099	8.9500	1545.6926	9.3847	8.2352
28.00	1766.6987	10.4293	8.4699	1804.5850	10.1614	8.8796	1591.1494	9.5896	8.2963
29.00	1737.5364	10.3483	8.3953	1798.2356	10.1988	8.8159	1624.9011	9.7411	8.3405
30.00	1708.3735	10.2613	8.3243	1787.0862	10.2151	8.7473	1653.2954	9.8681	8.3769
31.00	1679.2109	10.1695	8.2561	1774.5110	10.2188	8.6826	1676.8696	9.9734	8.4067
32.00	1650.0483	10.0712	8.1919	1759.7485	10.2095	8.6182	1693.5005	10.0475	8.4275
33.00	1620.8857	9.9695	8.1292	1743.0652	10.1881	8.5544	1706.3555	10.1047	8.4434
34.00	1591.7229	9.8643	8.0681	1724.6057	10.1561	8.4905	1714.4055	10.1405	8.4533
35.00	1562.5603	9.7555	8.0086	1705.8687	10.1165	8.4312	1718.6663	10.1594	8.4585
36.00	1533.3977	9.6441	7.9499	1685.5386	10.0676	8.3711	1719.6960	10.1640	8.4598
37.00	1504.2351	9.5299	7.8922	1664.1497	10.0108	8.3118	1717.5903	10.1546	8.4572
38.00	1475.0725	9.4121	7.8360	1642.9133	9.9495	8.2571	1713.8923	10.1337	8.4514
39.00	1445.9097	9.2932	7.7794	1620.2556	9.8788	8.2007	1705.7556	10.1020	8.4427
40.00	1416.7471	9.1720	7.7232	1597.3538	9.8034	8.1470	1696.6504	10.0615	8.4314
41.00	1387.5845	9.0469	7.6688	1573.9258	9.7225	8.0942	1685.5425	10.0120	8.4176
42.00	1358.4219	8.9211	7.6135	1550.1177	9.6365	8.0430	1672.8999	9.9557	8.4017
43.00	1329.2593	8.7954	7.5566	1525.9282	9.5464	7.9922	1658.6489	9.8921	8.3837
44.00	1300.0964	8.6658	7.5013	1501.3882	9.4520	7.9421	1643.1145	9.8226	8.3639
45.00	1270.9338	8.5349	7.4455	1476.3337	9.3538	7.8916	1626.5105	9.7483	8.3426
46.00	1241.7712	8.4033	7.3886	1451.0295	9.2522	7.8416	1608.6501	9.6682	8.3193
47.00	1212.6086	8.2704	7.3310	1425.9429	9.1478	7.7939	1590.0381	9.5846	8.2948
48.00	1183.4458	8.1361	7.2728	1400.1433	9.0400	7.7441	1570.5146	9.4967	8.2688
49.00	1154.2832	7.9999	7.2143	1374.1814	8.9295	7.6946	1550.0391	9.4043	8.2411
50.00	1125.1206	7.8622	7.1553	1348.1128	8.8167	7.6453	1529.1519	9.3099	8.2125
51.00	1095.9580	7.7239	7.0946	1321.9011	8.7016	7.5957	1507.6128	9.2123	8.1826
52.00	1066.7954	7.5847	7.0325	1295.8093	8.5848	7.5471	1485.5791	9.1122	8.1516
53.00	1037.6326	7.4430	6.9705	1269.0015	8.4650	7.4956	1462.8887	9.0089	8.1191
54.00	1008.4700	7.3001	6.9072	1242.4172	8.3442	7.4448	1439.7527	8.9033	8.0855
55.00	979.3074	7.1562	6.8424	1215.5359	8.2216	7.3923	1416.5479	8.7970	8.0513
56.00	950.1448	7.0121	6.7750	1188.5994	8.0969	7.3398	1392.7783	8.6879	8.0156
57.00	920.9822	6.8668	6.7060	1162.0427	7.9717	7.2886	1368.6604	8.5769	7.9788
58.00	891.8193	6.7189	6.6366	1135.0847	7.8443	7.2351	1344.3176	8.4644	7.9410
59.00	862.6567	6.5708	6.5644	1108.0330	7.7157	7.1804	1319.6724	8.3502	7.9020
60.00	833.4941	6.4221	6.4892	1081.0579	7.5861	7.1252	1294.9756	8.2354	7.8622

```
C     CHARACTERISTICS METHOD
C       RIVER ROUTING PROBLEM : RECTANGULAR CHANNEL WITH UNIFORM NORMAL
C         FLOW IS SUBJECTED TO A FLOOD WAVE AT THE UPSTREAM REACH OF
C         CHANNEL BEING CONSIDERED.  INITIAL UNIFORM DEPTH IS 6.0 FT.
C         THE FLOOD WAVE IS SUCH THAT THE DISCHARGE AT THE UPSTREAM POINT
C         INCREASES TO A MAXIMUN OF 2000 CFS IN A 20 MINUTE PERIOD AND
C         THEN DECREASES TO THE INITIAL UNIFORM FLOW IN A PERIOD OF 40
C         MINUTES.
C       DIMENSIONS ARE AS FOLLOWS : TIME (SECONDS),LENGTH (FT.),
C         DEPTH (FT.),VELOCITY (FT. PER SEC),DISCHARGE (CFS)
C       B IS THE WIDTH OF THE CHANNEL
C       SO IS THE CHANNEL SLOPE
C       RN IS THE MANNING ROUGHNESS COEFICIENT
C       TTL IS THE LENGTH OF CHANNEL REACH BEING INVESTIGATED
C       G IS THE ACCELERATION OF GRAVITY,32.2 FT. PER SEC PER SEC
C       YO IS THE INITIAL UNIFORM NORMAL DEPTH
C       NM IS THE NUMBER OF INCREMENTAL REACHES ALONG THE LENGTH OF RIVER
C       M IS EQUAL TO NM+1
C       ER IS THE ERROR TOLERANCE FOR ITERATIVE SOLUTIONS OF EQUATIONS
C       TTP IS THE TIME PERIOD BETWEEN QO AND QMAX
C       QMAX IS THE PEAK FLOOD DISCHARGE OF INFLOW HYDROGRAPH
C       TAP IS THE TIME PERIOD AFTER QMAX UNTIL QO
C       Q IS THE DISCHARGE AT THE UPSTREAM BOUNDARY POINT
C       QOUT IS THE DISCHARGE AT THE DOWNSTREAM BOUNDARY POINT
C       Y IS THE DEPTH OF FLOW
C       V IS THE VELOCITY OF FLOW
C       X IS THE DISTANCE FROM THE UPSTREAM BOUNDARY TO ANY POINT ALONG
C         THE RIVER
C      T IS THE TIME ASSOCIATED WITH ANY VALUES OF DISCHARGES,DEPTH, OR
C         VELOCITY CALCULATED
C       SFL AND SFR ARE THE FRICTION SLOPE OF POINTS ADJACENT TO THE
C         POINT BEING CALCULATED.  SFL AND SFR ARE EVALUATED AT THE TIME
C         INCREMENT BEFORE THAT FOR THE CURRENT CALCULATIONS
0001          DIMENSION X(2,21),T(2,21),Y(2,21),V(2,21)
0002          READ(1,950) RN,SO,TTL,G,TMAX
0003          READ(1,951) QMAX,TTP,TAP,ER,YO,B
0004          READ(1,953) NM,M
0005          WRITE(3,955) RN,SO,TTL,TMAX
0006          WRITE(3,956) QMAX,TTP,TAP,ER,YO,B
0007          WRITE(3,958) NM,M
0008          DX=TTL/NM
0009          GSO=G*SO
0010          CN=RN*RN/2.2082
0011          EF1=1./3.
0012          EF2=2./3.
0013          EF4=4./3.
0014          CM=1.486/RN*SQRT(SO)
C       CALCULATE THE INITIAL UNIFORM NORMAL DISCHARGE AND VELOCITY
0015          QO=CM*B*YO*(B*YO/(B+2.*YO))**EF2
0016          VO=QO/(B*YO)
0017          WRITE(3,960) YO,VO,QO
0018          WRITE(3,965)
0019          WRITE(3,967)
C       SPECIFY THE INITIAL CONDITIONS
0020          Y(1,1)=YO
0021          V(1,1)=VO
0022          T(1,1)=0.0
0023          X(1,1)=0.0
```

```
0024              DO 6 J=2,M
0025              Y(1,J)=YO
0026              V(1,J)=VO
0027              T(1,J)=0.0
0028            6 X(1,J)=X(1,J-1)+DX
0029              L=0
0030            9 IF(L) 10,25,10
0031           10 NI=3
0032              N=NM-1
0033              NCI=1
0034              NCE=M
          C        CALCULATE THE UPSTREAM BOUNDARY POINT
0035              F2=V(1,2)-SQRT(G*Y(1,2))
0036              X(2,1)=0.0
0037              T(2,1)=T(1,2)-X(1,2)/F2
0038              TH1=T(2,1)/60.0
0039              TD=T(2,1)-TTP
0040              IF(TD) 16,16,17
0041           16 Q=QO+(QMAX-QO)/TTP*T(2,1)
0042              GO TO 20
0043           17 IF(TD-TAP) 18,18,19
0044           18 Q=QMAX-(QMAX-QO)/TAP*TD
0045              GO TO 20
0046           19 Q=QO
0047           20 CONTINUE
0048              F6=SQRT(G/Y(1,2))
0049              SFR=CN*V(1,2)*V(1,2)/(B*Y(1,2)/(B+2.*Y(1,2)))**EF4
0050              FU=-V(1,2)+F6*Y(1,2)+(T(2,1)-T(1,2))*(G*SFR-GSO)
0051              Y(2,1)=FU/(2.*F6)+0.5*SQRT(FU*FU/(F6*F6)+4.*Q/(B*F6))
0052              V(2,1)=Q/(B*Y(2,1))
0053              GO TO 30
0054           25 NI=2
0055              N=NM
0056              NCI=2
0057              NCE=NM
          C        CALCULATE THE INTERIOR POINTS
0058           30 DO 100 J=NI,N,2
0059              J1=J-1
0060              J2=J+1
0061              F1=V(1,J1)+SQRT(G*Y(1,J1))
0062              F2=V(1,J2)-SQRT(G*Y(1,J2))
0063              F3=X(1,J1)-F1*T(1,J1)
0064              F4=X(1,J2)-F2*T(1,J2)
0065              T(2,J)=(F3-F4)/(F2-F1)
0066              X(2,J)=F4+F2*T(2,J)
0067              F5=SQRT(G/Y(1,J1))
0068              F6=SQRT(G/Y(1,J2))
0069              SFL=CN*V(1,J1)*V(1,J1)/(B*Y(1,J1)/(B+2.*Y(1,J1)))**EF4
0070              SFR=CN*V(1,J2)*V(1,J2)/(B*Y(1,J2)/(B+2.*Y(1,J2)))**EF4
0071              F7=V(1,J1)+F5*Y(1,J1)+(T(1,J1)-T(2,J))*(G*SFL-GSO)
0072              F8=V(1,J2)-F6*Y(1,J2)+(T(1,J2)-T(2,J))*(G*SFR-GSO)
0073              Y(2,J)=(F7-F8)/(F6+F5)
0074              V(2,J)=F8+F6*Y(2,J)
0075          100 CONTINUE
0076          108 IF(L) 110,120,110
0077          110 J=NM
          C        CALCULATE THE DOWNSTREAM BOUNDARY POINT
0078              F1=V(1,J)+SQRT(G*Y(1,J))
```

```
0079                SFL=CN*V(1,J)*V(1,J)/(B*Y(1,J)/(B+2.*Y(1,J)))**EF4
0080                X(2,M)=TTL
0081                T(2,M)=T(1,J)+(TTL-X(1,J))/F1
0082                TH2=T(2,M)/60.0
0083                F5=SQRT(G/Y(1,J))
0084                FD=-V(1,J)-F5*Y(1,J)+(T(2,M)-T(1,J))*(G*SFL-GSO)
                C       SOLVE NON-LINEAR EQUATION BY NEWTON'S ITERATION METHOD
0085                YPO=Y(2,N)
0086                DO 115 K1=1,25
0087                FBY=B*YPO/(B+2.*YPO)
0088                FB=FBY/YPO
0089                YP1=YPO-(CM*FBY**EF2+F5*YPO+FD)/(EF2*CM/FBY**EF1*FB*FB+F5)
0090                IF(ABS(YP1-YPO)-ER) 117,117,115
0091            115 YPO=YP1
0092            117 Y(2,M)=YP1
0093                V(2,M)=CM*(B*Y(2,M)/(B+2.*Y(2,M)))**EF2
0094                QOUT=V(2,M)*Y(2,M)*B
0095                WRITE(3,970) TH1,Q,Y(2,1),V(2,1),TH2,QOUT,Y(2,M),V(2,M)
0096                L=0
0097                GO TO 121
0098            120 L=1
                C       CHANGE RECENTLY CALCULATED ( T+DT ) VALUES TO ( T ) VALUES
0099            121 DO 122 J=NCI,NCE,2
0100                X(1,J)=X(2,J)
0101                T(1,J)=T(2,J)
0102                V(1,J)=V(2,J)
0103            122 Y(1,J)=Y(2,J)
0104                IF(T(2,NCE)-TMAX) 9,123,123
0105            123 STOP
0106            950 FORMAT(5F10.6)
0107            951 FORMAT(6F10.6)
0108            953 FORMAT(2I10)
0109            955 FORMAT(2X,'RN=',F10.4,5X,'SO=',F10.6,5X,'TTL=',F10.2,5X,'TMAX=',
                   1F10.2)
0110            956 FORMAT(2X,'QMAX=',F10.2,5X,'TTP=',F10.2,5X,'TAP=',F10.2,5X,'ER=',
                  1F10.6,5X,'YO=',F10.4,5X,'B=',F10.2)
0111            958 FORMAT(2X,'NM=',I5,5X,'M=',I5)
0112            960 FORMAT(2X,'INITIAL UNIFORM DEPTH=',F10.4,5X,'INITIAL VELOCITY=',
                   1F10.4,5X,'INITIAL DISCHARGE=',F10.4)
0113            965 FORMAT(19X,'UPSTREAM',55X,'DOWNSTREAM')
0114            967 FORMAT(6X,'TIME(MIN',1X,'DISCHARGE',2X,'DEPTH',4X,'VELOCITY',22X,
                  1'TIME(MIN)',1X,'DISCHARGE',2X,'DEPTH',4X,'VELOCITY')
0115            970 FORMAT(5X,F10.2,3F10.4,20X,F10.2,3F10.4)
0116                END
```

RN= 0.0200 SO= 0.001500 TTL= 10560.00 TMAX= 5400.00
QMAX= 2000.00 TTP= 1200.00 TAP= 2400.00 ER= 0.001000 YO= 6.0000 B= 20.00
NM= 20 M= 21
INITIAL UNIFORM DEPTH= 6.0000 INITIAL VELOCITY= 6.9458 INITIAL DISCHARGE= 833.4939

UPSTREAM				DOWNSTREAM			
TIME(MIN	DISCHARGE	DEPTH	VELOCITY	TIME(MIN)	DISCHARGE	DEPTH	VELOCITY
2.53	981.1133	6.3411	7.7361	0.84	833.4917	6.0000	6.9458
5.15	1134.0740	6.9202	8.1940	1.69	833.4902	6.0000	6.9458
7.74	1284.7888	7.5702	8.4859	2.53	833.4880	6.0000	6.9458
10.23	1430.3906	8.2028	8.7189	3.38	833.4846	6.0000	6.9458
12.65	1571.1152	8.8052	8.9215	4.22	833.4829	5.9999	6.9458
14.99	1707.6597	9.3814	9.1013	5.07	833.4792	5.9999	6.9458
17.27	1840.5959	9.9354	9.2628	5.91	833.4771	5.9999	6.9457
19.49	1970.3721	10.4703	9.4093	6.75	833.4756	5.9999	6.9457
21.67	1951.3264	10.7327	9.0906	7.60	833.4707	5.9999	6.9457
23.77	1890.0803	10.7590	8.7837	8.44	833.4712	5.9999	6.9457
24.45	1870.1409	10.7252	8.7185	10.81	838.7227	6.0266	6.9586
25.14	1850.2168	10.6863	8.6570	13.19	859.0237	6.1293	7.0075
25.82	1830.3005	10.6429	8.5987	15.49	903.8174	6.3542	7.1120
26.50	1810.3879	10.5954	8.5433	17.70	976.8652	6.7161	7.2726
27.18	1790.4702	10.5442	8.4903	19.85	1074.5999	7.1918	7.4710
27.87	1770.5447	10.4896	8.4395	21.97	1188.6577	7.7365	7.6822
28.55	1750.6057	10.4319	8.3906	24.05	1310.6575	8.3084	7.8876
29.24	1730.6484	10.3713	8.3434	26.10	1435.0784	8.8819	8.0787
29.92	1710.6689	10.3082	8.2976	28.19	1550.1440	9.4048	8.2413
30.61	1690.6653	10.2425	8.2532	30.27	1638.9558	9.8040	8.3586
32.53	1634.5857	10.0618	8.1228	30.96	1661.5081	9.9048	8.3874
34.44	1578.8638	9.8616	8.0051	31.65	1680.0366	9.9875	8.4107
36.27	1525.6221	9.6557	7.9001	32.34	1695.0452	10.0544	8.4294
38.01	1474.6909	9.4484	7.8039	33.04	1706.9424	10.1073	8.4441
39.72	1424.7949	9.2378	7.7118	33.73	1716.0872	10.1479	8.4554
41.44	1374.8079	9.0206	7.6204	34.43	1722.7656	10.1776	8.4635
43.18	1324.0557	8.7948	7.5275	35.12	1727.2375	10.1975	8.4690
44.95	1272.2532	8.5593	7.4320	35.82	1729.7063	10.2084	8.4720
46.85	1216.9724	8.3034	7.3282	36.52	1730.3752	10.2114	8.4728
48.83	1159.2476	8.0311	7.2173	37.22	1729.3950	10.2070	8.4716
49.51	1139.2930	7.9345	7.1793	39.19	1722.9048	10.1782	8.4637
50.21	1118.9929	7.8360	7.1401	41.16	1707.5200	10.1099	8.4448
50.92	1098.3762	7.7354	7.0997	43.04	1685.4597	10.0117	8.4175
51.63	1077.4617	7.6330	7.0579	44.85	1658.7305	9.8924	8.3839
52.36	1056.2622	7.5286	7.0150	46.62	1628.2837	9.7562	8.3449
53.10	1034.7974	7.4225	6.9707	48.40	1594.4507	9.6044	8.3006
53.84	1013.0806	7.3146	6.9251	50.22	1557.3848	9.4375	8.2511
54.60	991.1133	7.2048	6.8781	52.07	1517.2656	9.2560	8.1961
55.36	968.9082	7.0933	6.8297	54.06	1472.7607	9.0539	8.1333
56.13	946.4678	6.9800	6.7799	56.14	1424.5183	8.8336	8.0631
58.29	883.3059	6.6601	6.6313	56.86	1404.9409	8.7438	8.0339
60.49	833.4939	6.3636	6.5489	57.60	1385.1995	8.6531	8.0041
62.65	833.4939	6.2059	6.7153	58.34	1365.2693	8.5612	7.9736
64.77	833.4939	6.1185	6.8112	59.10	1345.1199	8.4681	7.9422
66.88	833.4939	6.0684	6.8675	59.87	1324.7517	8.3738	7.9101
69.02	833.4939	6.0390	6.9010	60.65	1304.1377	8.2780	7.8771
71.20	833.4939	6.0217	6.9207	61.45	1283.2979	8.1810	7.8432
73.43	833.4939	6.0118	6.9322	62.25	1262.2161	8.0825	7.8083
75.80	833.4939	6.0058	6.9391	63.06	1240.8882	7.9826	7.7725
78.28	833.4939	6.0027	6.9427	63.88	1219.3359	7.8812	7.7357
79.14	833.4939	6.0024	6.9430	66.20	1166.3586	7.6308	7.6424
80.01	833.4939	6.0022	6.9433	68.56	1110.1821	7.3629	7.5391
80.89	833.4939	6.0019	6.9435	70.81	1058.0203	7.1117	7.4386
81.78	833.4939	6.0017	6.9438	73.01	1011.7402	6.8869	7.3454
82.68	833.4939	6.0015	6.9440	75.18	971.9155	6.6918	7.2620
83.60	833.4939	6.0013	6.9442	77.36	938.3010	6.5258	7.1892
84.52	833.4939	6.0012	6.9444	79.57	910.5779	6.3880	7.1273
85.45	833.4939	6.0010	6.9446	81.83	888.4028	6.2771	7.0765
86.38	833.4939	6.0009	6.9447	84.22	870.7129	6.1882	7.0352
87.33	833.4939	6.0008	6.9449	86.71	857.4146	6.1212	7.0037
89.97	833.4939	6.0003	6.9454	87.57	853.7117	6.1025	6.9948
92.64	833.4939	6.0001	6.9457	88.44	850.6360	6.0869	6.9874
95.15	833.4939	6.0000	6.9457	89.32	848.0603	6.0739	6.9812
97.57	833.4939	6.0000	6.9458	90.21	845.8826	6.0629	6.9759

nonlinear equation is expressed by, $f(Y_P) = 0$. The first approximation for Y_P, that is, Y_{P1} is taken as Y_M. Calculation of the interior points continued as outlined in Eqs. 6-74 through 6-77 (pages 203–206).

6-5 Hydraulic Reservoir Routing

Application of hydraulic routing techniques in design of reservoirs is relatively new. The TVA[11] used an explicit method to: (1) investigate flow conditions at the cooling water facilities of a nuclear power plant; (2) determine velocity and stage variations in a narrow winding river below an existing hydroelectric station; (3) compute time-space variations in velocity, water surface elevation, and discharge in a reservoir subject to a proposed pumping schedule; (4) study the interrelationship of two connecting reservoirs; and (5) simulate the passage of a flood wave through a proposed reservoir. One advantage of the explicit scheme is its latitude in preselecting time and space intervals for which values of depth, velocity, and discharge are calculated. However, a distinct drawback occurs in restricting the time intervals for a stable solution.

The method of characteristics can overcome the limitation imposed on the small time intervals. However, values of depth, velocity, and discharge must be interpolated for selecting locations, a manageable procedure easily incorporated if desired.

Initial conditions within the reservoir may include steady-flow profile, a transient profile, or a flat pool. Boundary conditions vary from discharge relationships, or outlet controls—that is, overflow spillways, morning-glory spillways, gates, and the like. Additionally, local inflows, resistance coefficients and geometric properties of the channel and reservoir must be prescribed, generally from field-measured data.

An example of a general reservoir problem is presented as Example 6-6. It employs the method of characteristics and demonstrates routing a flood hydrograph through a reservoir of known physical dimensions.

Example 6-6 A trapezoidal-shaped reservoir of dimensions shown in Fig. 6-12 has an initial spillway flow of 100 cfs. A sudden flood results in a wave which increases the upstream flow to 2,500 cfs in a period of 1 hr after which it

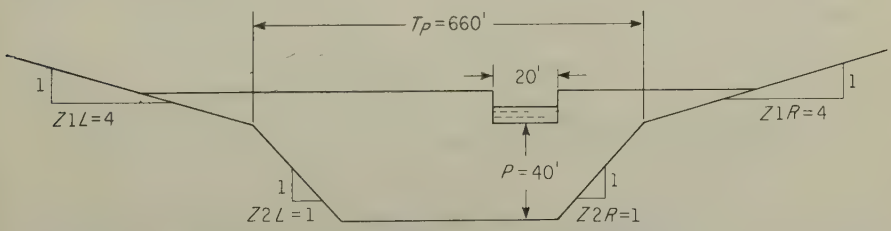


FIG. 6-12. Valley section for reservoir in Example 6-6.

decreases to the original 100 cfs in the next hour. The reservoir is assumed to have a roughness coefficient $n = 0.02$, a length of reservoir of $TTL = 5280$, and a bottom slope $S = 0.002$. The 20-ft-wide free overflow spillway is assumed to have a coefficient of discharge of 3.5; thus $Q_{\text{spillway}} = CLH^{3/2}$.

Solution:

The initial backwater profile can be calculated using Newton's Iteration technique as described in Example 6-5 to prescribe the initial conditions of flow prior to the flood. Computations can then be performed to obtain the upstream boundary point by Newton's iteration procedure remembering that cross-sectional area changes as depth of water in the reservoir exceeds the crest of the overflow spillway. The upstream boundary point, by use of Eq. 6-73, is

$$x_P - x_R = F_2(t_P - t_R)$$

where

$$F_2 = V_R - \sqrt{\frac{gT}{A_R}}$$

Since $x_P = 0$ at the upstream boundary,

$$t_P = t_R - x_R/F_2$$

Now, from Eq. 6-72,

$$V_P - V_R - \sqrt{\frac{gT}{A_R}}(Y_P - Y_R) + (t_P - t_R)(S_{f_R} - S)g = 0$$

and substituting $V_P = Q/A_P$ results in

$$\frac{Q}{A_P} - \sqrt{\frac{gT_R}{A_R}}Y_P + FU = 0$$

in which

$$FU = -V_R + F_6 Y_R + (t_P - t_R)(gS_{f_R} - gS)$$

and

$$F_6 = \sqrt{\frac{gT}{A_R}}$$

Next, if $Y_R > P$, that is, the crest of the spillway

$$A_P = A_c + T_P(Y_P - P) + (Y_P - P)^2\left(\frac{Z1R + Z1L}{2}\right)$$

with the following relationships defined as

$$A_c = [T_P - (Z2R + Z2L)P]P + \frac{P^2}{2}(Z2R + Z2L)$$

$$S_{f_R} = \frac{n^2}{2.208}\frac{V_R|V_R|}{R_R^{4/3}}$$

$$R_R = \frac{A_R}{P_c + (Y_R - P)(\sqrt{1 + Z1R^2} + \sqrt{1 + Z1L^2})}$$

$$A_R = A_c + T_P(Y_R - P) + (Y_R - P)^2\left(\frac{Z1R + Z1L}{2}\right)$$

$$P_c = T_P - P(Z2R + Z2L) + P(\sqrt{1 + Z2R^2} + \sqrt{1 + Z2L^2})$$

The interative solution for the depth y_P is

$$Y_{P_{k+1}} = Y_{P_k} - [Q + (FU - F_6 Y_{P_k})A_1]/[(FU - F_6 Y_{P_k})A_2 - F_6 A_1]$$

where

$$A_1 = A_c + T_P(Y_{P_k} - P) + (Y_{P_k} - P^2)\left(\frac{Z1R + Z1L}{2}\right)$$

$$A_2 = T_P + (Z1R + Z1L)(Y_P - P)$$

Then for the case $Y_R < P$,

$$Y_{P_{k+1}} = Y_{P_k} - [Q + (FU - F_6)A_1]/[(FU - F_6 y_{P_k})A_2 - F_6 A_1]$$

with

$$A_1 = T_P Y_{P_k} - \left(\frac{Z2R + Z2L}{2}\right) Y_{P_k}^2$$

$$A_2 = T_P - (Z2R + Z2L) Y_{P_k}$$

Adopting the continuity equation $Y_P = Q/A_1$ provides a means of determining the velocity. Interior points are calculated from Eqs. 6-74 through 6-77 as in Example 6-5. The downstream boundary, however, must again be computed through application of Newton's iteration technique.

$$t_P = t_L + \frac{TTL - x_L}{V_L - \sqrt{\frac{gA_L}{T}}}$$

and

$$V_P + \sqrt{\frac{gT}{A_L}} Y_P + FD = 0$$

where

$$FD = -V_L - \sqrt{\frac{gT}{A_L}} Y_L + (t_P - t_L)(S_{f_L} - S)g$$

and

$$S_{f_L} = \frac{n^2 V_L | V_L |}{2.21 R_L^{4/3}}$$

$$R_L = \frac{A_L}{T_P - (Z2R + Z2L) P + Y_L(\sqrt{1 + Z2R^2} + \sqrt{1 + Z2L^2})}$$

$$A_L = T_P Y_L - \frac{1}{2}(Z2R + Z2L) Y_L^2$$

but the free overflow discharge is

$$Q = 70.0(Y_{P_k} - P)^{3/2}$$

Substituting in the preceding equation produces

$$\frac{70(Y - P)^{3/2} + F_5 Y_{P_k}}{(A_c + T_p(Y_{P_k} - P)^2\left(\frac{Z1R + Z1L}{2}\right)} + FD = 0$$

where

$$F_5 = \sqrt{\frac{gT}{A_L}}$$

Thus the iterative solution for depth Y_P is

$$Y_{P_{k+1}} = Y_{P_k} - [70(Y_{P_k} - P)^{3/2} + (F_5 Y_{P_k} + FD)A_1] \div [110(Y_{P_k} - P)^{1/2} + (F_5 Y_{P_R} + FD)A_2 + F_5 A_1]$$

in which

$$A_1 = T_P Y_{P_k} - \frac{1}{2}(Z2R + Y2L)Y_{P_k}^2$$

Velocity V_P is computed from continuity $A_2 = [T_P - (Z2R + Z2L)Y_{P_k}]Y_{P_k}$. A Fortran IV program is reproduced on pages 211–217 to show use of these equations in hydraulic reservoir routing by the method of characteristics. Numerous other boundary conditions could be employed by manipulating the desired boundary equations.

6-6 Hydraulic Watershed Routing

A watershed is composed of two components of routed flow, the overland and channel flow portions. A watershed can thus be "built up" or described mathematically by portraying the various phases of flow of the effective rainfall throughout its boundaries. This delineation can lead to a complex computer routine so most applications of this technique have used simplifications in routing the overland flow phase of the watershed.

An investigation by Machmeier and Larson[12] employed this concept to develop hydrographs for a 21.35-sq-mi watershed. The smallest size unit used in the build up of the total watershed measured 800 ft by 1,750 ft or 0.05 sq mi. A diagram of the watershed is shown in Fig. 6-13 each channel representing a total area of 2.75 sq mi whose arrangement is set forth in Fig. 6-13a.

Typical cross-section dimensions of the main channels are given by

$$A = 6.79yM^{0.3} + 1.92y^2M^{0.06} \tag{6-80}$$

and

$$P = 6.79M^{0.3} + 2.0y[3.69M^{0.12} + 1]^{1/2} \tag{6-81}$$

```
C       CHARACTERISTICS METHOD
C       RESERVOIR ROUTING PROBLEM :  TRAPEZOIDAL RESERVOIR WITH INITIAL
C         STEADY FLOW OF 100 CFS IS SUBJECTED TO A FLOOD WAVE AT THE
C         UPSTREAM END OF THE RESERVOIR.  THE FLOOD INCREASES THE FLOW
C         AT THE UPSTREAM END FROM THE INITIAL STEADY FLOW TO A MAXIMUN
C         OF 2500 CFS IN 1 HR. AND THEN DECREASES TO THE INITIAL FLOW
C         DURING ANOTHER 1 HR. PERIOD.
C       DIMENSIONS ARE AS FLLLOWS : TIME(SECONDS),LENGTH(FEET),
C         DEPTH(FEET),VELOCITY(FEET PER SECOND),DISCHARGE(CFS)
C       QO IS THE INITIAL STEADY RESERVOIR DISCHARGE
C       QMAX IS THE PEAK FLOOD DISCHARGE OF INFLOW HYDROGRAPH
C       TTP IS THE TIME PERIOD BETWEEN QO AND QMAX
C       TAP IS THE TIME PERIOD AFTER QMAX UNTIL QO
C       SPL IS THE LENGTH OF THE RESERVOIR SPILLWAY CREST
C       DISCC IS THE SPILLWAY DISCHARGE COEFFICIENT
C       Z1R IS THE SIDE SLOPE OF THE RESERVOIR ABOVE THE ELEVATION OF THE
C           TOP OF THE DAM (RIGHT SIDE LOOKING UPSTREAM FROM THE DAM).
C       Z1L IS THE SAME AS Z1R EXCEPT IT IS FOR THE LEFT SIDE.
C       Z2R IS THE SIDE SLOPE OF THE RESERVOIR BELOW THE DAM ( LOOKING
C           UPSTREAM).
C       Z2L IS THE SAME AS Z2R EXCEPT IT IS FOR THE LEFT SIDE.
C       P IS THE HEIGHT OF THE DAM AND ALSO THE ELEVATION AT WHICH THE
C         SIDE SLOPES OF THE RESERVOIR CHANGE FROM Z1 TO Z2
C       TP IS THE WIDTH OF THE RESERVOIR AT THE TOP OF THE DAM.
C       TTL IS THE LENGTH OF THE RESERVOIR.
C       YA AND VA ARE THE DEPTH AND VELOCITY OF THE RESERVOIR AT A
C           CROSS-SECTION LOCATED UPSTREAM FROM THE DAM BEYOND ANY
C           SIGNIFICANT SURFACE DRAWDOWN.
C       T(Y) CALCULATES THE TOP WIDTH FOR A DEPTH ( Y )
C       A(Y) CALCULATES THE CROSS-SECTIONAL AREA FOR A DEPTH ( Y )
C       R(Y) CALCULATES THE HYDRAULIC RADIUS FOR A DEPTH ( Y )
C       B IS THE WIDTH OF THE RESERVOIR BOTTOM
C       SO IS THE RESERVOIR BOTTOM SLOPE
C       RN IS THE MANNING ROUGHNESS COEFICIENT
C       ALPHA IS THE VELOCITY DISTRIBUTION ENERGY COEFFICIENT
C       G IS THE ACCELERATION OF GRAVITY,32.2 FT. PER SEC PER SEC
C       NM IS THE NUMBER OF INCREMENTAL REACHES ALONG THE LENGTH OF RIVER
C       ER IS THE ERROR TOLERANCE FOR ITERATIVE SOLUTIONS OF EQUATIONS
C       O IS THE DISCHARGE AT THE UPSTREAM BOUNDARY POINT
C       QOUT IS THE DISCHARGE AT THE DOWNSTREAM BOUNDARY POINT
C       Y IS THE DEPTH OF FLOW
C       V IS THE VELOCITY OF FLOW
C       X IS THE DISTANCE FROM THE UPSTREAM BOUNDARY TO ANY POINT ALONG
C         THE RESERVOIR
C       T IS THE TIME ASSOCIATED WITH ANY VALUES OF DISCHARGE ,DEPTH,OR
C         VELOCITY CALCULATED
C       SFL AND SFR ARE THE FRICTION SLOPE OF POINTS ADJACENT TO THE
C         POINT BEING CALCULATED.  SFL AND SFR ARE EVALUATED AT THE TIME
C         INCREMENT BEFORE THAT FOR THE CURRENT CALCULATIONS.
C       COMPUTED VALUES FOR THE INLET AND OUTLET ARE PRINTED EACH 20 TH
C         TIME THEY ARE COMPUTED.
0001          COMMON/AAA/X(2,21),T(2,21),V(2,21),Y(2,21),TTL,QO,P,EF1,EF2,EF4,
             1EF10,EF13,AP,PP,TP,Z1R,Z1L,ZH1R,ZH1L,Z2R,Z2L,ZH2R,ZH2L,YA,VA,SO,
             2CN,ALPHA,B,TMAX,DX,G,M
0002          A1(Y)=AP+TP*(Y-P)+(Y-P)*(Y-P)/2.*(Z1R+Z1L)
0003          P1(Y)=PP+(Y-P)*(ZH1R+ZH1L)
0004          T1(Y)=TP+(Y-P)*(Z1R+Z1L)
0005          A2(Y)=B*Y+Y*Y/2.*(Z2R+Z2L)
```

```
0006              P2(Y)=B+Y*(ZH2R+ZH2L)
0007              T2(Y)=B+Y*(Z2R+Z2L)
0008              READ(1,950) RN,SO,TTL,TMAX,G,NM
0009              READ(1,951) QO,QMAX,TTP,TAP,ER
0010              READ(1,952) TP,ALPHA,SPL,DISCC,P
0011              READ(1,953) Z1R,Z1L,Z2R,Z2L
0012              WRITE(3,955) RN,SO,TTL,TMAX,NM
0013              WRITE(3,956) QO,QMAX,TTP,TAP,ER
0014              WRITE(3,957) TP,ALPHA,SPL,DISCC,P
0015              WRITE(3,958) Z1R,Z1L,Z2R,Z2L
0016              ZH1R=SQRT(Z1R*Z1R+1.)
0017              ZH1L=SQRT(Z1L*Z1L+1.)
0018              ZH2R=SQRT(Z2R*Z2R+1.)
0019              ZH2L=SQRT(Z2L*Z2L+1.)
0020              B=TP-(Z2R+Z2L)*P
0021              AP=B*P+P*P/2.*(Z2R+Z2L)
0022              PP=B+P*(ZH2R+ZH2L)
0023              DISCF=DISCC*SPL
0024              DX=TTL/NM
0025              GSO=G*SO
0026              CN=RN*RN/2.2082
0027              M=NM+1
0028              EF1=1./3.
0029              EF2=2./3.
0030              EF4=4./3.
0031              EF10=10./3.
0032              EF13=13./3.
0033              CM=1.486/RN*SQRT(SO)
          C        CALCULATE THE GRADUALLY VARIED FLOW PROFILE ALONG THE RESERVOIR
          C           AND SPECIFY INITIAL CONDITIONS
0034              YOD=P*(QO/DISCF)**EF2
          C        SOLVE NON-LINEAR EQUATION BY NEWTON'S ITERATION METHOD
0035              YAO=YOD
0036              DO 5 KO=1,15
0037              YP=YAO-P
0038              YPS=SQRT(YP)
0039              YA1=YAO-(QO-DISCF*YP*YPS)/(-1.5*DISCF*YPS)
0040              IF(ABS(YA1-YAO)-ER) 6,6,5
0041            5 YAO=YA1
0042            6 YA=YA1
0043              VA=QO/A1(YA)
0044              WRITE(3,960) YA,VA,QO
0045              CALL GVFP
0046              WRITE(3,961)
0047              DO 7 J=1,M
0048              WRITE(3,962) X(1,J),Y(1,J),V(1,J)
0049            7 T(1,J)=0.0
0050              WRITE(3,965)
0051              WRITE(3,967)
0052              KT=20
0053              L=0
0054            9 IF(L) 10,27,10
0055           10 NI=3
0056              NCI=1
0057              NCE=M
0058              N=NM-1
          C        CALCULATE THE UPSTREAM BOUNDARY POINT
0059              YR=Y(1,2)
```

```
0060                VR=V(1,2)
0061                IF(VR-P) 11,11,12
0062             11 F2=VR-SQRT(G*A2(YR)/T2(YR))
0063                GO TO 13
0064             12 F2=VR-SQRT(G*A1(YR)/T1(YR))
0065             13 X(2,1)=0.0
0066                T(2,1)=T(1,2)-X(1,2)/F2
0067                TH1=T(2,1)/60.0
0068                TD=T(2,1)-TTP
0069                IF(TD) 16,16,17
0070             16 Q=QO+(QMAX-QO)/TTP*T(2,1)
0071                GO TO 20
0072             17 IF(TD-TAP) 18,18,19
0073             18 Q=QMAX-(QMAX-QO)/TAP*TD
0074                GO TO 20
0075             19 Q=QO
0076             20 CONTINUE
0077                IF(YR-P) 21,21,24
0078             21 F6=SQRT(G*T2(YR)/A2(YR))
0079                SFR=CN*VR*VR*(P2(YR)/A2(YR))**EF4
0080                FU=-VR+F6*YR+(T(2,1)-T(1,2))*(G*SFR-GSO)
                 C      SOLVE NON-LINEAR EQUATION BY NEWTON'S ITERATION METHOD
0081                YPO=YR
0082                DO 22 KO=1,25
0083                FAD1=B+YPO*(Z2R+Z2L)
0084                YP1=YPO-(Q/A2(YPO)-F6*YPO+FU)/(-Q*FAD1/(A2(YPO)*A2(YPO))-F6)
0085                IF(ABS(YP1-YPO)-ER) 23,23,22
0086             22 YPO=YP1
0087             23 Y(2,1)=YP1
0088                V(2,1)=Q/A2(YP1)
0089                GO TO 30
0090             24 F6=SQRT(G*T1(YR)/A1(YR))
0091                SFR=CN*VR*VR*(P1(YR)/A1(YR))**EF4
0092                FU=-VR+F6*YR+(T(2,1)-T(1,2))*(G*SFR-GSO)
                 C      SOLVE NON-LINEAR EQUATION BY NEWTON'S ITERATION METHOD
0093                YPO=YR
0094                DO 25 KO1=1,25
0095                YP=YPO-P
0096                FAD1=TP+(YPO-P)*(Z1R+Z1L)
0097                YP1=YPO-(Q/A1(YPO)-F6*YPO+FU)/(-Q*FAD1/(A1(YPO)*A1(YPO))-F6)
0098                IF(ABS(YP1-YPO)-ER) 26,26,25
0099             25 YPO=YP1
0100             26 Y(2,1)=YP1
0101                V(2,1)=Q/A1(YP1)
0102                GO TO 30
0103             27 NI=2
0104                NCI=2
0105                NCE=NM
0106                N=NM
                 C      CALCULATE THE INTERIOR POINTS
0107             30 DO 100 J=NI,N,2
0108                J1=J-1
0109                J2=J+1
0110                YR=Y(1,J2)
0111                VR=V(1,J2)
0112                YL=Y(1,J1)
0113                VL=V(1,J1)
0114                IF(YR-P) 32,32,33
```

```
0115           32 F2=VR-SQRT(G*A2(YR)/T2(YR))
0116              F6=SQRT(G*T2(YR)/A2(YR))
0117              SFR=CN*VR*VR*(P2(YR)/A2(YR))**EF4
0118              GO TO 34
0119           33 F2=VR-SQRT(G*A1(YR)/T1(YR))
0120              F6=SQRT(G*T1(YR)/A1(YR))
0121              SFR=CN*VR*VR*(P1(YR)/A1(YR))**EF4
0122           34 IF(YL-P) 35,35,36
0123           35 F1=VL+SQRT(G*A2(YL)/T2(YL))
0124              F5=SQRT(G*T2(YL)/A2(YL))
0125              SFL=CN*VL*VL*(P2(YL)/A2(YL))**EF4
0126              GO TO 37
0127           36 F1=VL+SQRT(G*A1(YL)/T1(YL))
0128              F5=SQRT(G*T1(YL)/A1(YL))
0129              SFL=CN*VL*VL*(P1(YL)/A1(YL))**EF4
0130           37 F3=X(1,J1)-F1*T(1,J1)
0131              F4=X(1,J2)-F2*T(1,J2)
0132              T(2,J)=(F3-F4)/(F2-F1)
0133              X(2,J)=F4+F2*T(2,J)
0134              F7=V(1,J1)+F5*Y(1,J1)+(T(1,J1)-T(2,J))*(G*SFL-GSO)
0135              F8=V(1,J2)-F6*Y(1,J2)+(T(1,J2)-T(2,J))*(G*SFR-GSO)
0136              Y(2,J)=(F7-F8)/(F6+F5)
0137              V(2,J)=F8+F6*Y(2,J)
0138         100 CONTINUE
0139         108 IF(L) 110,120,110
0140         110 J=NM
             C    CALCULATE THE DOWNSTREAM BOUNDARY POINT
0141              YL=Y(1,J)
0142              VL=V(1,J)
0143              F1=VL+SQRT(G*A1(YL)/T1(YL))
0144              F5=SQRT(G*T1(YL)/A1(YL))
0145              SFL=CN*VL*VL*(P1(YL)/A1(YL))**EF4
0146              X(2,M)=TTL
0147              T(2,M)=T(1,J)+(TTL-X(1,J))/F1
0148              TH2=T(2,M)/60.0
0149              FD=-VL-F5*YL+(T(2,M)-T(1,J))*(G*SFL-GSO)
             C    SOLVE NON-LINEAR EQUATION BY NEWTON'S ITERATION METHOD
0150              YPO=Y(2,N)
0151              DO 115 K1=1,25
0152              YP=YPO-P
0153              YPS=SQRT(YP)
0154              FBD1=TP+YP*(Z1R+Z1L)
0155              YP1=YPO-(DISCF*YPS*YP/A1(YPO)+F5*YPO+FD)/(DISCF*YPS/A1(YPO)*
                 1(1.5-YP*FBD1/A1(YPO))+F5)
0156              IF(ABS(YP1-YPO)-ER) 117,117,115
0157         115 YPO=YP1
0158         117 Y(2,M)=YP1
0159              V(2,M)=DISCF*(YP1-P)**1.5/A1(YP1)
0160              IF(KT-20) 119,118,119
0161         118 QOUT=V(2,M)*A1(YP1)
0162              WRITE(3,970) TH1,Q,Y(2,1),V(2,1),TH2,QOUT,Y(2,M),V(2,M)
0163              KT=1
0164              L=0
0165              GO TO 121
0166         119 KT=KT+1
0167              L=0
0168              GO TO 121
0169         120 L=1
```

```
FORTRAN IV G LEVEL  20                  MAIN              DATE = 71314        18/28/08              PAGE 0005

C       CHANGE RECENTLY CALCULATED ( T+DT ) VALUES TO ( T ) VALUES
0170       121 DO 122 J=NCI,NCE,2
0171           X(1,J)=X(2,J)
0172           T(1,J)=T(2,J)
0173           Y(1,J)=Y(2,J)
0174       122 V(1,J)=V(2,J)
0175           IF(T(2,NCI)-TMAX) 9,299,299
0176       299 STOP
0177       950 FORMAT(5F10.6,I10)
0178       951 FORMAT(5F10.6)
0179       952 FORMAT(7F10.6)
0180       953 FORMAT(4F10.6)
0181       955 FORMAT(2X,'RN=',F10.4,5X,'SO=',F10.6,5X,'TTL=',F10.2,5X,'TMAX=',
              1F10.2,5X,'NM=',I5)
0182       956 FORMAT(2X,'QO=',F10.2,5X,'QMAX=',F10.2,5X,'TTP=',F10.2,5X,'TAP=',
              1F10.2,5X,'ER=',F10.6)
0183       957 FORMAT(2X,'TP=',F10.2,5X,'ALPHA=',F10.2,5X,'SPL=',F10.2,5X,'DISCC=
              1',F10.3,5X,'P=',F10.2)
0184       958 FORMAT(5X,'Z1R=',F10.2,5X,'Z1L=',F10.2,5X,'Z2R=',F10.2,5X,'Z2L=',
              1F10.2)
0185       960 FORMAT(2X,'INITIAL OUTLET DEPTH=',F10.4,5X,'INITIAL OUTLET VELOCIT
              1Y=',F10.4,5X,'INITIAL OUTLET DISCHARGE=',F10.4)
0186       961 FORMAT(2X,'STEADY STATE CONDITIONS:',10X,'X-DISTANCE',9X,'DEPTH',
              18X,'VELOCITY')
0187       962 FORMAT(30X,3F15.4)
0188       965 FORMAT(21X,'INLET',60X,'OUTLET')
0189       967 FORMAT(7X,'TIME(MIN)','DISCHARGE',2X,'DEPTH',4X,'VELOCITY',23X,
              1'TIME(MIN)','DISCHARGE',2X,'DEPTH',4X,'VELOCITY')
0190       970 FORMAT(5X,F10.2,3F10.4,20X,F10.2,3F10.4)
0191           END
```

```
FORTRAN IV G LEVEL  20                GVFP              DATE = 71314        18/28/08              PAGE 0001

0001                  SUBROUTINE GVFP
                C       GVFP ( GRADUALLY VARIED FLOW PROFILE ) IS A SUBROUTINE TO COMPUTE
                CCC       DEPTHS AND VELOCITIES OF FLOW AT SPECIFIED LOCATIONS ALONG
                C         A STEADY STATE GRADUALLY VARIED FLOW.
0002                  COMMON/AAA/X(2,21),T(2,21),V(2,21),Y(2,21),TTL,QO,P,EF1,EF2,EF4,
                     1EF10,EF13,AP,PP,TP,Z1R,Z1L,ZH1R,ZH1L,Z2R,Z2L,ZH2R,ZH2L,YA,VA,SO,
                     2CN,ALPHA,B,TMAX,DX,G,M
0003                  GA1(W)=AP+TP*(W-P)+(W-P)*(W-P)/2.*(Z1R+Z1L)
0004                  GP1(W)=PP+(W-P)*(ZH1R+ZH1L)
0005                  GDA1(W)=TP+(W-P)*(Z1R+Z1L)
0006                  GA2(W)=B*W+W*W/2.*(Z2R+Z2L)
0007                  GP2(W)=B+W*(ZH2R+ZH2L)
0008                  GDA2(W)=B+W*(Z2R+Z2L)
0009                  ER=0.00001
0010                  ALF=ALPHA/(2.*G)
0011                  Y(1,M)=YA
0012                  V(1,M)=VA
0013                  X(1,M)=TTL
0014                  F1=ALF*QO*QO
0015                  F2=CN*QO*QO*DX/2.
0016                  DSX=TTL
0017                  MM=M
0018                  DDY=0.0
0019                5 YAO=YA-DDY
0020                  IF(YA-P) 7,6,6
0021                6 F3=SO*DX-YA-F1/GA1(YA)**2.-F2*GP1(YA)**EF4/GA1(YA)**EF10
0022                  GO TO 8
0023                7 F3=SO*DX-YA-F1/GA2(YA)**2.-F2*GP2(YA)**EF4/GA2(YA)**EF10
0024                8 DO 14 K=1,15
0025                  IF(YAO-P) 12,10,10
0026               10 YA1=YAO-(YAO+F1/GA1(YAO)**2.-F2*GP1(YAO)**EF4/GA1(YAO)**EF10+F3)/
                     1(1.-2.*F1*GDA1(YAO)/GA1(YAO)**3.-F2*EF2*GP1(YAO)**EF1/GA1(YAO)**
                     2EF13*(2.*(ZH1R+ZH1L)*GA1(YAO)-5.*GDA1(YAO)*GP1(YAO)))
0027                  GO TO 13
0028               12 YA1=YAO-(YAO+F1/GA2(YAO)**2.-F2*GP2(YAO)**EF4/GA2(YAO)**EF10+F3)/
                     1(1.-2.*F1*GDA2(YAO)/GA2(YAO)**3.-F2*EF2*GP2(YAO)**EF1/GA2(YAO)**
                     2EF13*(2.*(ZH2R+ZH2L)*GA2(YAO)-5.*GDA2(YAO)*GP2(YAO)))
0029               13 IF(ABS(YA1-YAO)-ER) 15,15,14
0030               14 YAO=YA1
0031               15 MM=MM-1
0032                  DDY=YA-YA1
0033                  DSX=DSX-DX
0034                  X(1,MM)=DSX
0035                  Y(1,MM)=YA1
0036                  IF(YA1-P) 17,16,16
0037               16 V(1,MM)=QO/GA1(YA1)
0038                  GO TO 18
0039               17 V(1,MM)=QO/GA2(YA1)
0040               18 YA=Y(1,MM)
0041                  VA=V(1,MM)
0042                  IF(MM-1) 20,20,5
0043               20 RETURN
0044                  END
```

```
RN=    0.0200      SO=   0.002000     TTL=   5280.00     TMAX=  10800.00     NM=   20
QO=     100.00     QMAX=    2500.00     TTP=   3600.00     TAP=   3600.00     ER=  0.001000
TP=     660.00     ALPHA=      1.00     SPL=     20.00     DISCC=    3.500        P=     40.00
   Z1R=      4.00      Z1L=      4.00      Z2R=      1.00      Z2L=      1.00
INITIAL OUTLET DEPTH=    41.2684    INITIAL OUTLET VELOCITY=    0.0039    INITIAL OUTLET DISCHARGE=  100.0000
STEADY STATE CONDITIONS:
```

X-DISTANCE	DEPTH	VELOCITY
0.0	30.7084	0.0053
264.0000	31.2364	0.0052
528.0000	31.7644	0.0051
792.0000	32.2924	0.0051
1056.0000	32.8204	0.0050
1320.0000	33.3484	0.0049
1584.0000	33.8764	0.0048
1848.0000	34.4044	0.0047
2112.0000	34.9324	0.0047
2376.0000	35.4604	0.0046
2640.0000	35.9884	0.0045
2904.0000	36.5164	0.0044
3168.0000	37.0444	0.0044
3432.0000	37.5724	0.0043
3696.0000	38.1004	0.0042
3960.0000	38.6284	0.0042
4224.0000	39.1564	0.0041
4488.0000	39.6844	0.0041
4752.0000	40.2124	0.0040
5016.0000	40.7404	0.0040
5280.0000	41.2684	0.0039

	INLET			OUTLET		
TIME(MIN)	DISCHARGE	DEPTH	VELOCITY	TIME(MIN)	DISCHARGE	DEPTH	VELOCITY
0.28	111.3357	30.7090	0.0059	0.25	99.9998	41.2684	0.0039
5.62	324.7297	30.7194	0.0173	5.59	101.1768	41.2784	0.0039
10.95	538.0654	30.7488	0.0287	10.92	104.6819	41.3077	0.0041
16.28	751.2932	30.7976	0.0399	16.25	110.5658	41.3563	0.0043
21.61	964.3567	30.8649	0.0511	21.57	118.7943	41.4228	0.0046
26.93	1177.2097	30.9507	0.0623	26.90	129.6592	41.5082	0.0050
32.24	1389.7981	31.0551	0.0732	32.21	143.2280	41.6117	0.0055
37.55	1602.0845	31.1761	0.0841	37.52	159.6167	41.7324	0.0061
42.85	1814.0161	31.3146	0.0948	42.81	179.2083	41.8714	0.0069
48.14	2025.5728	31.4701	0.1053	48.10	201.9385	42.0265	0.0077
53.42	2236.7019	31.6419	0.1156	53.38	228.2163	42.1987	0.0087
58.68	2447.3657	31.8303	0.1257	58.65	258.0596	42.3864	0.0098
63.94	2342.4756	32.0196	0.1195	63.90	290.3877	42.5818	0.0109
69.18	2132.7659	32.1911	0.1082	69.14	319.6682	42.7525	0.0120
74.41	1923.4792	32.3375	0.0971	74.37	345.3154	42.8979	0.0129
79.64	1714.5237	32.4609	0.0862	79.60	367.5969	43.0212	0.0137
84.85	1505.8542	32.5623	0.0755	84.81	386.4294	43.1236	0.0144
90.07	1297.3831	32.6451	0.0649	90.03	401.9836	43.2068	0.0149
95.27	1089.0889	32.7080	0.0543	95.23	413.7549	43.2691	0.0153
100.48	880.9092	32.7513	0.0439	100.44	422.1350	43.3131	0.0156
105.68	672.8206	32.7753	0.0335	105.64	426.5693	43.3363	0.0158
110.88	464.7686	32.7803	0.0231	110.84	427.4624	43.3409	0.0158
116.08	256.7085	32.7662	0.0128	116.04	424.8496	43.3273	0.0157
121.29	100.0000	32.7363	0.0050	121.25	418.8760	43.2960	0.0155
126.49	100.0000	32.7002	0.0050	126.45	412.0562	43.2602	0.0153
131.70	100.0000	32.6647	0.0050	131.66	405.4060	43.2250	0.0150
136.91	100.0000	32.6295	0.0050	136.87	398.9153	43.1905	0.0148
142.12	100.0000	32.5956	0.0050	142.08	392.7324	43.1574	0.0146
147.33	100.0000	32.5628	0.0050	147.29	386.3982	43.1234	0.0144
152.55	100.0000	32.5298	0.0050	152.51	380.1658	43.0897	0.0141
157.77	100.0000	32.4965	0.0050	157.72	374.1973	43.0573	0.0139
162.99	100.0000	32.4646	0.0050	162.94	368.1040	43.0240	0.0137
168.21	100.0000	32.4337	0.0050	168.16	362.2380	42.9918	0.0135
173.43	100.0000	32.4028	0.0050	173.39	356.5552	42.9604	0.0133
178.65	100.0000	32.3721	0.0050	178.61	350.9080	42.9291	0.0131

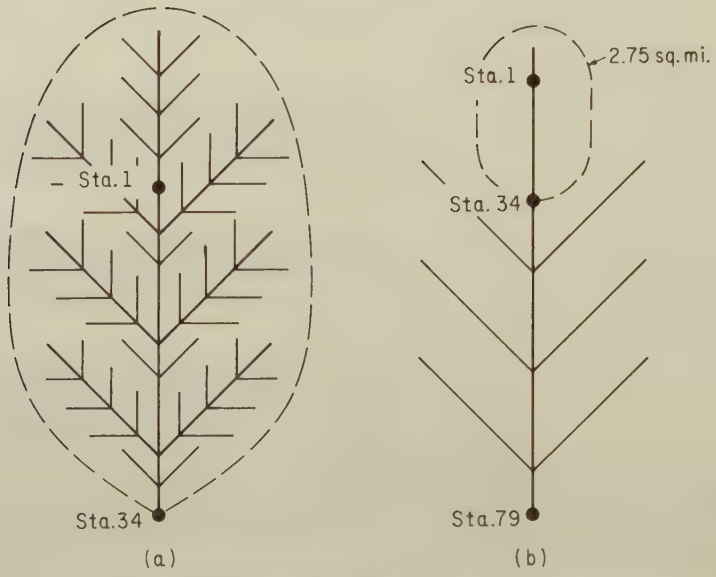


FIG. 6-13. Diagram of model watershed showing: (a) complete channel system for single 2.75-sq mi subwatershed; and (b) arrangement of 2.75-sq mi subwatersheds. (After Ref. 12.)

where A = cross-sectional area, sq ft
y = depth of cross section
M = drainage area above point in watershed
P = wetted perimeter

Equations 6-80 and 6-81 assist in identifying cross-sectional characteristics based only upon a known location within the watershed. Additional channel information is necessary in the form of the definition of channel slope at any point. This relationship is given by

$$S = 0.003155M^{-0.2}$$

where S = slope, ft/ft
M = drainage area above point in watershed

The smallest input in the total watershed is from a 0.35-sq-mi subwatershed. This unit is composed of seven 0.05-sq-mi units and its outflow was obtained by lagging the outflow hydrographs according to their respective travel times to the outlet. The outflow for this 0.35-sq-mi unit is shown in Fig. 6-14.

Basic equations for hydraulic routing, Eqs. 6-35 and 6-46, are used to formulate the solution to this watershed-routing problem. They can be reduced to

$$(AV)_x + A_t = 0 \tag{6-82}$$

and

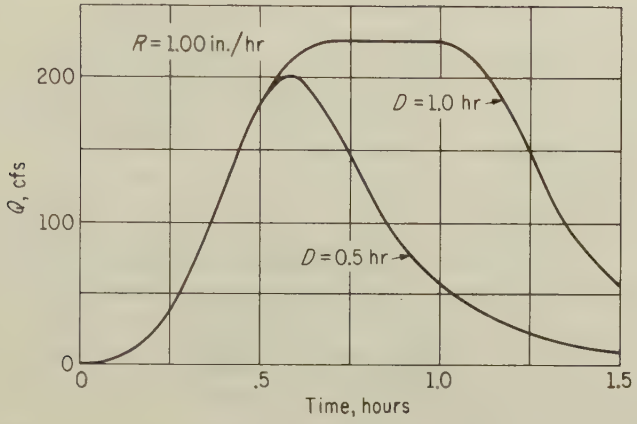


FIG. 6-14. Hydrographs for a 0.35-sq-mi watershed with supply rate of 1.00 in./hr. and two different durations. (After Ref. 12.)

$$V_t + VV_x = g(S - S_f) \tag{6-83}$$

in which the subscript indicates partial derivatives with respect to x or t. Since routing is to be performed only past the 0.35 sq-mi watershed, the lateral inflow term i is zero and the expression, $g\partial y/\partial x$ in Eq. 6-47 is assumed to be neglibible. Simultaneous solution of $Q = AV$, and the upstream boundary condition, insures that the effect of depth and velocity at the next station downstream enters the calculations for depth and velocity at the upstream boundary.

An explicit routing is performed for this watershed having a main stream length of 7.433 mi. The reaches and contributing areas are listed in Table 6-1. Results of the routings for various durations and supply rates are displayed in Figs. 6-15 and 6-16.

TABLE 6-1 Station Numbers and Reach Characteristics for Model Watershed (After Ref. 12)

Reach no (1)	Station no (2)	Watershed area for reach, mi (3)	Length of reach, sq mi (4)
1	1–6	0.35	0.332
2	7–17	1.15	0.665
3	18–28	1.95	0.665
4	29–47	2.75	1.197
5	48–51	8.25	0.522
6	52–59	8.95	1.218
7	60–63	14.45	0.522
8	64–71	15.15	1.218
9	72–75	20.65	0.522
10	76–79	21.35	0.522

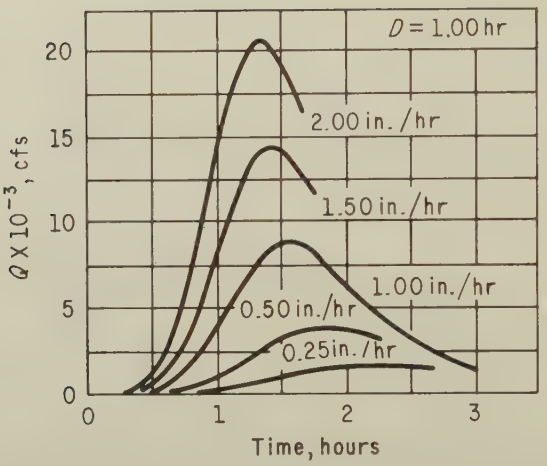


FIG. 6-15. Model watershed hydrographs for various supply rates and a duration of 1.00 hr. (After Ref. 12.)

Needless to say, numerous applications of hydraulic routing techniques appear in the literature. Each is generally structured for a specific situation. The need to perform hydraulic watershed routing is frequently undertaken in conjunction with a simulation study as will be further discussed in Chapter 10. Material presented here and in Chapter 10 is by no means exhaustive, but rather is presented so that an interested student can understand the structuring processes of hydrologic modeling.

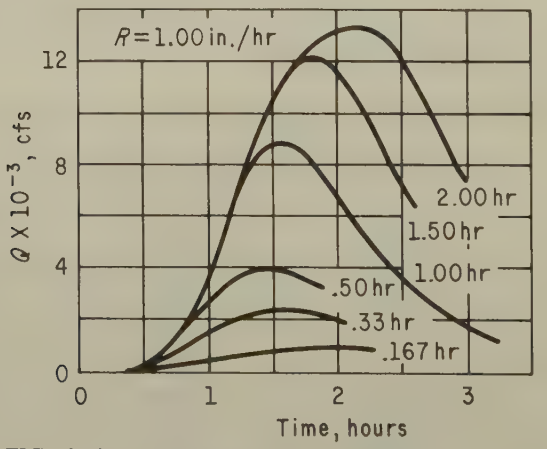


FIG. 6-16. Model watershed hydrographs for various durations and a supply rate of 1.00 in./hr. (After Ref. 12.)

References

1. Graeff, "Traité d'hydraulique." Paris: (1883), p. 438–443.

2. G. T. McCarthy, "The Unit Hydrographs and Flood Routing," unpublished manuscript, North Atlantic Division, U.S. Army Corps of Engineers, June 1938.

3. Chow, V. T., *Open Channel Hydraulics*. New York: McGraw-Hill Book Company, 1959.

4. U.S. Weather Bureau, "Elements of River Forecasting," Tech. Memo, WBTMHYDRO-4 U.S. Department of Commerce, October 1967.

5. Nash, J. E., "The form of the Instantaneous Unit Hydrograph, *Int. Assoc. Sci. Hydrol.*, Pub. 45, Vol. 3 (1957), pp. 14–121.

6. Zoch, R. T., "On the Relation Between Rainfall and Stream Flow," *Monthly Weather Review*, Vol. 62 (1934), pp. 315–322.

7. Overton, D. E., "Muskingum Flood Routing of Upland Streamflow," Vol. IV, No. 3 J. Hydrol., (1966), pp. 185–200.

8. Terstriep, M. L., and J. B. Stall, "Urban Runoff by Road Research Laboratory Method," *Proc. A.S.C.E., J. Hydr. Div.*, November 1969.

9. Harbaugh, T. E., "Numerical Techniques for Spatially Varied Unsteady Flow," University of Missouri Water Resources Center, Report No. 3, 1967.

10. Fread, D. L., and T. E. Harbaugh, "Gradually Varied Flow Profiles by Newton's Iteration Technique," *J. Hydrol.*, Vol. 2 (1971), pp. 129–139.

11. Garrison, J. M., J. Granju, and J. T. Price, "Unsteady Flow Simulation in Rivers and Reservoirs," *Proc. A.S.C.E., J. Hydr. Div.*, September 1969, pp. 1559–1576.

12. Machmeier, R. E., and C. L. Larson, "Runoff Hydrographs for Mathematical Watershed Model," *Proc. A.S.C.E., J. Hydr. Div.*, November 1968, pp. 1453–1474.

Problems

6-1. Discuss the main differences between hydrologic and hydraulic routing techniques.

6-2. Given the following data, route a storm hydrograph through a full reservoir and plot on the same graph the inflow and resulting outflow hydrograph for the Green-Acre Branch watershed. Bottom of the rectangular spillway is placed at elevation 980.0. Given:

(a) Area = 0.64 sq mi
(b) Length = 1.10 mi
(c) $L_c = 0.53$
(d) $C_t = 2.00$
(e) $C_p = 0.62$
(f) Outflow = $CLH^{3/2}$; $C = 3.5$, L = 10 ft
(g) Storage-elevation curve table (see next page):

Elevation (ft)	Storage $10^{-4}(\text{ft})^3$	Total storage $10^{-4}(\text{ft})^3$
960		0
	40	
970		40
	210	
980		250
	590	
990		840
	1,240	
1,000		2,080

Find:

(a) 15-min unit hydrograph by Snyder's Method.

(b) 30-min unit hydrograph using the lag method.

(c) Storm hydrograph which results from 1.8 in. rain for first 30 min and 0.63 in. for the next 15 min.

(d) Develop a $2S/\Delta t + O$ vs. O curve using a routing period of 15 min and the outflow and storage curves provided.

(e) Route the storm hydrograph through the reservoir assuming it is full to the bottom of the spillway El. 980.

(f) Indicate maximum height of water in the reservoir.

(g) At what elevation should the top of the dam be placed to obtain 5 ft of freeboard?

6-3. Repeat Prob. 6-2 with the reservoir initially empty.

6-4. Select a rural watershed in your geographic region which has rainfall-runoff records. Use the Muskingham method of watershed routing to find K and x.

6-5. Verify Eq. 6-47.

6-6. Verify Eq. 6-57.

6-7. Rewrite Eqs. 6-47 and 6-57 for the case of no lateral inflow.

6-8. Verify Eqs. 6-66 through 6-69.

6-9. Rework Example 6-2 with values of mannings $n = 0.01$.

6-10. Rework Examples 6-2 and 6-3 for a trapezoidal channel with 2:1 side slopes.

6-11. Assume that Example 6-4 has a further restriction imposed by the downstream river channel. Rework this example to provide a maximum outflow of 325 cfs.

7

GROUNDWATER HYDROLOGY

7-1 Introduction

The amount of water stored below ground in the United States exceeds by a significant amount all above ground storage in streams, rivers, reservoirs and lakes including the Great Lakes.[6] This enormous reservoir sustains streamflow during precipitation-free periods and constitutes the major source of fresh water for many arid localities. Figure 7-1 indicates the distribution and nature of primary groundwater areas of the United States.

Quantification of the volume and rate of flow of groundwater in various regions is an exceedingly difficult task because volumes and flow rates are determined to a considerable extent by the geology of the region. The character and arrangement of rocks and soils are important factors, and these are often highly variable within a groundwater reservoir. An additional difficulty is the inability to directly measure many critical geologic and hydraulic reservoir characteristics.

In spite of these predicaments, hydrologists are continually developing new techniques for measurement and analysis which are contributing to an extensive body of knowledge in the field of groundwater hydrology. Many practical problems can be adequately solved by employing these techniques. This chapter presents the fundamentals of flow in a porous medium and shows how they are applied to the solution of various hydrologic issues.

7-2 Groundwater Flow—General

Understanding movement of groundwater requires a knowledge of time and space dependency of the flow, nature of the porous medium and fluid, and the boundaries of the flow system.

Patterns show areas underlain by aquifers generally capable of yielding to individual wells 50 gpm or more of water containing not more than 2,000 ppm of dissolved solids (includes some areas where more highly mineralized water is actually used)
Watercourses in which ground water can be replenished by perennial streams
Buried valleys not now occupied by perennial streams
Unconsolidated and semiconsolidated aquifers
Consolidated – rock aquifers
Both unconsolidated and consolidated – rock aquifers
Not known to be underlain by aquifers that will generally yield as much as 50 gpm to wells

FIG. 7-1. Groundwater areas in the United States. (From H. E. Thomas, "Water," *The Yearbook of Agriculture*, 1955.)

Groundwater flows are usually three-dimensional. Unfortunately the solution of such problems by analytic methods is intensely complex unless the system is symmetric.[2] In other cases, space dependency in one of the coordinate directions may be so slight that assumption of two-dimensional flow is satisfactory. Many problems of practical importance fall into this class. Sometimes one-dimensional flow can be assumed, thus further simplifying the solution.

Fluid properties such as velocity, pressure, temperature, density, and viscosity often vary in time and space. When time-dependency occurs, the issue is termed an *unsteady-flow problem* and solutions are usually difficult. On the other hand, situations where space dependency alone exists, are *steady-flow problems*. Only homogeneous (single-phase) fluids will be considered herein. For a discussion of multiple-phase flow, Ref. 2 is recommended.

Boundaries to groundwater-flow systems may be fixed geologic structures or free water surfaces which are dependent for their position on the state of the flow. A hydrologist must be able to define these boundaries mathematically if he is to solve groundwater-flow problems.

Porous media through which groundwaters flow may be classified as isotropic, anisotropic, heterogeneous, homogeneous, or several possible combinations of these. An *isotropic* medium has uniform properties in all directions from a given point. *Anisotropic* media have one or more properties which depend on a given direction. For example, permeability of the medium might be greater along a horizontal plane than along a vertical one. *Heterogeneous* media have nonuniform properties of anisotropy or isotropy, while *homogeneous* media are uniform in their characteristics.

7-3 Subsurface Distribution of Water

Groundwater distribution may be generally catagorized into zones of aeration and saturation. The saturated zone is one in which all voids are filled with water under hydrostatic pressure. In the zone of aeration, the interstices are filled partly with air, partly with water. The saturated zone is commonly called the *groundwater zone*. The zone of aeration may ideally be subdivided into several subzones. Todd classifies these as follows.[4]

1. *Soil-water zone*. A soil-water zone begins at the ground surface and extends downward through the major root band. Its total depth is variable and dependent upon soil type and vegetation. The zone is unsaturated except during periods of heavy infiltration. Three categories of water classification may be encountered in this region: hygroscopic water, which is adsorbed from the air; capillary water, held by surface tension; and gravitational water, which is excess soil water draining through the soil.

2. *Intermediate zone.* This belt extends from the bottom of the soil-water zone to the top of the capillary fringe and may change from non-existence to several hundred feet in thickness. The zone is essentially a connecting link between a near-ground surface region and the near-water-table region through which infiltrating fluids must pass.

3. *Capillary zone.* A capillary zone extends from the water table to a height determined by the capillary rise which can be generated in the soil. The capillary-band thickness is a function of soil texture and may fluctuate not only from region to region but also within a local area.

4. *Saturated zone.* In the saturated zone, groundwater fills the pore spaces completely and porosity is therefore a direct measure of storage volume. Part of this water (specific retention) cannot be removed by pumping or drainage because of molecular and surface-tension forces. Specific retention is the ratio of volume of water retained against gravity drainage to gross volume of the soil.

Water that can be drained from a soil by gravity is known as the *specific yield.* It is expressed as the ratio of volume of water which can be drained by gravity to gross volume of the soil. Values of specific yield depend upon soil-particle size, shape and distribution of pores, and degree of compaction of the soil. Average values for alluvial aquifers range from 10 to 20 percent. Meinzer and others have developed procedures for determining specific yield.[46]

7-4 Geologic Considerations

Determination of groundwater volumes and flow rates requires a thorough knowledge of the geology of a groundwater basin. In bedrock areas, hydrologic characteristics of the rocks—that is, their location, size, orientation and ability to store or transmit water—must be known. In unconsolidated rock areas, basins often contain hundreds to thousands of feet of semiconsolidated to unconsolidated fill deposits which originated from erosion of headwater areas. Such fills often contain extensive quantities of stored water. Charateristics of these basin fills must be evaluated.

A knowledge of the distribution and nature of geohydrologic units such as *aquifers*, *aquicludes*, and *aquitards* is essential to proper planning for development or management of groundwater supplies. In addition, bedrock-basin boundaries must be located and an evaluation made of their leakage characteristics.

An *aquifer* is a water-bearing stratum or formation which is capable of transmitting water in quantities sufficient to permit development. Aquifers may be considered as falling into two categories, confined and unconfined, depending upon whether or not a water table or free surface exists under atmospheric pressure. Storage volume within an aquifer is changed when-

ever water is recharged to or discharged from an aquifer. In the case of an unconfined aquifer this may be easily determined as

$$\Delta S = S_y \Delta V \tag{7-1}$$

where ΔS = change in storage volume
S_y = average specific yield of aquifer
ΔV = volume of aquifer lying between original water table and water table at some later specified time

For saturated, confined aquifers, pressure changes produce only slight modifications in storage volume. In this case, weight of the overburden is supported partly by hydrostatic pressure and somewhat by solid material in the aquifer. When hydrostatic pressure in a confined aquifer is reduced by pumping or other means, the load on the aquifer increases, causing its compression with the result that some water is forced out. Decreasing the hydrostatic pressure also causes a small expansion which in turn produces an additional release of water. For confined aquifers, water yield is expressed in terms of a *storage coefficient* S_c, defined as the volume of water an aquifer takes in or releases per unit surface area of aquifer per unit change in head normal to the surface. Figure 7-2 illustrates the classifications of aquifers.

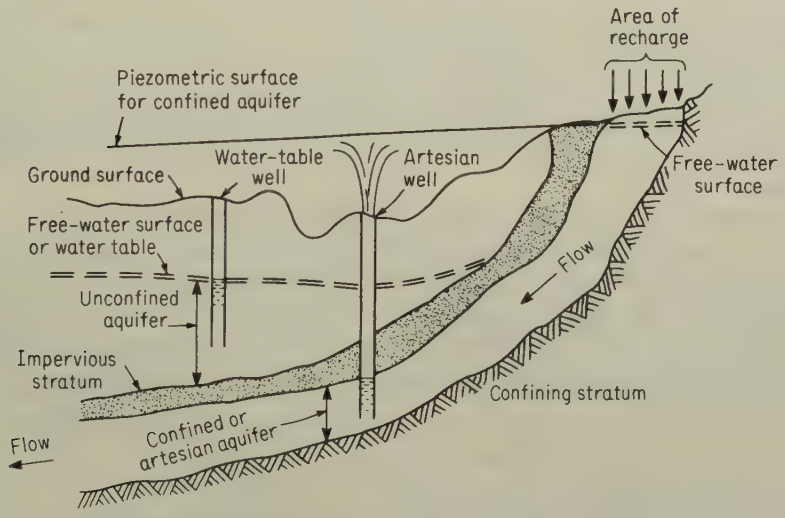


FIG. 7-2. Aquifer classifications.

In addition to water-bearing strata exhibiting satisfactory rates of yield, there are also nonwater-bearing and impermeable strata which may contain large quantities of water but whose transmission rates are not high enough to permit effective development. An *aquifuge* is a formation impermeable and devoid of water.

7-5 Fluctuations in Groundwater Level

Any circumstance that alters the pressure imposed on underground water will also cause a variation in groundwater level. Seasonal factors, changes in stream and river stages, evapotranspiration, atmospheric pressure changes, winds, tides, external loads, various forms of withdrawal and recharge, and earthquakes all may produce fluctuations in water table level or the piezometric surface, depending upon whether the aquifer is free or confined.[4] It is important that an engineer concerned with development and utilization of groundwater supplies be aware of these factors. He should also be able to evaluate their importance relative to operation of a specific groundwater basin.

7-6 Groundwater-Surfacewater Relationships

Notwithstanding that water-resource development has often been based on the predominant use of either surface or groundwaters, it must be emphasized that these two components of the total water resource are interdependent. Changes in one component can have far-reaching effects on the other. Coordinated development and management of the combined resource is critical. Linkage between surface and groundwaters should be investigated in all regional studies so that adverse effects can be noted if they exist and opportunities for joint management understood.

In Chapter 3 it was shown how surface stream flows are sustained by the groundwater resource, and it was also pointed out that groundwaters are replenished by infiltration derived from precipitation on the earth's surface.

Underground reservoirs often are extensive and can serve to store water for a multitude of uses. If withdrawals from these reservoirs consistently exceed recharge, *mining* occurs and ultimate depletion of the resource results. By properly coordinating use of surface and groundwater supplies, optimum regional water-resource development seems most likely to be assured. Several studies directed toward this coordinated use have been initiated.[1,5,18]

7-7 Hydrostatics

Water located in pore spaces of a saturated medium is under pressure (called *pore pressure*) and can be determined by inserting a piezometer in the medium at a point of interest. If location A (Fig. 7-3) is considered, it can be seen that pore pressure is given by

$$p = h_a\gamma \qquad (7\text{-}2)$$

where p is the pore pressure (gauge pressure), h_a the head measured from

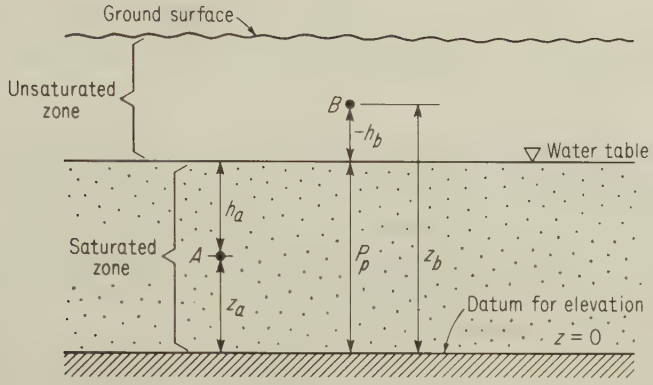


FIG. 7-3. Definition sketch showing hydrostatic pressures in a porous medium.

the point to the water table, and γ is the specific weight of water. Pore pressure is considered positive or negative, depending upon whether the pressure head is measured above (positive) or below (negative) the point under consideration. If an arbitrary datum is established, total head or piezmetric head above the datum is

$$P_p = z + h \tag{7-3}$$

where P_p is known as the piezometric potential. In Fig. 7-3 this is equal to $h_a + z_a$ for point A in the saturated zone and $z_b - h_b$ for point B in the unsaturated zone. Term h_a is the pore pressure of A while $-h_b$ denotes tension or vacuum (negative pore pressure) at B.

7-8 Groundwater Flow

Analogies can be drawn between flow in pipes under pressure and in fully saturated confined aquifers. Flow of groundwater with a free surface is also similar to that in an open channel. A major difference is the geometry of a groundwater-system flow channel as compared with common hydraulic pipe flow or channel systems. The problem can easily be recognized by envisioning a discharging cross section composed of a number of small openings, each with its own geometry, orientation, and size so that the flow velocity issuing from each pore varies in both magnitude and direction. Difficulties in analyzing such systems are apparent. Computations are usually based on macroscopic averages of fluid and medium properties over a given cross-sectional area.

Unknown quantities to be determined in groundwater-flow problems are density, pressure, and velocity if constant temperature conditions are assumed to exist. In general, water is considered incompressible, so the number of working variables is reduced. An exception to this is discussed

later relative to the storage coefficient for a confined aquifer. Primary emphasis here will be placed on flow of water in a saturated porous medium.

7-9 Darcy's Law

Darcy's law for fluid flow through a permeable bed is stated as[19]

$$Q = -KA\,dh/dx \tag{7-4}$$

where A is the total cross-sectional area including the space occupied by the porous material, K the hydraulic conductivity of the material, and Q the flow across the control area A. In Eq. 7-4,

$$h = z + p/\gamma + C \tag{7-5}$$

where h is the piezometric head, z the elevation above a datum, p the hydrostatic pressure and C is an arbitrary constant. If specific discharge $q = (Q/A)$ is substituted in Eq. 7-4,

$$q = -K \frac{d}{dx} \left(z + \frac{p}{\gamma} \right) \tag{7-6}$$

Note that q also equals the porosity n multiplied by pore velocity V_p. Darcy's law is widely used in groundwater flow problems. Several applications will be illustrated in later sections.

Darcy's law is limited in applicability to cases where the Reynolds number is on the order of one. For Reynolds numbers less than one, Darcy's law may be considered valid. Deviations from Darcy's law have been shown to occur at Reynolds numbers as low as 2, depending upon such factors as grain size and shape. The Reynolds number N_R is defined herein as

$$N_R = \rho q d/\mu \tag{7-7}$$

where q = specific discharge
d = mean grain diameter

For many conditions of practical importance (zones lying adjacent to collecting devices are an exception) Darcy's law has been found to apply.

Of special interest is the fact that the Darcy equation is analogous to Ohm's law

$$i = (1/R)E \tag{7-8}$$

where i is the current, R the resistance, and E the voltage. Current and velocity are analogous as are K, $1/R$, E, and dh/dx. Similarity of the two equations is the basis for electric analog models of groundwater-flow systems.[4,38]

Example 7-1 Water temperature in an aquifer is 60°F and rate of water movement = 1.2 ft/day. Average particle diameter in the prous medium is 0.08 in. Find the Reynolds number and indicate whether Darcy's law is applicable.

Equation 7-7 gives the Reynolds number as

$$N_R = \rho q d/\mu$$

This may also be written as

$$N_R = qd/\upsilon$$

From Table 1 in Appendix B, υ is found to be 1.21×10^{-5} sq ft/sec. Converting the velocity q into units of ft/sec, $q = 1.2/86{,}400 = 1.39 \times 10^{-5}$. Mean grain diameter in feet = 0.08/12 = 0.0067. Substituting these values in the equation,

$$N_R = \frac{1.39 \times 10^{-5} \times 0.0067}{1.21 \times 10^{-5}}$$

$$= 0.0077$$

Since $N_R < 1.0$, Darcy's law does apply.

7-10 Permeability

Hydraulic conductivity K is an important parameter, often separated into two components, one related to the medium the other to the fluid. The product

$$k = Cd^2 \tag{7-9}$$

called the *specific* or *intrinsic permeability*, is a function of the medium only. In Eq. 7-9 d represents mean grain diameter of the particles; C is a constant shape factor associated with packing, size distribution, and other factors.[2,4] Using this definition, hydraulic conductivity (also known as *coefficient of permeability*) can be written as

$$K = k\gamma/\mu \tag{7-10}$$

Dimensions of intrinsic permeability are L^2. Since values of k given as ft^2 or cm^2 are extremely small, a unit of measure known as the *darcy* has been widely adopted.

$$1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2 \quad \text{or} \quad 1.062 \times 10^{-11} \text{ ft}^2$$

Several ways of expressing hydraulic conductivity are reported in the literature. The U.S. Geological Survey has defined the standard coefficient of permeability K_s as the number of gallons per day of water passing through one square foot of medium under a unit hydraulic gradient at a temperature of 60°F. Another measure, called the *field coefficient of permeability* K_f, is defined as

$$K_f = K_s(\mu_{60}/\mu_f) \tag{7-11}$$

where μ_{60} = dynamic viscosity of water at 60°F

μ_f = dynamic viscosity at prevailing field temperature

The temperature effect on density is neglected since it is usually quite small over the range of groundwater temperatures encountered in practice.

It is often convenient to use the coefficient of transmissivity

$$T = K_f b \tag{7-12}$$

where K_f is the field hydraulic conductivity and b the saturated depth of the aquifer.

Table 7-1 gives values of the intrinsic permeability and standard coefficient of permeability for several classes of materials. Considerable

TABLE 7-1 Some Values of the Standard Coefficient of Permeability and Intrinsic Permeability for Several Classes of Materials

Material	Approximate range K_s gal/day/ft^2	Approximate range k darcys
Clean gravel	10^6–10^4	10^5–10^3
Clean sands; mixtures of clean gravels and sands	10^4–10	10^3–1
Very fine sands; silts; mixtures of sands, silts, clays; stratified clays	10–10^{-3}	1–10^{-4}
Unweathered clays	10^{-3}–10^{-4}	10^{-4}–10^{-5}

variation within divisions can occur, hence a careful geologic survey should accompany all groundwater studies.

Example 7-2 Laboratory tests of an aquifer material give a standard coefficient of permeability $K_s = 3.78 \times 10^2$ gpd/sq ft. If prevailing field temperature is 50°F, find the field coefficient of permeability K_f.

Using Eq. 7-11,

$$K_f = K_s(\mu_{60}/\mu_f)$$

From Table 1, Appendix B, the kinematic viscosity at 60°F = 1.21×10^{-5} sq ft/sec and at 50°F it is 1.41×10^{-5} sq ft/sec. For constant density,

$$K_f = \frac{3.78 \times 10^2 \times 1.21 \times 10^{-5}}{1.41 \times 10^{-5}}$$

and

$$K_f = 3.24 \times 10^2 \text{ gpd/sq ft}$$

7-11 Velocity Potential

Potential theory is directly applicable to groundwater-flow computations. The *velocity potential* ϕ is a scalar function of time and space. Potential is defined by

$$\phi(x,y,z) = -K\left(z + \frac{p}{\gamma}\right) + C \tag{7-13}$$

where C is an arbitrary constant. By definition, its derivative with respect to any given direction is the velocity of flow in that direction. Thus it may be written that

$$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y} \qquad w = \frac{\partial \phi}{\partial z} \tag{7-14}$$

where u, v, and w are velocities in the x, y, and z directions respectively and K is assumed constant. In vector notation this becomes

$$V = \text{grad}\,\phi = \nabla\phi \tag{7-15}$$

with V the combined velocity vector and

$$\text{grad}\,\phi = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z} = \nabla\phi \tag{7-16}$$

7-12 Hydrodynamic Equations

Determination of values for variables u, v, w, and h is the target of most groundwater-flow problems. The first three variables are the specific discharge components in the x, y, and z directions respectively, while h is the total head at a specified point in the flow domain. To effect a solution, four equations involving these variables are needed. These are the equations of motion in each direction plus the continuity equation.

The equations of motion are based on Newton's second law,

$$F = ma \tag{7-17}$$

where F is the force, m the mass, and a the acceleration. Considering forces acting on a fluid element, accelerations in the three coordinate directions may be determined according to Eq. 7-17. If frictionless flow is assumed, the body forces plus the surface force (pressure) must be equivalent to the total force in each direction. In the manner of Harr,[3] the following equations (Euler's equations) in the three coordinate directions are obtained:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{7-18}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} \tag{7-19}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{7-20}$$

where X, Y, and Z are body forces per unit mass in each coordinate direction. For steady flow [u, v, w, and h, $\neq f(t)$] the first terms in the left-hand side of each equation vanish. With laminar groundwater flow in the range of validity of Darcy's law, velocities are small (often on the order of 5 ft/yr to 5 ft/day).[4] Thus for steady laminar flow, Eqs. 7-18 through 7-20 reduce to

$$X = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad Y = \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad Z = \frac{1}{\rho}\frac{\partial p}{\partial z} + g \tag{7-21}$$

In most groundwater-flow problems the velocity head is negligible and thus p may be given as $\rho g(h - z)$. Then Eq. 7-21 becomes

$$X = g\frac{\partial h}{\partial x} \qquad Y = g\frac{\partial h}{\partial y} \qquad Z = g\frac{\partial h}{\partial z} \tag{7-22}$$

Remembering that Darcy's law defines $\partial h/\partial x = -u/K$, etc., it follows that

$$X = -gu/K \qquad Y = -gv/K \qquad Z = -gw/K \tag{7-23}$$

For steady laminar flow, the body forces are linear functions of velocity and Eqs. 7-18 to 7-20 may be written as

$$g\frac{\partial h}{\partial x} = -g\frac{u}{K} \tag{7-24}$$

$$g\frac{\partial h}{\partial y} = -g\frac{v}{K} \tag{7-25}$$

$$g\frac{\partial h}{\partial z} = -g\frac{w}{K} \tag{7-26}$$

where

$$\left.\begin{aligned} u &= -K\partial h/\partial x \\ v &= -K\partial h/\partial y \\ w &= -K\partial h/\partial z \end{aligned}\right\} \tag{7-27}$$

This demonstrates that the equations of motion fit Darcy's law for steady laminar flow.

The continuity equation may be stated as[20]

$$\frac{\partial \rho}{\partial t} + \partial\frac{(\rho u)}{\partial x} + \partial\frac{(\rho v)}{\partial y} + \partial\frac{(\rho w)}{\partial z} = 0 \tag{7-28}$$

This equation is valid for a compressible fluid with time dependent properties. In steady compressible flow the first term becomes zero, and for steady incompressible flow the equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7-29}$$

Now since $u = \partial\phi/\partial x$ etc., Eq. 7-29 becomes

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{7-30}$$

which is known as the *Laplace equation*. With steady-state laminar flow, groundwater motion is completely described by the continuity equation subject to appropriate boundary conditions.

If hydraulic conductivity K is constant, Eq. 7-30 can be written as

$$\nabla^2 h = 0 \tag{7-31}$$

the expression of steady incompressible flow in a homogeneous, isotropic porous medium.

For unsteady flow, the compressibility of both aquifer and water are

pertinent. Consider a small element of porous medium which has a volume $\Delta x \Delta y \Delta z$. Then the term in a continuity equation representing change in storage is defined by

$$\frac{\partial(\rho n \, \Delta x \, \Delta y \, \Delta z)}{\partial t} \tag{7-32}$$

Presupposing that compressive forces are predominant in the vertical (z) direction, lateral changes can be neglected. Thus in terms of the element described, only Δz is considered variable. A storage expression written as the sum of three terms involving partial derivatives of the variables Δz, ρ, and porosity n is[2]

$$\frac{\partial(\rho n \, \Delta x \, \Delta y \, \Delta z)}{\partial t} = \left(n\rho \frac{\partial(\Delta z)}{\partial t} + \rho \Delta z \frac{\partial n}{\partial t} + n \Delta z \frac{\partial \rho}{\partial t} \right) \Delta x \, \Delta y \tag{7-33}$$

The three elements on the right can be expressed in terms of pore pressure p, aquifer compressibility α, and fluid compressibility β.[2,4]

Fluid compressibility is defined as the reciprocal of its bulk modulus of elasticity. It is given by[4]

$$\beta = -\frac{\partial V/V}{\partial p} \tag{7-34}$$

where V is the volume and p the pore pressure. If the piezometric surface of a confined aquifer is lowered a distance of one unit, the amount of water released from a column of aquifer of unit horizontal cross-sectional area is defined as the storage coefficient S. This is analogous to the specific yield S_y of an unconfined aquifer. Obviously in Eq. 7-34 S is equivalent to ∂V. Further, if the aquifer column is of height b, $V = b$. The change in pressure ∂p is equivalent to the negative product of the change in head (one unit) and specific weight of water. Making these substitutions in Eq. 7-34,

$$\beta = S/\gamma b \tag{7-35}$$

Now if the aquifer material is considered elastic, that is, if Δz and n can be modified, the volume change can be expressed in terms of alteration in density of the material due to difference in packing. Thus

$$(\partial V/V) = -(\partial \rho/\rho) \tag{7-36}$$

Introducing Eqs. 7-35 and 7-36 into Eq. 7-34 gives

$$\partial \rho = \frac{\rho S}{b\gamma} \partial p \tag{7-37}$$

Next, substituting this expression for $\partial \rho$ in Eq. 7-28,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = -\frac{\rho S}{b\gamma} \frac{\partial p}{\partial t} \tag{7-38}$$

The left-hand side of this equation can be expanded to

$$\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\left(u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right) \tag{7-39}$$

The second term is normally very small compared with the first and can be neglected. Validity of this assumption improves as the flow angle decreases. Using Eq. 7-39 and the foregoing assumption, Eq. 7-38 becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=-\frac{S}{b\gamma}\frac{\partial p}{\partial t} \tag{7-40}$$

or if isotropic conditions prevail,

$$K\nabla^2 h=\frac{S}{b\gamma}\frac{\partial p}{\partial t} \tag{7-41}$$

since from Eq. 7-27 $u=-K\partial h/\partial x$, etc. Inserting γh for p and the transmissivity T for Kb produces

$$\nabla^2 h=\frac{S}{T}\frac{\partial h}{\partial t} \tag{7-42}$$

which is the general equation for unsteady flow in a confined aquifer of constant thickness b.

Storage coefficient S and transmissivity are commonly called the *formation constants* of a confined aquifer. For an unconfined aquifer Eq. 7-42 reverts to

$$\nabla^2 h=\frac{S}{Kb}\frac{\partial h}{\partial t} \tag{7-43}$$

since b is a function of change in head. The unsteady flow equation for an unconfined aquifer is nonlinear in form. Solution of such an equation is discussed by Jacob.[22] Where variations in saturated thickness of unconfined aquifers are minor, Eq. 7-42 may be used as an approximation.[4]

For unconfined aquifers, the right-hand side of Eq. 7-43 is often negligible so that the equation

$$\nabla^2 h=0 \tag{7-31}$$

is frequently valid for both steady and unsteady flow.

7-13 Flowlines and Equipotential Lines

Many problems of practical interest in groundwater hydrology can be considered two-dimensional flow problems. The equation of continuity for steady incompressible flow in an isotropic medium then becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7-44}$$

and

$$\nabla^2 h=\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0 \tag{7-45}$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{7-46}$$

The Laplace equation is satisfied by two conjugate harmonic functions ϕ and ψ.[3,4] Curves $\phi(x, y)$ = constant are orthogonal to the curves $\psi(x, y)$ = constant. The function $\phi(x, y)$ is the velocity potential, the function $\psi(x, y)$ is known as the *stream function* and is defined by

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \tag{7-47}$$

Substituting Eq. 7-47 into Eq. 7-44 yields

$$\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial^2\psi}{\partial y\,\partial x} = 0 \tag{7-48}$$

It has already been shown that

$$u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}$$

so we can write

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \tag{7-49}$$

These are known as the *Cauchy-Riemann equations*. The stream function satisfies both the equation of continuity and the equations of Cauchy-Riemann. It can also be shown that the Laplace equation is satisfied and therefore[2,3]

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \tag{7-50}$$

Refer now to Fig. 7-4. If V is a velocity vector tangent to a particle flow path 3–4, then it can be decomposed into two components u and v.[20] By geometry of the figure

$$\frac{v}{u} = \frac{dy}{dx} = \tan\alpha \tag{7-51}$$

and thus

$$v\,dx - u\,dy = 0 \tag{7-52}$$

If Eqs. 7-47 are substituted into Eq. 7-51, then

$$\frac{\partial\psi}{\partial x}\,dx + \frac{\partial\psi}{\partial y}\,dy = 0 \tag{7-53}$$

The total differential $d\psi$ is equal to zero, and ψ must be a constant. A series of curves $\psi(x, y)$ equal to a succession of constants can be drawn and will be tangent at all points to the velocity vectors. These curves trace the flow path of a fluid particle and are known as *streamlines* or *flowlines*. An important property of the stream function is demonstrated with the

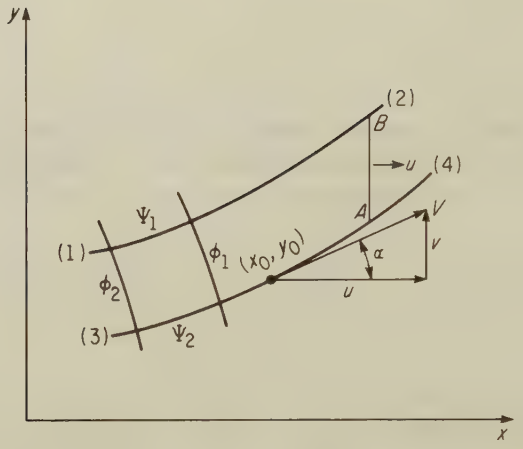


FIG. 7-4. Definition sketch for a stream function.

aid of Fig. 7-4. Consider the flow crossing a vertical section AB between streamlines defined as ψ_1 and ψ_2. If the discharge across the section is designated as Q, it is apparent that

$$Q = \int_{\psi_2}^{\psi_1} u \, dy \tag{7-54}$$

or

$$Q = \int_{\psi_2}^{\psi_1} d\psi \tag{7-55}$$

and

$$Q = \psi_1 - \psi_2 \tag{7-56}$$

Equation 7-56 illustrates the important property that flow between two streamlines is constant. Streamline spacing reveals the relative magnitudes of flow velocities between them. Higher values are associated with narrower spacings, and vice versa.

Referring again to Fig. 7-4, curves designated as ϕ_1 and ϕ_2 called *equipotential lines* are determined by velocity potentials $\phi(x, y)$ = constant. These curves intersect flowlines at right angles, illustrated in the following way. The total differential $d\phi$ is given by

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy \tag{7-57}$$

Substituting for terms $\partial\phi/\partial x$ and $\partial\phi/\partial y$ their equivalents u and v makes

$$u \, dx + v \, dy = 0 \tag{7-58}$$

and

$$\frac{dy}{dx} = -\frac{u}{v} \tag{7-59}$$

Thus equipotential lines are normal to flowlines. The system of flowlines and equipotential lines forms a flow net.

One significant point of difference between ϕ and ψ functions is that equipotential lines exist only when flow is irrotational. For two-dimensional flow the condition of irrotationality is said to exist when the z component of vorticity ζ_z is zero, or

$$\zeta_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{7-60}$$

Proof of this is given by Eskinazi.[20] Substituting for u and v in Eq. 7-60 in terms of ϕ,

$$\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} = 0 \tag{7-61}$$

This indicates that when the velocity potential exists, the criterion for irrotationality is satisfied.

Once either streamlines or equipotential lines in a flow domain are determined the other is automatically known because of relationships Eq. 7-49. Thus

$$\psi = \int \left(\frac{\partial \phi}{\partial x}\, dy - \frac{\partial \phi}{\partial y}\, dx\right) \tag{7-62}$$

and

$$\phi = \int \left(\frac{\partial \psi}{\partial y}\, dx - \frac{\partial \psi}{\partial x}\, dy\right)$$

It is enough then to determine only one of the functions since the other can be obtained using relations Eq. 7-62. The complex potential given by

$$w = \phi + i\psi \tag{7-63}$$

where i is the square root of -1 is widely used in analytic flow net analyses.[2,3] Of special importance is the fact that

$$\nabla^2 w = \nabla^2 \phi + i \nabla^2 \psi = 0 \tag{7-64}$$

satisfies the conditions of continuity and irrotationality simultaneously.

Equations presented in this section have been limited to the case of two-dimensional flow. Extension to three dimensions would be obtained in a similar fashion.

7-14 Boundary Conditions

To solve groundwater-flow problems it is necessary that appropriate boundary conditions be specified. Some of the more commonly encountered ones are described in this section. More comprehensive discussions will be found elsewhere.[13,23]

Boundary conditions discussed can be categorized as follows: impervious boundaries; surfaces of seepage; constant head boundaries; and lines of seepage (free surfaces).

Impervious boundaries may be man-made objects such as concrete dams, rock strata, or soil strata which are highly impervious. In Fig. 7-5

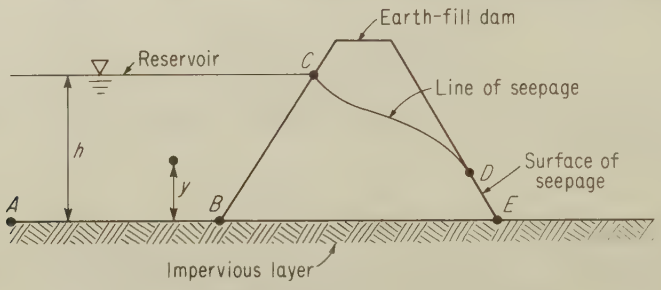


FIG. 7-5. Some common boundary conditions.

the impervious boundary AB represents such a limit. Since flow cannot cross an impervious boundary, velocity components normal to it vanish and the impervious boundary is a streamline. In other words, at the boundary, ψ = constant.

Next look at the upstream face of the earth dam BC. At any point of elevation y along BC the pressure can be assumed hydrostatic, or

$$p = \gamma(h - y) \tag{7-65}$$

The definition of a velocity potential states that

$$\phi = -K\left(\frac{p}{\gamma} + y\right) + C \tag{7-66}$$

Substituting for pressure in Eq. 7-66 yields

$$\phi = -K\left(\frac{\gamma(h - y)}{\gamma} + y\right) + C \tag{7-67}$$

and

$$\phi = -Kh + C \tag{7-68}$$

Thus for a constant reservoir level h and an isotropic medium,

$$\phi = \text{constant}$$

and surface BC, often termed a *reservoir boundary*, is an equipotential line.

The free surface or line of seepage CD in Fig. 7-5 is seen to be a boundary between the saturated and unsaturated zones. Since flow does not occur across this boundary, it is obviously also a streamline. Pressure along this free surface must be constant, and therefore along CD

$$\phi + Ky = \text{constant} \tag{7-69}$$

This is a linear relationship in ϕ, and therefore equal vertical falls along CD must be associated with successive equipotential drops. One important groundwater-flow problem is to determine the location of the line of seepage.

The surface of seepage DE of Fig. 7-5 represents the location at which water seeps through the downstream face of the dam and trickles toward point E. Pressure along DE is atmospheric. The surface of seepage is neither a flowline or an equipotential line.

7-15 Flow Nets

Flow nets, or graphical representations of families of streamlines and equipotential lines, are widely used in groundwater studies to determine quantities, rates and directions of flow. Use of flow nets is limited to steady incompressible flow at constant viscosity and density for homogeneous media or for regions which can be compartmentalized into homogeneous segments. Darcy's law must be applicable to the flow conditions.

The manner in which a flow net can be used in problem solving is best explained with the aid of Fig. 7-6. This diagram shows a portion of a flow

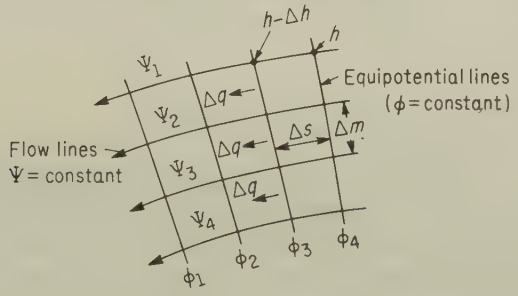


FIG. 7-6. Segment of an orthogonal flow net.

net constructed so that each unit bounded by a pair of streamlines and equipotential lines is approximately square. The reason for this will be clear later.

A flow net can be determined exactly if functions ϕ and ψ are known beforehand. This is often not the case, and as a result, graphically constructed flow nets have been much used. Preparation of a flow net requires application of the concept of square elements and adherence to boundary conditions. Graphical flow nets are usually difficult for a beginner to create but with reasonable practice an acceptable net can be drawn. Various mechanical methods for graphical flow net construction are presented in the literature and will not be discussed here.[3,23]

After a flow net has been constructed, it can be analyzed using geometry of the net and by applying Darcy's law.

Remembering that $h = (p/\gamma + z)$, Fig. 7-6 shows the hydraulic gradient G_h between two equipotential lines is given by

$$G_h = \Delta h/\Delta s \tag{7-70}$$

Then applying Darcy's law, in the manner of Todd[4] the flow increment between adjacent streamlines is

$$\Delta q = K\Delta m(\Delta h/\Delta s) \tag{7-71}$$

where Δm represents the cross-sectional area for a net of unit width normal to the plane of the diagram. If the flow net is constructed in an orthogonal manner and composed of approximately square elements,

$$\Delta m \approx \Delta s$$

and

$$\Delta q = K\,\Delta h \tag{7-72}$$

Now if there are n equipotential drops between the equipotential lines, it is evident that

$$\Delta h = \frac{h}{n}$$

where h is the total head loss over the n spaces. If the flow is divided into m sections by the flow lines, then the discharge per unit width of the medium will be

$$Q = \sum_{i=1}^{m} \Delta q = Kmh/n \tag{7-73}$$

When the medium's hydraulic conductivity is known, the discharge can be computed using Eq. 7-73 and a knowledge of flow-net geometry.

Where the flow net has a free surface or line of seepage, the entrance and exit conditions given on Fig. 7-5 will be useful. A more comprehensive discussion of these conditions is given in Ref. 24.

Some trouble arises in flow net construction at locations where the velocity becomes infinite or vanishes. Such points are known as *singular points* and according to DeWiest may be placed in three separate categories.[2] In the first classification flowlines and equipotential lines do not intersect at right angles. Such a situation often occurs when a boundary coincides with a flowline. Point A in Fig. 7-7 is an example.

The second classification has a discontinuity along the boundary which abruptly changes the slope of the streamline. In Fig. 7-8, points A, B, and C represent such discontinuities. At points A and C the velocity is infinite, while at point B it is zero. If the angle of discontinuity measured in a counterclockwise direction inside the flow field is less than 180°, the velocity is zero; if larger than 180°, it is infinite. The angle at A is 270°, for example.

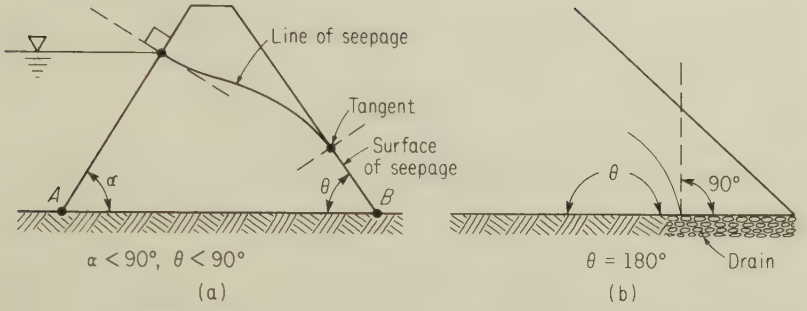


FIG. 7-7. Some entrance and exit conditions for the line of seepage. (After Casagrande, Ref. 24.)

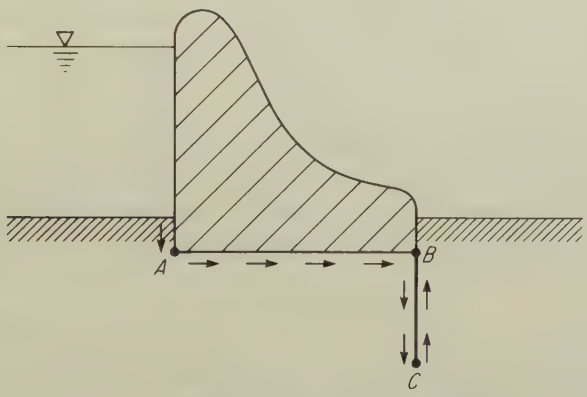


FIG. 7-8. Flowline slope discontinuities.

The third category includes the case where a source or sink exists in the flow net. Under these circumstances the velocity is infinite, since squares of the flow net approach zero size as the source or sink is approached. Wells and recharge wells represent sinks and sources in a practical sense and will be discussed later.

7-16 Variable Hydraulic Conductivity

It is common for flow within a porous medium of one hydraulic conductivity to enter another region with a different permeability. When such a boundary is crossed, flowlines are refracted. The change in direction which occurs can be determined as a function of the two permeabilities involved in the manner of Todd and DeWiest.[2,4] Figure 7-9 illustrates this.

Consider two soils of permeabilities K_1 and K_2 which are separated by the boundary LR shown on the figure. Direction of the flowlines before and after crossing the boundary is defined by angles θ_1 and θ_2.

For continuity to be preserved, the velocity components in media K_1

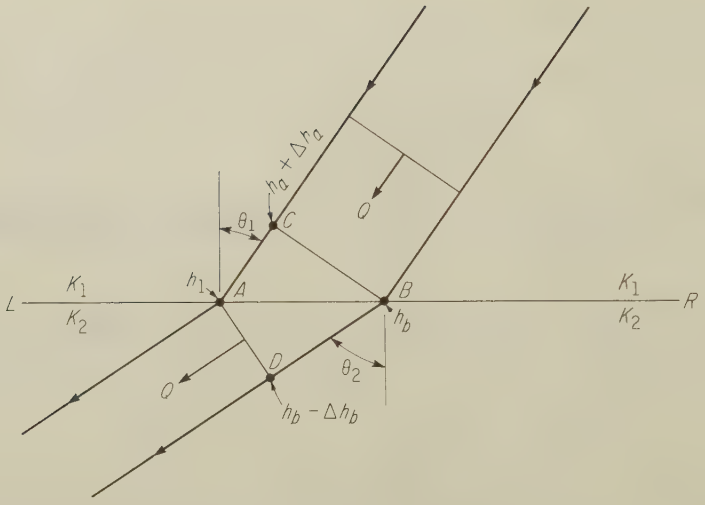


FIG. 7-9. Flowline refraction.

and K_2 which are normal to the boundary must be equal, since the cross-sectional area at the boundary is AB for a unit depth. Using Darcy's law and noting the equipotential drops h_a and h_b,

$$K_1 \frac{\Delta h_a}{AC} \cos \theta_1 = K_2 \frac{\Delta h_b}{BD} \cos \theta_2 \tag{7-74}$$

From the geometry of the figure it is apparent that

$$AC = AB \sin \theta_1$$
$$BD = AB \sin \theta_2$$

The head loss between A and B is shown on the figure to be equal to both Δh_a and Δh_b and since there can be only a single value,

$$\Delta h_a = \Delta h_b$$

Introducing these expressions in Eq. 7-74 produces

$$\frac{K_1}{\tan \theta_1} = \frac{K_2}{\tan \theta_2} \tag{7-75}$$

For refracted flow in a saturated porous medium, the ratio of the tangents of angles formed by the intersection of flowlines with normals to the boundary is given by the ratio of hydraulic conductivities. As a result of refraction, the flow net on the K_2 side of the boundary will no longer be squares if the equipotential line spacing DB is maintained. To adjust the net on the K_2 side, the relation

$$\frac{\Delta h_b}{\Delta h_a} = \frac{K_1}{K_2} \tag{7-76}$$

can be used where $\Delta h_b \neq \Delta h_a$.

Equipotential lines are also refracted in crossing permeability boundaries. The relationship for this is

$$\frac{K_1}{K_2} = \frac{\tan \alpha_2}{\tan \alpha_1} \tag{7-77}$$

where α is the angle between the equipotential line and a normal to the boundary of permeability.[2]

7-17 Anisotropy

In many cases hydraulic conductivity is dependent upon the direction of flow within a given layer of soil. This condition is said to be anisotropic. Sedimentary deposits often fit this aspect, with flow occurring more readily along the plane of deposition than across it. Where the permeability within a plane is uniform but very small across it as compared to that along the plane, a flow net can still be used after proper adjustments are made. A discussion of this is given elsewhere.[2,3,33] Nonhomogeneous aquifers require special consideration but may sometimes be analyzed by using representative or average parameters. A detailed study is outside the scope of this book.[2,3,38]

7-18 Dupuit's Theory

Groundwater-flow problems in which one boundary is a free surface can be analyzed on the basis of Dupuit's theory of unconfined flow. This theory is founded on two assumptions made by Dupuit in 1863.[14] First, if the line of seepage is only slightly inclined, streamlines may be considered horizontal and correspondingly, equipotential lines will be essentially vertical. Second, slopes of the line of seepage and the hydraulic gradient are equal. Where field conditions are known to be satisfactorily represented by these assumptions, results obtained according to Dupuit's theory compare very favorably with those arrived at by more rigorous techniques.

Figure 7-10 is useful in translating the foregoing assumptions into a mathematical statement. Consider an element given in the figure which has a base area $dx\,dy$ and a vertical height h. Writing the continuity equation in the x direction and considering steady flow to be the case,

$$\text{Inflow}_{x_0} = \text{velocity}_{x_0} \times \text{area}_{x_0} \tag{7-78}$$

The velocity at $x = 0$ is given by Darcy's law as

$$u_{x_0} = -K \frac{\partial h}{\partial x} \tag{7-79}$$

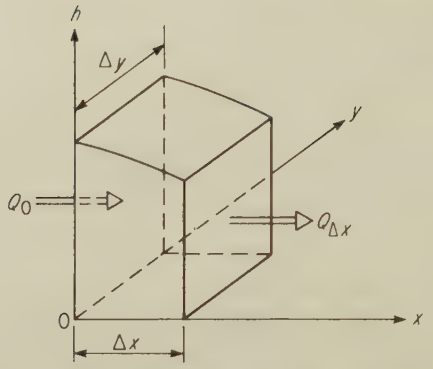


FIG. 7-10. Definition sketch for development of Dupuit's equation.

Thus the discharge across the element at $x = 0$ is

$$Q_0 = -K \frac{\partial h}{\partial x} h \, dy \tag{7-80}$$

Outflow at $x = dx$ is obtained by a Taylor's series expansion as

$$Q_{dx} = -K \frac{\partial h}{\partial x} h \, dy + dx \frac{\partial}{\partial x} \left(-K \frac{\partial h}{\partial x} h \, dy \right) + \cdots \tag{7-81}$$

Subtracting outflow from inflow if K is considered constant, we obtain

$$I_x - O_x = K \, dx \, dy \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) \tag{7-82}$$

or

$$I_x - O_x = \frac{K \, dx \, dy}{2} \frac{\partial}{\partial x} \left(\frac{\partial h^2}{\partial x} \right) \tag{7-83}$$

where dx and dy are considered fixed lengths. A similar consideration in the y direction yields

$$I_y - O_y = \frac{K \, dx \, dy}{2} \frac{\partial}{\partial y} \left(\frac{\partial h^2}{\partial y} \right) \tag{7-84}$$

Assuming there is no movement in the vertical direction, these are the only components of inflow and outflow. Further, still dealing with steady flow, the change in storage must be zero. As a result,

$$\frac{K \, dx \, dy}{2} \frac{\partial}{\partial x} \left(\frac{\partial h^2}{\partial x} \right) + \frac{K \, dx \, dy}{2} \frac{\partial}{\partial y} \left(\frac{\partial h^2}{\partial y} \right) = 0 \tag{7-85}$$

and since $(K \, dx \, dy)/2$ is constant, this reduces to

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0 \tag{7-86}$$

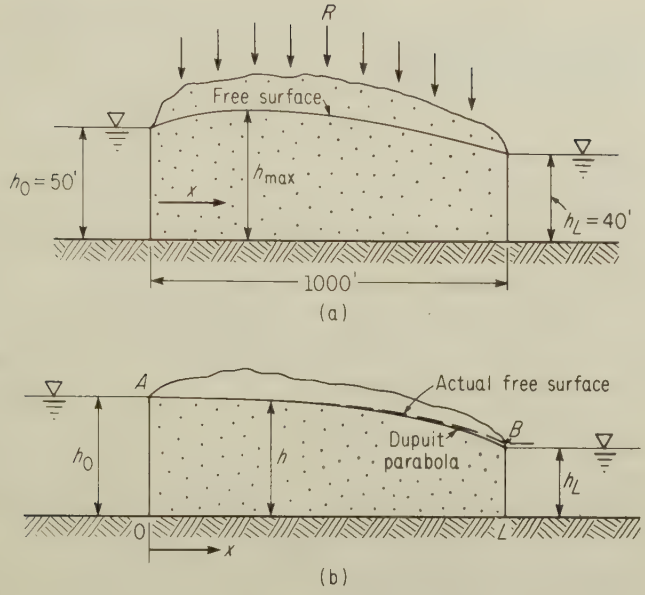


FIG. 7-11. Steady flow in a porous medium between two water bodies: (a) free surface with infiltration; and (b) free surface without infiltration.

or

$$\nabla^2 h^2 = 0 \tag{7-87}$$

Consequently according to Dupuit's assumptions, Laplace's equation for the function h^2 must be satisfied.[25]

In the particular case where recharge is occurring as a result of infiltrated water reaching the water table, a simple adjustment may be made to Eq. 7-86. If recharge intensity (dimensionally LT^{-1}), is specified as R, then total recharge to the element of Fig. 7-10 will be $R\,dx\,dy$ and the continuity equation for steady flow becomes,

$$K\frac{dx\,dy}{2}\left(\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2}\right) + R\,dx\,dy = 0 \tag{7-88}$$

or more simply,

$$\nabla^2 h^2 + \frac{2}{K}R = 0 \tag{7-89}$$

Now applying Dupuit's theory to the flow problem illustrated on Fig. 7-11b, and assuming one-dimensional flow in the x direction only, the discharge per unit width of the aquifer given by Darcy's law is

$$Q = -Kh\frac{dh}{dx} \tag{7-90}$$

In this instance h is the height of the line of seepage at any position x along the impervious boundary. For the one-dimensional example considered here, Eq. 7-86 becomes

$$\frac{d^2h^2}{dx^2} = 0 \tag{7-91}$$

Upon integration,

$$h^2 = ax + b \tag{7-92}$$

where a and b are constants.

Then for boundary conditions at $x = 0, h = h_0$,

$$b = h_0^2 \tag{7-93}$$

Differentiation of Eq. 7-92 yields

$$2h\frac{dh}{dx} = a \tag{7-94}$$

Also from Darcy's equation, $hdh/dx = -Q/K$. Making this substitution,

$$a = -2Q/K \tag{7-95}$$

and inserting values of the constants in Eq. 7-92,

$$h^2 = -2\frac{Q}{K}x + h_0^2 \tag{7-96}$$

This is the equation of a free surface. It is a parabola (often called *Dupuit's parabola*). If the existance of a surface of seepage at B is ignored and noting that at $x = L, h = h_L$, Eq. 7-96 becomes

$$h_L^2 = -\frac{2QL}{K} + h_0^2 \tag{7-97}$$

or

$$Q = \frac{K}{2L}(h_0^2 - h_L^2) \tag{7-98}$$

which is known as the *Dupuit equation*.

Example 7-3 Refer to Fig. 7-11a. Given the dimensions shown and a recharge intensity R of 0.01 ft/day, find the discharge at $x = 1{,}000$ ft using Dupuit's equation. Assume $K = 8$.

Solution:

Note that

$$\frac{dQ}{dx} = R$$

or

$$Q = Rx + C$$

at $x = 0$,

$$Q = Q_0$$

Therefore

$$Q = R_x + Q_0$$

Also

$$Q = -Kh\frac{dh}{dx}$$

$$-Kh\frac{dh}{dx} = Rx + Q_0$$

Integrating,

$$\frac{-Kh^2}{2}\Bigg|_{h_0}^{h_L} = \frac{Rx^2}{2}\Bigg|_0^L + Q_0 x\Bigg|_0^L$$

and inserting the limits,

$$\frac{-K(h_L^2 - h_0^2)}{2} = \frac{RL^2}{2} + Q_0 L$$

$$Q_0 = \frac{K(h_0^2 - h_L^2)}{2L} - \frac{RL}{2}$$

Then since $Q = Rx + Q_0$,

$$Q = R\left(x - \frac{L}{2}\right) + \frac{K(h_0^2 - h_L^2)}{2L}$$

$$R = 0.01 \times 7.5 = 0.075 \text{ gal/day/ft}^2$$

$$Q = 0.075(1000 - 500) + \frac{8(50^2 - 40^2)}{2000}$$

$$= 0.075 \times 500 + \frac{8 \times 900}{2000}$$

$$= 37.5 + 3.6$$

$$= 41.1 \text{ gpd/sq ft}$$

7-19 Methods for Developing Groundwater Supplies

Development of groundwater supplies is accomplished mainly through wells or infiltration galleries. Many factors are involved in the performance of these collection works and a thorough knowledge of groundwater-flow mechanics and regional geology is essential. Some groundwater-flow problems can be solved by applying relatively simple mathematical tools. Other problems require more rigorous analyses. Graphical studies and model analyses are also widely employed.

Flow to Wells

A well system can be considered as composed of three elements—the well structure, pump, and discharge piping.[15] The well itself contains an

open section through which water enters and a casing to transport the flow to the ground surface. The open section is usually a perforated casing or slotted metal screen permitting water to enter and at the same time preventing collapse of the hole. Occasionally gravel is placed at the bottom of the well casing around the screen.

When a well is pumped, water is removed from the aquifer immediately adjacent to the screen. Flow then becomes established at locations some distance from the well in order to replenish this withdrawal. Due to flow resistance offered by the soil, a head loss results and the piezometric surface adjacent to the well is depressed, producing a cone of depression (Fig. 7-12), which spreads until equilibrium is reached and steady-state conditions are established.

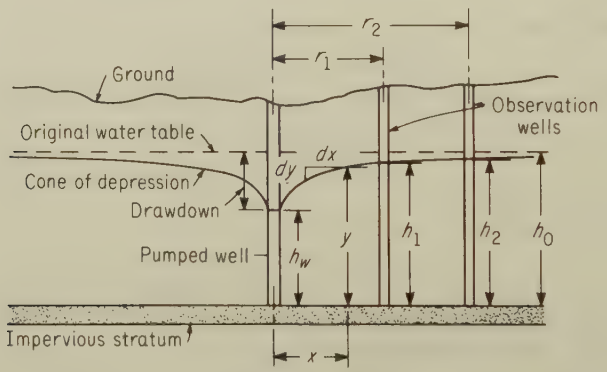


FIG. 7-12. Well in an unconfined aquifer.

The hydraulic characteristics of an aquifer (which are described by the storage coefficient and aquifer permeability) can be determined by laboratory or field tests. The three most commonly used field methods are application of tracers, use of field permeameters, and aquifer performance tests.[4] A discussion of aquifer performance tests will be given here along with the development of flow equations for wells.[11,15,16]

Aquifer performance tests may be either (1) equilibrium or (2) non-equilibrium tests. In (1) the cone of depression must be stabilized for a flow equation to be derived. For (2) the derivation includes a condition that steady-state conditions have not been reached. Adolph Thiem published the first performance tests based on equilibrium conditions in 1906.[8]

7-20 Steady Unconfined Radial Flow toward a Well

The basic equilibrium equation for an unconfined aquifer can be derived using the notation of Fig. 7-12. Here flow is assumed to be radial, the original water table is considered to be horizontal, the well is pre-

sumed to fully penetrate the aquifer of infinite aereal extent, and steady-state conditions must prevail. Then flow toward the well at any distance x away must equal the product of the cylindrical element of area at that section and the flow velocity. With Darcy's law this becomes

$$Q = 2\pi xyK_f \frac{dy}{dx} \tag{7-99}$$

where $2\pi xy$ = area at any section
$K_f\, dy/dx$ = flow velocity
Q = discharge, ft^3/sec

Integrating over the limits specified,

$$\int_{r_1}^{r_2} Q \frac{dx}{x} = 2\pi K_f \int_{h_1}^{h_2} y\, dy \tag{7-100}$$

$$Q \log_e \frac{r_2}{r_1} = \frac{2\pi K_f (h_2^2 - h_1^2)}{2} \tag{7-101}$$

and

$$Q = \frac{\pi K_f (h_2^2 - h_1^2)}{\log_e (r_2/r_1)} \tag{7-102}$$

This equation can then be solved for K_f, yielding

$$K_f = \frac{1055 Q \log_{10}(r_2/r_1)}{h_2^2 - h_1^2} \tag{7-103}$$

where $\log_e$ has been converted to the $\log_{10}$, K_f is in gallons per day per square foot, Q in gallons per minute, and r and h are in feet. If the drawdown is small compared with the total aquifer thickness, an approximate formula for discharge of the pumped well can be obtained by inserting h_w for h_1 and the height of the aquifer for h_2 in Eq. 7-102.

Example 7-4 An 18-in. well fully penetrates an unconfined aquifer of 100-ft depth. Two observation wells located 100 ft and 235 ft from the pumped well are known to have drawdowns of 22.2 ft and 21 ft respectively. If the flow is steady and K_f = 1,320 gpd/ft^2, what would be the discharge?

Solution:
Equation 7-102 is applicable, and for the given units this is

$$Q = \frac{K(h_2^2 - h_1^2)}{1055 \log_{10}(r_2/r_1)}$$

$$\log_{10}(r_2/r_1) = \log_{10}(235/100) = 0.37107$$

$$h_2 = 100 - 21 = 79 \text{ ft}$$

$$h_1 = 100 - 22.2 = 77.8 \text{ ft}$$

$$Q = \frac{1320(79^2 - 77.8^2)}{1055 \times 0.37107}$$

$$= 634.44 \text{ gpm}$$

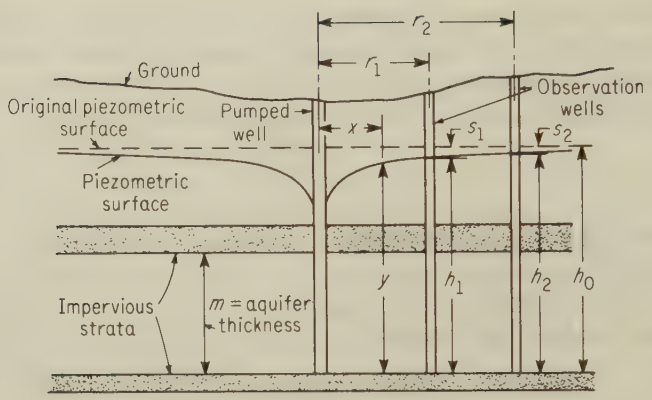


FIG. 7-13. Radial flow to a well in a confined aquifer.

7-21 Steady Confined Radial Flow toward a Well

The basic equilibrium equation for a confined aquifer can be obtained in a similar manner, using the notation of Fig. 7-13. The same assumptions apply. Mathematically the flow in cubic feet per second is found from:

$$Q = 2\pi x m K_f \frac{dy}{dx} \tag{7-104}$$

Integrating,

$$Q = 2\pi K_f m \frac{h_2 - h_1}{\log_e (r_2/r_1)} \tag{7-105}$$

The coefficient of permeability may be determined by rearranging Eq. 7-105 to the form

$$K_f = \frac{528 Q \log_{10} (r_2/r_1)}{m(h_2 - h_1)} \tag{7-106}$$

in which Q is in gallons per minute, K_f is the permeability in gallons per day per square feet, and r and h are measured in feet.

Example 7-5 Determine the permeability of an artesian aquifer being pumped by a fully penetrating well. The aquifer is 90 ft thick and composed of medium sand. The steady-state pumping rate is 850 gpm. Drawdown of an observation well 50 ft away is 10 ft; in a second observation well 500 ft away it is 1 ft.

Solution:

$$K_f = \frac{528 Q \log_{10} (r_2/r_1)}{m(h_2 - h_1)}$$

$$= \frac{528 \times 850 \times \log_{10} (10)}{90 \times (10 - 1)}$$

$$= 554 \text{ gpd/ft}^2$$

7-22 Well in a Uniform-Flow Field

For a steady-state well in a uniform-flow field where the original piezometric surface is not horizontal, a somewhat different situation from that previously assumed prevails. Consider the artesian aquifer shown in Fig. 7-14. The heretofore assumed circular area of influence becomes dis-

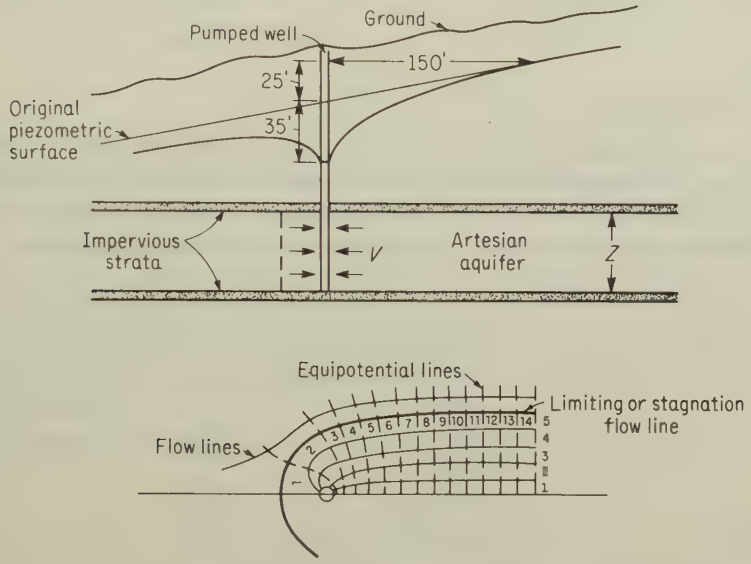


FIG. 7-14. Well in a uniform flow field and flow net definition.

torted in this case. A solution is possible by applying potential theory; by graphical means; or, if the slope of the piezometric surface is very slight, Eq. 7-105 may be employed without serious error.

Referring to the definition sketch of Fig. 7-14, a graphical solution to this type of problem will be discussed. First an orthogonal flow net consisting of flowlines and equipotential lines must be constructed. Formation should be performed so that the completed flow net will be composed of a number of elements which approach little squares in shape. Once the net is complete, it can be analyzed by considering the net geometry and using Darcy's law in the manner of Todd.[4]

Example 7-6 Find the discharge to the well of Fig. 7-14 by using an applicable flow net. Consider the aquifer to be 35 ft thick, $K_f = 3.65 \times 10^{-4}$ fps, and other dimensions as shown.

Solution:
Using Eq. 7-73,

$$q = \frac{Kmh}{n}$$

where $h = (35 + 25) = 60$ ft

$m = 2 \times 5 = 10$

$n = 14$

$$q = \frac{3.65 \times 10^{-4} \times 60 \times 10}{14}$$

$= 0.0156$ cfs per unit thickness of the aquifer

Total discharge Q is thus

$$Q = 0.0156 \times 35 = 0.55 \text{ cfs, or } 245 \text{ gpm}$$

7-23 Well Fields

When more than one unit in a well field is pumped, there is a composite effect on the free-water surface. This consequence is illustrated by Fig. 7-15 in which the cones of depression are seen to overlap. The drawdown at a given location is equal to the sum of the individual drawdowns.

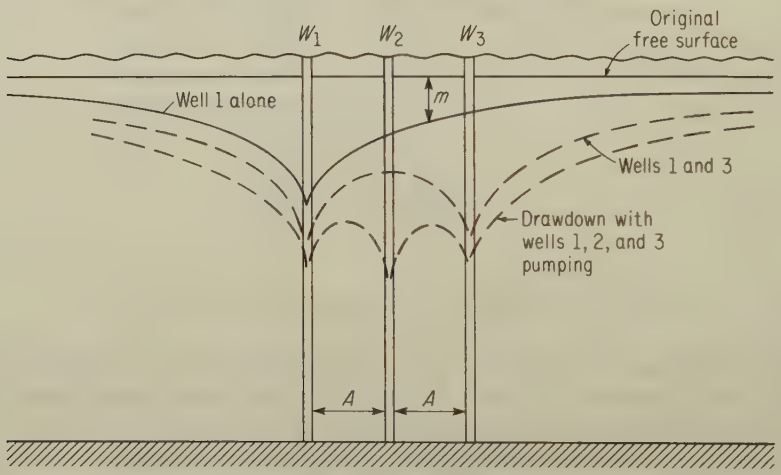


FIG. 7-15. Combined effect of pumping several wells at equal rates.

If within a particular well field, pumping rates of the pumped wells are known, the composite drawdown at a point can be determined. In like manner, if the drawdown at one point is known the well flows can be calculated.

If the drawndown at a given point is designated as m, and subscripts $1, 2, \ldots, n$ are used to relate this drawdown to a particular well (for example, m_1 refers to the drawdown for W_1) for the total drawdown m_T at some location,[4]

$$m_T = \sum_{i=1}^{n} m_i \qquad (7\text{-}107)$$

The number of wells, their rate of pumping, and well-field geometry and characteristics determine the total drawdown at a specified location.

Again considering Eq. 7-102,

$$h_0^2 - h^2 = \frac{Q}{\pi K} \log_e (r_0/r) \tag{7-108}$$

it can be seen that the drawdown for a well pumped at rate Q can be computed if h_0, r_0, and r are known. It follows then from Eq. 7-107 that total drawdown for n pumped wells in an unconfined aquifer is given by

$$h_0^2 - h^2 = \sum_{i=1}^{n} \frac{Q_i}{\pi K} \log_e \left(\frac{r_{0i}}{r_i}\right) \tag{7-109}$$

where h_0 is the original height of the water table, h in this case is the combined effect height of the water table after pumping n wells, Q_i is the flow rate of the ith well, r_{0i} is distance of the ith well to a location at which the drawdown is considered negligible, and r_i is the distance from well i to the point at which the drawdown is being investigated. Todd indicates that values of r_0 used in practice often range from 500 to 1,000 ft.[4] The impact of this assumption is softened because Q in Eq. 7-108 is not very sensitive to r_0. Equation 7-109 should be used only where drawdowns are relatively small.

For flow in a confined aquifer the expression for combined drawdown becomes

$$h_0 - h = \sum_{i=1}^{n} \frac{Q_i}{2\pi Km} \log_e \left(\frac{r_{0i}}{r_i}\right) \tag{7-110}$$

Equations for well flow covering a variety of particular well-field patterns are reported in the literature.[4] Those given here are applicable for steady flow in a homogeneous isotropic medium.

7-24 The Method of Images

Some groundwater-flow problems subjected to boundary conditions negating direct use of radial-flow equations can be transformed into infinite systems fitting these equations by applying the method of images.[15,22,49]

When a stream is located near a pumped well and the stream and aquifer are interconnected, the drawdown curve of a pumped well may be affected as shown on Fig. 7-16. Another boundary condition often affecting drawdown of a well is an impervious formation that limits extent of the aquifer. The cone of depression of a pumped well is not affected until the boundary is intersected. After that, the shape of the drawdown curve will be changed by the boundary. Boundary effects can frequently be evaluated by means of so-called "image wells." The boundary condition is replaced by either a recharging or a discharging well which is pumped or recharged at a rate equivalent to that of the pumped well.

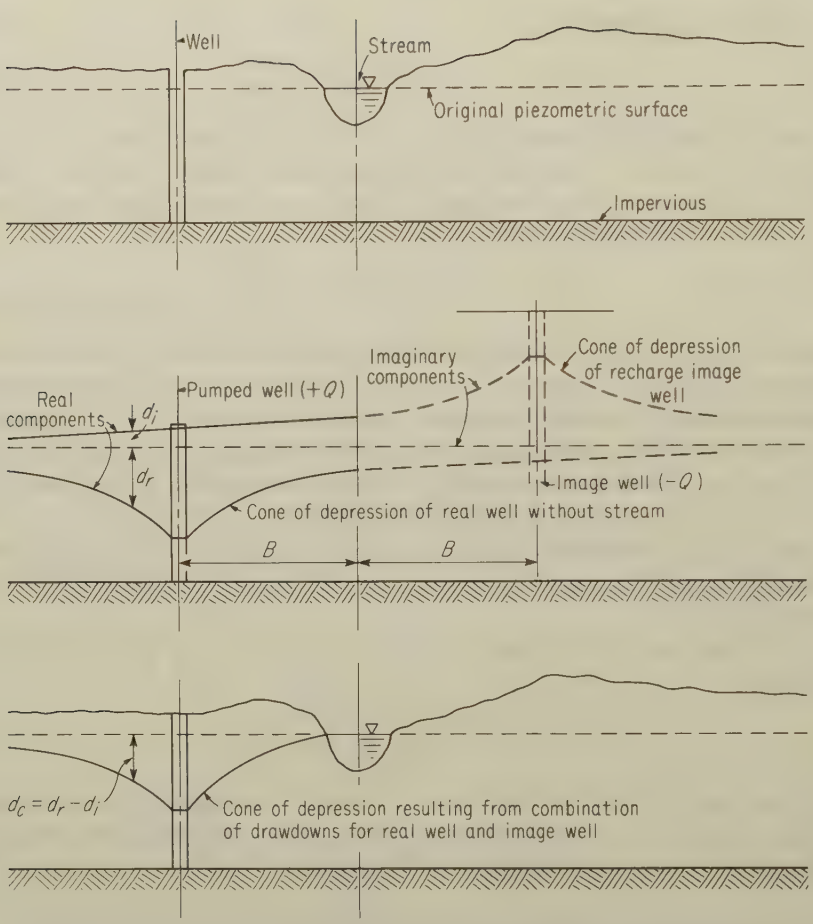


FIG. 7-16. Drawdown in a pumping well whose aquifer is connected to a stream.

That is, in an infinite aquifer, drawdowns of the real and image wells would be identical. The image well is located at a distance from the boundary equal to that of the real well but on the opposite side (Fig. 7-16). Streams are replaced by recharge wells while impermeable boundaries are supplanted by pumped image wells. Computations for the case of a well and impervious boundary directly follow the procedures outlined under the section on well fields. For the well and stream system, the recharge image well is considered to have a negative discharge. The heads are then added according to this sign convention.

The procedure for combining drawdown curves of real and image wells to obtain an actual drawdown curve is illustrated graphically for the example shown in Fig. 7-16. More detailed information on other cases can be found elsewhere.[2,49]

7-25 Unsteady Flow

When a new well is first pumped, a large portion of the discharge comes directly from the storage volume released as the cone of depression develops. Under these circumstances the equilibrium equations overestimate permeability and therefore yield of the well. Where steady-state conditions are not encountered—as is usually the situation in practice—a nonequilibrium equation must be used. Two approaches can be taken, the rather rigorous method of C. V. Theis, or a simplified procedure such as that proposed by Jacob.[9,10]

In 1935 Theis published a nonequilibrium approach which takes into consideration time and storage characteristics of the aquifer.[9] His method utilizes an analogy between heat transfer described by the Biot-Fourier law, and groundwater flow to a well. Theis states that the drawdown(s) in an observation well located at a distance r from the pumped well is given by

$$s = \frac{114.6Q}{T} \int_u^\infty \frac{e^{-u}}{u}\, du \tag{7-111}$$

where T = transmissibility
Q = discharge, gpm

and

$$u = \frac{1.87r^2S_c}{Tt} \tag{7-112}$$

where S_c = storage coefficient
T = transmissibility, gpd/ft
t = time in days since start of pumping

The integral in Eq. 7-111 is usually known as the *well function* of u and commonly written as $W(u)$. It may be evaluated from the infinite series

$$W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} + \cdots \tag{7-113}$$

The basic assumptions employed in the Theis equation are essentially the same as those in the Thiem equation except for the nonsteady-state condition. Some values of this function are given in Table 7-2.

Equations 7-111 and 7-112 can be solved by comparing a log-log plot of u vs. $W(u)$ known as a *type curve*, with a log-log plot of the observed data r^2/t vs. s. In plotting type curves, $W(u)$ and s are ordinates, u and r^2/t are abscissas. The two curves are superimposed and moved about until segments coincide. In this operation the axes must remain parallel. A coincident point is then selected on the matched curves and both plots marked. The type curve then yields values of u and $W(u)$ for the desired point. Corresponding values of s and r^2/t are determined from a plot of

TABLE 7-2 Values of $W(u)$ for Various Values of u*

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

*After Wenzel, Ref. 26.

the observed data. Inserting these values in Eqs. 7-111 and 7-112 and rearranging, values for transmissibility T and storage coefficient S_c can be found.

Often this procedure can be shortened and simplified. When r is small and t large, Jacob found that values of u are generally small.[10] Thus terms in the series of Eq. 7-113 beyond the second one become negligible and the expression for T becomes

$$T = \frac{264Q(\log_{10}t_2 - \log_{10}t_1)}{h_0 - h} \tag{7-114}$$

which further can be reduced to

$$T = 264Q/\Delta h \tag{7-115}$$

where Δh = drawdown per log cycle of time $[(h_0 - h)/(\log_{10}t_2 - \log_{10}t_1)]$
Q = well discharge, gpm

h_0 and h are defined on Fig. 7-13, and T is the transmissibility in gallons per day per foot. Field data on drawdown $(h_0 - h)$ vs. t are drafted on semilogarithmic paper. The drawdown is plotted on an arithmetic scale, Fig. 7-17. This plot forms a straight line whose slope permits computing formation constants using Eq. 7-115 and

$$S_c = \frac{0.3Tt_0}{r^2} \tag{7-116}$$

with t_0 the time corresponding to zero drawdown.

Example 7-7 Using the following data, find the formation constants for an aquifer using a graphical solution to the Theis equation. Discharge equals 540 gpm.

Distance from pumped well, ft, r	r^2/t	Average drawdown, ft, s
50	1,250	3.04
100	5,000	2.16
150	11,250	1.63
200	20,000	1.28
300	45,000	.80
400	80,000	.51
500	125,000	.33
600	180,000	.22
700	245,000	.15
800	320,000	.10

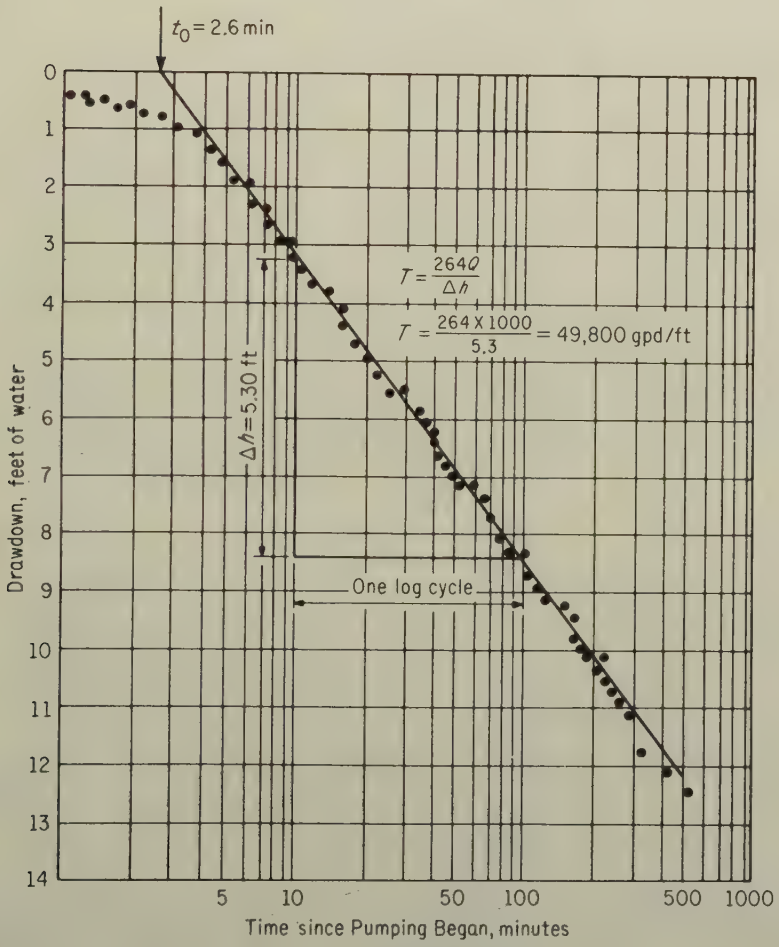


FIG. 7-17. Pumping test data.

Solution:

Plot s vs. r^2/t and $W(u)$ vs. u as shown on Fig. 7-18. Determine the match point as noted and compute S_c and T using Eqs. 7-111 and 7-112,

$$T = \frac{114.6Q}{s} W(u)$$

$$= \frac{114.6 \times 540}{1.28} \times 1.9 = 99{,}500 \text{ gpd/ft}$$

$$S_c = \frac{uT}{1.87r^2/t}$$

$$= \frac{0.09 \times 99{,}500}{1.87 \times 20{,}000} = 0.240$$

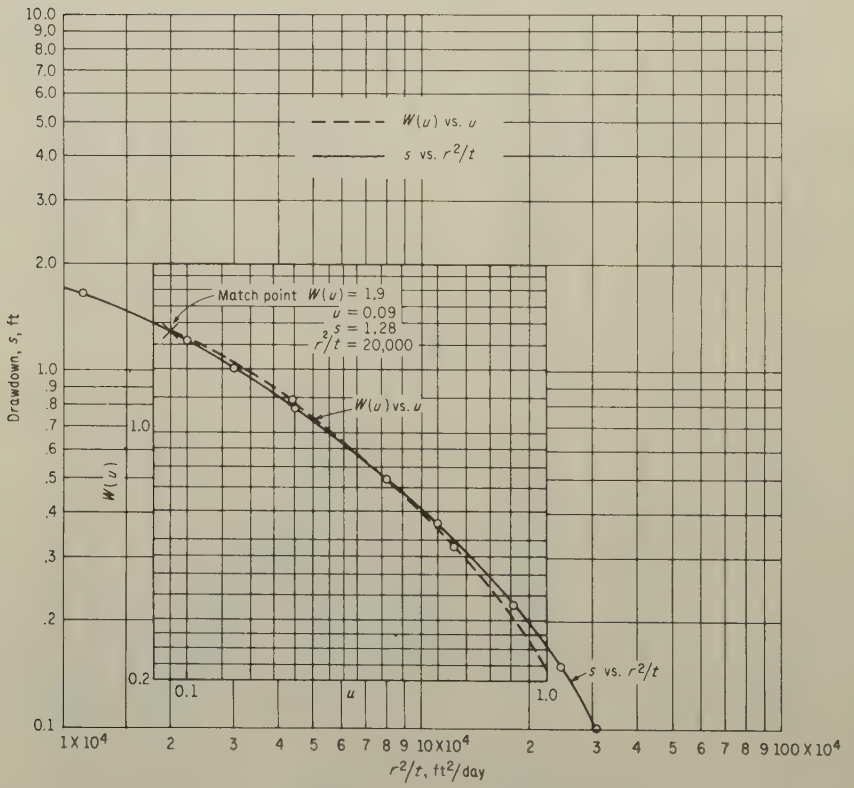


FIG. 7-18. Graphical solution to Theis' equation.

Example 7-8 Using the data given in Fig. 7-17, find the coefficient of transmissibility T and storage coefficient S_c for an aquifer. Given Q = 1,000 gpm and r = 300 ft.

Solution:

Find the value of Δh from the graph, 5.3 ft. Then by Eq. 7-115.

$$T = \frac{264Q}{\Delta h} = \frac{264 \times 1000}{5.3}$$

$$= 49{,}800 \text{ gpd/ft}$$

Using Eq. 7-116,

$$S_c = \frac{0.3Tt_0}{r^2}$$

Note from Fig. 7-16 that $t_0 = 2.6$ min. Converting to days, this becomes

$$t_0 = 1.81 \times 10^{-3} \text{ days}$$

and

$$S_c = \frac{0.3 \times 49{,}800 \times 1.81 \times 10^{-3}}{(300)^2}$$

$$= 0.0003$$

7-26 Leaky Aquifers

The foregoing analyses have dealt with free aquifers or those confined between impervious strata. In reality, many cases exist wherein the confining strata are not completely impervious and water is actually transferred from them to the productive aquifer. The flow regime is altered and computations must include leakage. Since about 1930, leaky aquifers have been the subject of research by investigators such as De Glee, Jacob, Hantush, DeWiest, Walton, and others.[27–36,38] A thorough treatment of their work is beyond the scope of this book, and interested readers should consult the indicated references.

7-27 Partially Penetrating Wells

In many actual situations there is only partial penetration of the well. The question then arises as to the applicability of procedures developed previously for full penetration.

Numerous studies of this problem have been conducted.[13,37,39] In 1957 Hantush reported that steady flow to a well just penetrating an infinite leaky aquifer becomes very nearly radial at a distance from the well of about 1.5 the aquifer thickness.[39] As depth of penetration increases, the approach to radial flow becomes increasingly apparent. Therefore, computation of drawdowns for partially penetrating wells are made using equations for total penetration with relative safety—providing the distance from the pumped well is greater than 1.5 times the aquifer thickness. At points closer to the well, it is frequently possible to use a flow net or other relationships developed for this region.[2,4,38]

7-28 Salt-Water Intrusion

Contamination of fresh groundwater by intrusion of salt water often presents a serious quality problem. Islands and coastal regions are particularly vulnerable. Aquifers located inland sometimes contain highly saline waters as well. Fresh water is lighter than salt water (specific gravity of the latter is about 1.025) and forms a fresh-water layer above the underlying salt water. This equilibrium is disturbed when an aquifer is pumped, since salt water replaces the fresh water removed. Under equilibrium conditions, a drawdown of 1 ft in a fresh-water table corresponds to a rise of about 40 ft by salt water. Wells subjected to salt-water intrusion obviously have limited pumping rates.

Recharge wells have been drilled in coastal areas to maintain a head sufficient to preclude sea-water intrusion, a practice employed effectively in Southern California.

7-29 Computers and Numerical Methods in Groundwater Hydrology

Many advances in the application of computers and numerical methods have taken place since the late 1940's.[41–45]

Electric analogs solve a wide variety of groundwater-flow problems,[2,43,45] and consist essentially of a resistance-capacitance network. Table 7-3 indicates the manner in which components of the electric analog and the actual flowfield are related.

TABLE 7-3 Elements of a Groundwater Reservoir and an Electric Analog Compared

Groundwater reservoir component	Corresponding electric analog component
Hydraulic conductivity	Resistivity
Aquifer storage	Capacitance
Head	Voltage
Volumetric flow rate	Amperage

Digital computers have also proven to be versatile tools for use in groundwater studies.[2,44,47] The applicable mathematical model is usually written in finite-difference or finite-element form.

7-30 Groundwater-Basin Development

To utilize groundwater resources efficiently while simultaneously permitting maximum development of the resource, equilibrium must be established between withdrawals and replenishments. Economic, legal, political, social, and water-quality aspects require full consideration.

Lasting supplies of groundwater will be assured only when long-term withdrawals are balanced by recharge during the corresponding period. The potential of a groundwater basin can be assessed by employing the water-budget equation,

$$\Sigma I - \Sigma O = \Delta S$$

where the inflow ΣI includes all forms of recharge, total outflow ΣO includes every kind of discharge, and ΔS represents change in storage during the accounting period. The most significant forms of recharge and discharge are those listed in Table 7-4.

TABLE 7-4 Some Important Forms of Recharge and Discharge

Recharge	Discharge
Seepage from streams, ponds, lakes	Seepage to lakes, streams, springs
Subsurface inflows	Subsurface outflows
Infiltrated precipitation	Evapotranspiration
Waters recharged artificially	Pumping or other artificial means of collection

A groundwater hydrologist must be able to estimate the quantity of water that can be economically and safely produced from a groundwater basin in a specified time period. He should also be competent to evaluate the consequences of imposing various rates of withdrawal on an underground supply.

Development of groundwater basins should be based on careful study since groundwater resources are finite and exhaustible. If the various types of recharge balance withdrawals from a basin over a period of time, no difficulty will be encountered. Excessive drafts, however, can deplete underground water supplies to a point where economic development is not feasible. Mining of water will ultimately deplete the entire supply.

References

1. Clark, J. W., W. Viessman, Jr., and M. J. Hammer, *Water Supply and Pollution Control*, 2nd ed. Scranton, Pa.: Intext Educational Publishers, 1965.

2. DeWiest, R. J. M., *Geohydrology*. New York: John Wiley & Sons, Inc., 1965.

3. Harr, M. E., *Groundwater and Seepage*. New York: McGraw-Hill Book Company, 1962.

4. Todd, D. K., *Groundwater Hydrology*. New York: John Wiley & Sons, Inc., 1960.

5. Buras, Nathan, "Conjunctive Operation of Dams and Aquifers," A.S.C.E., *J. Hydr. Div. Proc.*, Vol. 89, No. HY6 (November 1963).

6. Ferris, J. G., "Ground Water," *Mech. Eng.*, January 1960.

7. Thomas, H. E., "Underground Sources of Water," in *Water, The Yearbook of Agriculture.* Washington, D. C.: U. S. Department of Agriculture, 1955.

8. Thiem, G., *Hydrologische Methodern.* Leipzig: Gebhardt, 1906, p. 56.

9. Theis, C. V., "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage," *Trans. Am. Geophys. Union*, Vol. 16 (1935), pp. 519–524.

10. Cooper, H. H., Jr. and C. E. Jacob, "A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well-field History," *Trans. Am. Geophys. Union*, Vol. 27 (1946), pp. 526–534.

11. Hoffman, John F., "How Underground Reservoirs Provide Cool Water for Industrial Uses," *Heating, Piping, and Air Conditioning*, October 1960.

12. Meinzer, O. E., "Outline of Methods for Estimating Groundwater Supplies," U.S. Geological Survey, Water-Supply Paper 638-C, Washington, D. C., 1932.

13. Muskat, M., *The Flow of Homogeneous Fluids Through Porous Media.* Ann Arbor, Mich.: J. W. Edwards, Inc., 1946.

14. Dupuit, Jules, *Etudes théoriques et pratiques sur le mouvement des eau dans les canaux de couverts et à travers les terrains perméables*, 2nd ed. Paris: Dunod, 1863.

15. Hoffman, John F., "Field Tests Determine Potential Quantity, Quality of Ground Water Supply," *Heating, Piping and Air Conditioning*, August 1961.

16. Hoffman, John F., "Well Location and Design," *Heating, Piping and Air Conditioning*, August 1963.

17. Richter, R. C., and R. Y. D. Chun, "Artificial Recharge of Ground Water Reservoirs in California," *Proc. A.S.C.E. J. Irrigation and Drainage Div.*, Vol. 85, No. IR4 (December 1959)

18. Clendenen, F. B., "A Comprehensive Plan for the Conjunctive Utilization of a Surface Reservoir with Underground Storage for Basin-Wide Water Supply Development: Solano Project California," *Doctorate* Eng. thesis, University of California, Berkeley, 1959.

19. Darcy, Henri, *Les fontaines publiques de la ville de Dijon.* Paris: V. Dalmont, 1856.

20. Eskinazi, Salamon, *Principles of Fluid Mechanics.* Boston: Allyn and Bacon, Inc., 1962.

21. Kaplan, W., *Advanced Calculus.* Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1952.

22. Jacob, C. E., "Flow of Groundwater," in Hunter Rouse (ed.), *Engineering Hydraulics.* New York: John Wiley & Sons, Inc., 1950.

23. Taylor, D. W., *Fundamentals of Soil Mechanics.* New York: John Wiley & Sons Inc., 1948.

24. Casagrande, A., "Seepage Through Dams," in *Contributions to Soil Mechanics, 1925–1940.* Boston: Boston Society of Civil Engineers, 1940.

25. Polubarinova-Kochina, P. Ya, "Theory of Groundwater Movement," Princeton, N.J.: Princeton University Press, 1962.

26. Wenzel, L. K., "Methods for Determining Permeability of Water Bearing Materials with Special Reference to Discharging Well Methods," U.S. Geological Survey, Water-Supply Paper 887, Washington, D. C., 1942.

27. De Glee, G. J., *Over Grondwaterstromingen by Waterontrekking by middel van Plutten.* Delft: T. Waltman, Jr., 1930, p. 175.

28. Jacob, C. E., "Radial Flow in a Leaky Artesian Aquifer," *Trans. Am. Geophys. Union*, Vol. 27 (1946), pp. 198–205.

29. Hantush, M. S., "Plain Potential Flow of Groundwater with Linear Leakage," Ph.D. dissertation, University of Utah, 1949.

30. Hantush, M. S., and C. E. Jacob, "Nonsteady Radial Flow in an Infinite Leaky Aquifer and Nonsteady Green's Functions for an Infinite Strip of Leaky Aquifer," *Trans. Am. Geophys. Union*, Vol. 36, (1955), pp. 95–112.

31. Hantush, M. S., and C. E. Jacob, "Flow to an Eccentric Well in a Leaky Circular Aquifer," *J. Geophys. Res.*, Vol. 65 (1960), pp. 3425–3431.

32. Hantush, M. S., "Analysis of Data from Pumping Tests in Leaky Aquifers," *Trans. Am. Geophys. Union*, Vol. 37 (1956), pp. 702–714.

33. Hantush, M. S., "Modification of the Theory of Leaky Aquifers," *J. Geophys. Res.*, Vol. 65 (1960), pp. 3713–3725.

34. DeWiest, R. J. M., "On the Theory of Leaky Aquifers," *J. Geophys. Res.*, Vol. 66 (1961), pp. 4257–4262.

35. DeWiest, R. J. M., "Flow to an Eccentric Well in a Leaky Circular Aquifer with Varied Lateral Replenishment," *Geofis. Pura e Aplic.*, Vol. 54 (1963), pp. 87–102.

36. Walton, W. C., "Leaky Artesian Aquifer Conditions in Illinois," Report of Investigation No. 39, Illinois State Water Survey, 1960.

37. Kirkham, D., "Exact Theory of Flow into a Partially Penetrating Well," *J. Geophys. Res.*, Vol. 64 (1959), pp. 1317–1327.

38. Walton, William C., *Groundwater Resource Evaluation.* New York: McGraw-Hill Book Company, 1970.

39. Hantush, M. S., "Nonsteady Flow to a Well Partially Penetrating an Infinite Leaky Aquifer," *Proc. Isaqi Sci. Soc.*, Vol. 1 (1957), pp. 10–19.

40. Todd, David K., "Ground Water Has to be Replenished," *Chem. Eng. Prog.*, Vol. 59, No. 11 (November 1963).

41. Stallman, R. W., "From Geologic Data . . . To Aquifer Analog Models," *Geotimes*, Vol. V, No. 5 (April 1961).

42. Remson, Irwin, Charles A. Appel, and Raymond A. Webster, "Ground-Water Models Solved by Digital Computer," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 91, No. HY 3 (May 1965).

43. Stallman, Robert W., "Electric Analog of Three-Dimensional Flow to Wells and Its Application to Unconfined Aquifers," United States Department of the Interior Geological Survey, Open File Report, July 26, 1961.

44. Fayers, F. J., and J. W. Sheldon, "The Use of a High-Speed Digital Computer in the Study of the Hydrodynamics of Geologic Basins," *J. Geophys. Res.*, Vol. 67, No. 6 (June 1962), pp. 2421–2431.

45. Brown, Russell H., "Progress in Ground Water Studies with the Electric-Analog Model," *J. A.W.W.A.*, (August 1962), pp. 943–958.

46. Meinzer, O. E., "The Occurrence of Groundwater in the United States," U.S. Geological Survey, Water-Supply Paper No. 489, 1923.

47. Robertson, J. M., *Hydrodynamics in Theory and Application*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965.

48. Shahbazi, M., and D. K. Todd, "Analytic Techniques For Determining Ground Water Flow Fields," Water Resources Center Contribution No. 117, Hydraulic Laboratory, University of California, Berkeley, August 1967.

49. Wisler, C. O., and E. F. Brater, *Hydrology*. New York: John Wiley & Sons Inc., 1959.

Problems

7-1. What is the Reynolds number for flow in a soil when the water temperature is 54°F, velocity 0.7 ft/day, and mean grain diameter 0.07 in.?

7-2. A 12-in. well fully penetrates a confined aquifer 110 ft thick. The coefficient of permeability is 630 gpd/ft.^{2} Two test wells located 40 ft and 120 ft away show a difference in drawdown between them of 9 ft. Find the rate of flow delivered by the well.

7-3. Determine the permeability of an artesian aquifer being pumped by a fully penetrating well. The aquifer composed of medium sand is 100 ft thick. Steady-state pumping rate is 1,300 gpm. The drawdown in an observation well 70 ft away is 13 ft, and in a second observation well 500 ft away is 1.3 ft. Find K_f in gallons per day per square foot.

7-4. Consider a confined aquifer with a coefficient of transmissibility T = 700 ft^3/day/ft. At t = 5 min the drawdown = 5.6 ft; at 50 min, s = 23.1 ft; at 100 min, s = 28.2 ft. The observation well is 60 ft from the pumping well. Find the discharge of the well.

7-5. Assume an aquifer being pumped at a rate of 315 gpm is confined and pumping test data are given below. Find the coefficient of transmissibility T and storage coefficient S. Assume r = 60 ft.

Time since pumping started, min	1.3	2.5	4.2	8.0	11.0	100.0
Drawdown s, ft	4.6	8.1	9.3	12.0	15.1	29.0

7-6. Given the following data:

$$Q = 60{,}000 \text{ ft}^3\text{/day} \qquad t = 30 \text{ days}, r = 1 \text{ ft}$$
$$T = 650 \text{ ft}^3\text{/day/ft} \qquad S_c = 6.4 \times 10^{-4}$$

Assume this to be a nonequilibrium problem. Find the drawdown s. Note for

$$u = 8.0 \times 10^{-9} \qquad W(u) = 18.06$$
$$u = 8.2 \times 10^{-9} \qquad W(u) = 18.04$$
$$u = 8.6 \times 10^{-9} \qquad W(u) = 17.99$$

7-7. Water temperature in an aquifer is 60°F and rate of water movement 1.2 ft/day. Average particle diameter in a porous medium is 0.08 in. Find the Reynolds number and indicate whether Darcy's law applies.

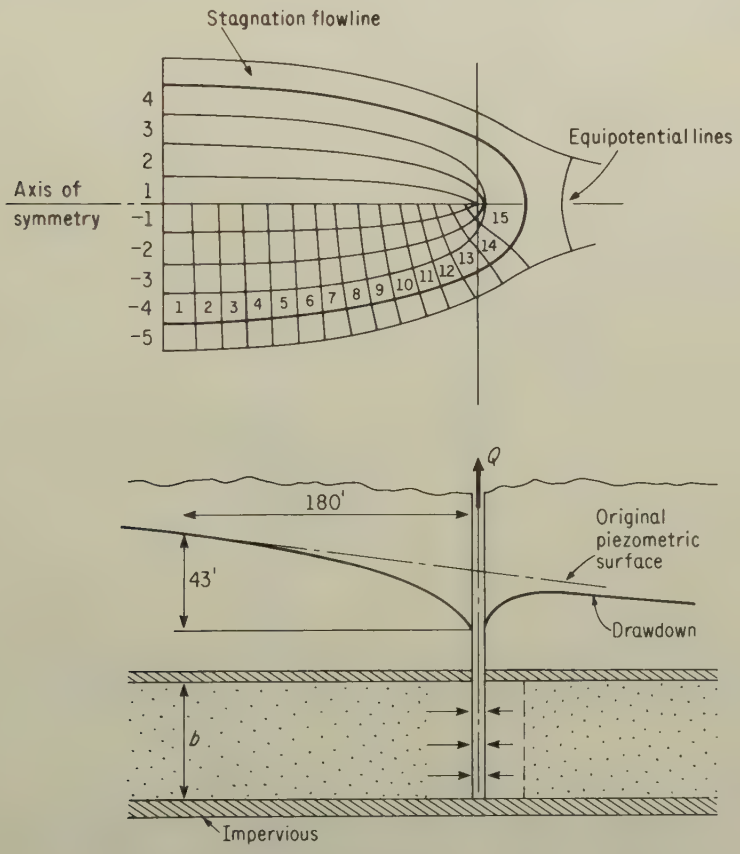
7-8. A laboratory test of a soil gives a standard coefficient of permeability

$K_s = 3.78 \times 10^2$ gpd/ft^2. If the prevailing field temperature is 50°F, find the field coefficient of permeability K_f.

7-9. An 18-in. well fully penetrates an unconfined aquifer 85 ft deep. Two observation wells located 90 ft and 235 ft from the pumped well are known to have drawdowns of 22.8 ft and 21 ft respectively. If flow is steady and $K_f = 1{,}320$ gpd/ft^2, what would be the discharge?

7-10. A confined aquifer 80 ft deep is being pumped under equilibrium conditions at a rate of 760 gpm. The well fully penetrates the aquifer. Water levels in observation wells 150 ft and 230 ft are 96 ft and 98 ft respectively. Find the field coefficient of permeability.

7-11. Given the following well and flow net data, find the discharge using a flow net solution. The well is fully penetrating and the confined aquifer 45 ft thick. $K_f = 2.87 \times 10^{-4}$ ft/sec.



PROB. 7-11

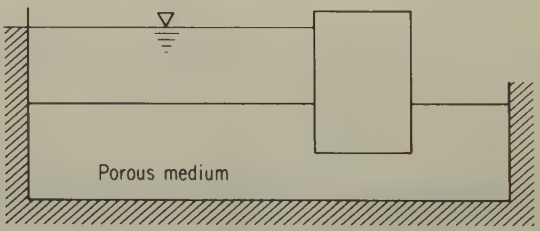
7-12. A well is pumped at the rate of 610 pgm under nonequilibrium conditions. For the data listed, find the formation constants S and T. Use the Theis method.

r^2/t	Average drawdown h, ft
1,250	3.24
5,000	2.18
11,250	1.93
20,000	1.28
45,000	0.80
80,000	0.56
125,000	0.38
180,000	0.22
245,000	0.15
320,000	0.10

7-13. Given a well pumping at a rate of 600 pgm. An observation well is located at r = 200 ft. Find S and T using the Jacob method for the following test data.

Drawdown, ft	Time, min
0.43	26
0.94	78
1.08	99
1.20	131
1.34	173
1.46	218
1.56	266
1.63	303
1.68	331
1.71	364
1.85	481
1.93	573
2.00	661
2.06	732
2.12	843
2.15	926
2.20	1034
2.23	1134
2.28	1272
2.30	1351
2.32	1419
2.36	1520
2.38	1611

7-14. By employing a finite-difference method, find and plot the flowline for ψ = 0.5.



PROB. 7-14

8

MINOR STRUCTURE DESIGN

Accurate prediction of peak discharge rates or synthesizing complete discharge hydrographs for a minor structure are two of the more challenging aspects of engineering hydrology. Minor types of hydraulic structures range from small crossroad culverts, levees, drainage ditches, urban storm-drain systems, and airport drainage structures to the spillway appurtanences of small dams. All require varying amounts of hydrologic design information. Generally a hydrologist is required to provide peak rates of discharge for a design frequency, a stage height at a design frequency, or synthesize a complete discharge hydrograph for a design storm. Other information such as sedimentation rates, low-flow frequency analysis, abstractions, and reservoir yield studies are also a part of the hydrologist's notebook of skills.

Regional analysis of data generally provides information on rainfall or streamflow frequencies. From this information, hydraulic design proportions the size of minor structures. Regional hydrologic techniques are classified into two basic approaches: (1) regionalized rainfall frequency, and (2) regionalized streamflow frequency. These two methods produce peak rates of discharge and often are used to synthesize hydrographs for design purposes. The main disadvantage of the second method results from the relatively short record lengths employed in deriving regionalized frequency curves for small drainage areas.

The regionalized rainfall-frequency approach overcomes the restraint of short record length but introduces another obstacle, that of estimating the abstractions to get rainfall excess or direct runoff (see Chapter 2). This difficulty is also present in unit hydrograph methods. Direct surface runoff must be accurately appraised in lagging techniques to build design storm hydrographs. Thus the peak discharge or design-storm hydrograph

does not necessarily represent the same frequency level as the rainfall used to generate it.

8-1 Frequency Levels in Minor Structures

General Structures

The design frequency, generally encountered in minor structure design and shown in Table 8-1, can and does vary. An example is the design

TABLE 8-1 Minor Structure Design Frequencies

Type of minor structure		Frequency
Highway cross-road drainage		
0–400	ADT	10 yr
400–1,700	ADT	10–25 yr
1,700–5,000	ADT	25 yr
5,000–	ADT	50 yr
Airfields		5 yr
Storm drainage		2–10 yr
Levees		2–50 yr
Drainage ditches		5–50 yr

frequency of a culvert which under cases of excessive backwater could effectively halt interstate traffic. Due to economic consequences of such an event, these structures are generally designed for higher frequencies.

Small Dams

Small dams have not been included in Table 8-1. They generally receive special attention if constructed in populated areas where failure could cause loss of life. When this possibility occurs, the design storm for portions of the dam is established by use of the *Probable maximum precipitation* or PMP. The PMP is defined as the reasonable maximization of the meteorological factors that operate to produce a maximum storm.

Its estimate is based on an investigation by the Environmental Science Service Administration[1,2] conducted to establish the maximum possible amount of precipitable water which could be achieved throughout the United States. Point values of PMP for a 6-hr storm are shown in Fig. 8-1. This chart is helpful in selecting the worst possible conditions to be expected for a given region in the United States.

Small dams customarily are designed for two or more levels of frequency to provide an emergency spillway and insure an adequate allowable freeboard. Figure 8-2 shows a typical small dam with normal and minimum freeboard. These normal values, for earth dams with riprap protection on the upstream slope, has the freeboard based upon wave runup caused by storm winds with 100-mph wind velocites. Minimum

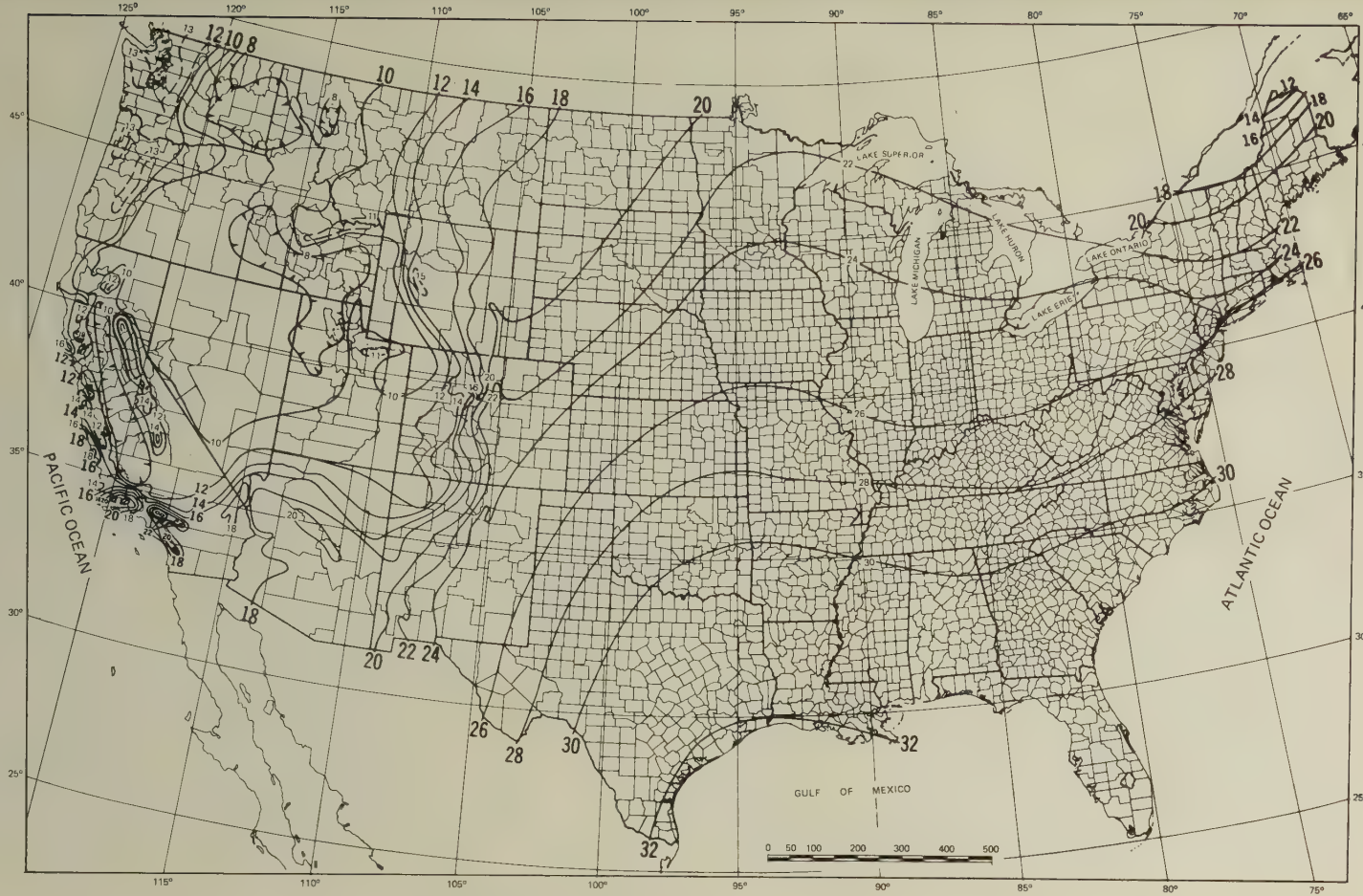


FIG. 8-1. PMP for 6-hr precipitation (in.). (U. S. Weather Bureau, NOAA.)

Fetch miles	N.F.*	M.F.*
<1	4	3
1	5	4
2.5	6	5
5	8	6
10	10	7

*After U.S. Bureau of Reclamation.

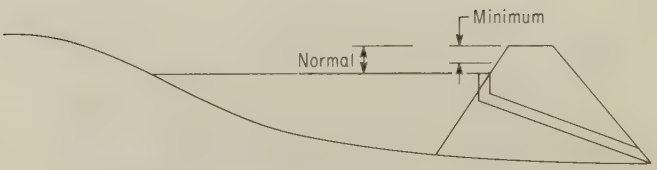


FIG. 8-2. Typical freeboard requirements.

freeboard values pertain to wind velocities of 50 mph. If smooth concrete rather than riprap is used on the upstream face the freeboard values shown should be increased 50 percent.[3]

Design frequency requirements are altered to fit planned or forseeable use of the structures. The SCS[4] classifies structures into three groups:

Class (a). Structures located in rural or agricultural areas where failure may damage farm buildings, agricultural land, or township or county roads.

Class (b). Structures located in predominately rural or agricultural areas where failure may damage isolated homes, main highways or minor railroads or cause interruption of use or service of relatively important public utilities.

Class (c). Structures located where failure may cause loss of life, serious damage of homes, industrial and commercial buildings, important public utilities, main highways, or railroads.

The physical size of a small dam may range to over 100 ft in height but generally is restricted to structures retarding less than 25,000 acre-ft of storage at the emergency spillway crest.

Rainfall-frequency levels for use in synthesizing a design storm hydrograph of both the emergency spillway and freeboard pool level are given in Table 8-2 for the three classifications noted. These values are based upon 6-hr rainfall depths for (a) the 100-yr frequency (Fig. 8-3), and (b) the PMP (Fig. 8-1). Design storm depths for all watersheds having a

TABLE 8-2 Small-Dam Frequency Requirements

Classification	Emerg. spillway design	Freeboard pool design
(a)	P^*_{100}	$P_{100} + .12(\text{PMP} - P_{100})$
(b)	$P_{100} + .12(\text{PMP} - P_{100})$	$P_{100} + .40(\text{PMP} - P_{100})$
(c)	$P_{100} + .26(\text{PMP} - P_{100})$	PMP

*P_{100} represents a 6-hr rainfall depth for the 100-yr frequency event. (After Soil Conservation Service.)

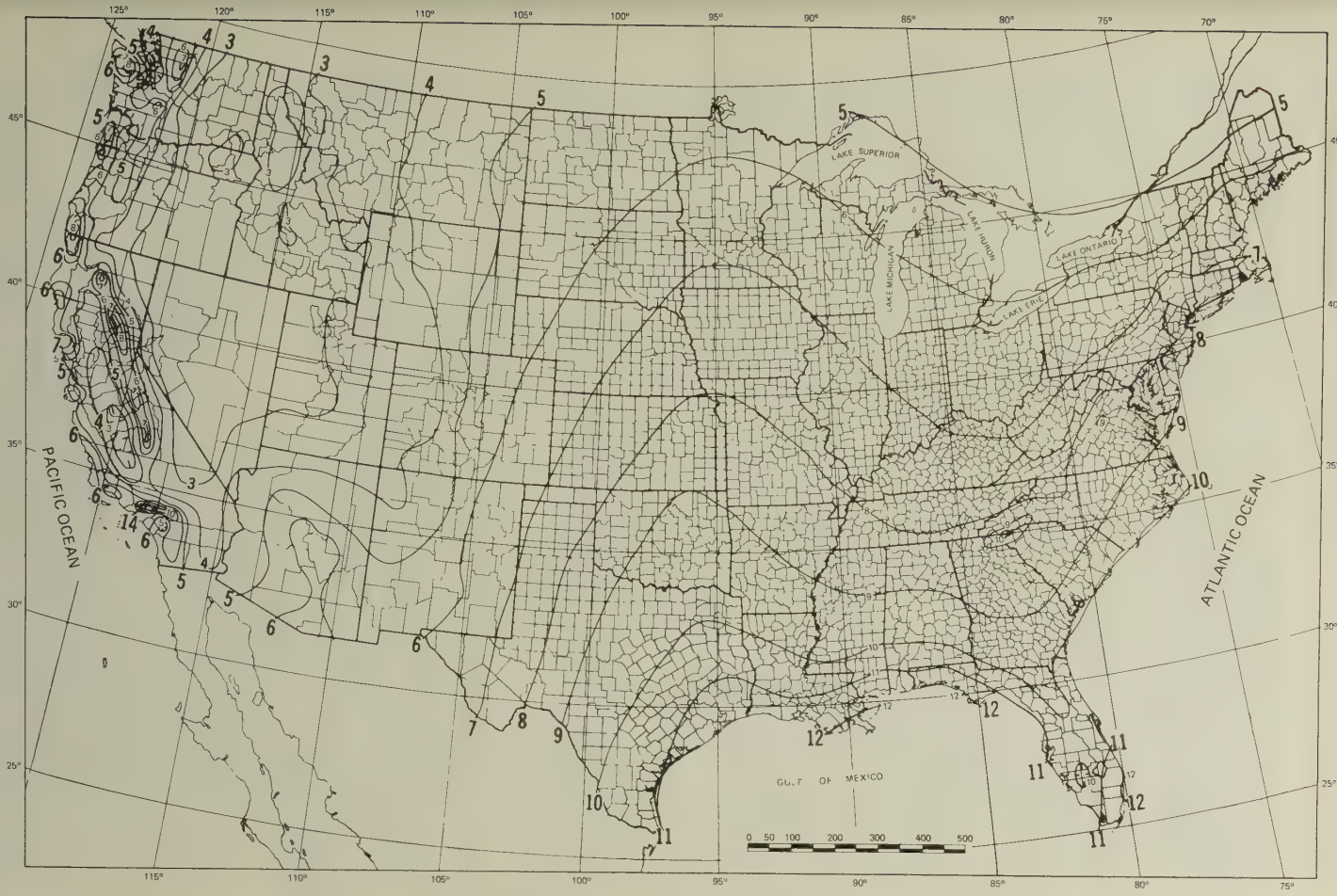


FIG. 8-3. 100-yr frequency 6-hr precipitation (in.). (U. S. Weather Bureau, NOAA.)

time of concentration less than six hours are established in Table 8-2. For those watersheds with greater time of concentration, adjustments are made to the derived storm to account for the greater amounts of direct runoff in a longer period of time.

8-2 Physiographic Data

Design of any structure requires a certain degree of data, if only a field estimate of the drainage area and a description of terrain type and cover. The following material identifies some general data types and sources.

The hydrologic study for any minor structure always requires a reliable topographic map. U.S. Geological Survey topographic maps are usually available. Mapping of the United States is almost complete with 15-min quadrangles and many of these areas are mapped by $7\frac{1}{2}$-min quadrangles. County maps and aerial photos can also be utilized to advantage in making preliminary studies of the watershed.

Based on an area map, a careful investigation of the watersheds drainage behavior must be made. Additional information can be obtained from Geological Survey maps which depict predominant rock formations. Soil types, infiltration and errosive characteristics of soils can be secured from U.S. Soil Conservation Districts or University Extension Divisions.

Topographic map detail necessarily shifts with the type and purpose of structure being designed. Careful field reconaissance always increases understanding of an area's hydrology no matter how insignficant the structure may be.

8-3 Hydrologic Data

The battle in hydrologic design is that of getting adequate data for the region under study. Considerable data can be acquired from previously published reports issued by governmental agencies and/or universities. The following is a list of federal agencies which gather hydrologic data in a usable form:

1. Agricultural Research Service
2. Soil Conservation Service
3. Forest Service
4. U.S. Army Corps of Engineers
5. National Oceanic and Atmospheric Administration
6. Bureau of Reclamation
7. Department of Transportation
8. U.S. Geological Survey, Topographic Division
9. U.S. Geological Survey, Water Resources Division

Additional data can often be procured from Departments of State Governments, Interstate Compact Commissions and Regional and local agencies.

8-4 SCS Method

A small dam probably presents the best example of minor structure design, since it involves development of a design-storm hydrograph and use of hydrologic or hydraulic routing techniques to size the spillway. Routing techniques have been covered in Chapter 6, and the unit hydrographs in Chapter 3, now application of the various hydrograph methods in preparing a design-storm hydrograph is presented. Routing procedures to be used with the developed design-storm hydrograph are the same as those covered in Chapter 6.

A design hydrograph for both the emergency spillway design and freeboard pool design can be found through a procedure evolved by the U.S. Soil Conservation Service. The method is based upon previous routing of an assumed triangular unit hydrograph. Tabular solution provides a design-inflow hydrograph which can be used in selecting the spillway proportions and sizing the reservoir storage for flood detention.

The procedure consists of computing the direct runoff by use of curves developed by SCS founded on field studies of the amount of measured runoff from numerous soil cover combinations. A runoff curve number is extracted from Table 8-3. Selection of the runoff curve number is

TABLE 8-3 Runoff Curve Numbers for Hydrologic Soil-Cover Complexes (After Ref. 5)[a,b]

Land use or cover	Treatment or practice	Hydrologic condition	Hydrologic soil group			
			A	B	C	D
Row crops	Straight row	Poor	72	81	88	91
	Straight row	Good	67	78	85	89
	Contoured	Poor	70	79	84	88
	Contoured	Good	65	75	82	86
Small grain	Straight row	Poor	65	76	84	88
	Contoured	Good	61	73	81	84
Legumes or rotation	Contoured	Good	55	69	78	83
Native pasture or range		Fair	49	69	79	84
		Good	39	61	74	80
Woods		Fair	36	60	73	79
		Good	25	55	70	77

[a] Complete table in the *Soil Conservation Service Engineering Handbook*, Section 4A, lists 128 curve numbers for various combinations of cover, treatment, hydrologic condition, and soil.

[b] These runoff curve numbers assume antecedent moisture conditions of 0.5 to 1.1" in the previous 5 days for dormant season and 1.4 to 2.1" in the previous 5 days for the growing season.

dependent upon antecedent conditions and the types of cover. Soils are classified A, B, C, or D according to the following criteria:

A. (Low runoff potential.) Soils having high infiltration rates even when thoroughly wetted and consisting chiefly of deep, well to excessively drained sands or gravels. They have a high rate of water transmission.

B. Soils having moderate infiltration rates when thoroughly wetted and consisting chiefly of moderately deep to deep, moderately well to well drained soils with moderately fine to moderately coarse textures. They have a moderate rate of water transmission.

C. Soils having slow infiltration rates when thoroughly wetted and consisting chiefly of soils with a layer that impedes downward movement of water, or soils with moderately fine to fine texture. They have a slow rate of water transmission.

D. (High runoff potential.) Soils having very slow infiltration rates when thoroughly wetted and consisting chiefly of clay soils with a high swelling potential, soils with a permanent high water table, soils with a claypan or clay layer at or near the surface, and shallow soils over nearly impervious material. They have a very slow rate of water transmission.

Procedures are available for finding runoff curve numbers for other antecedent conditions.[5] A design-storm hydrograph is then synthesized by referring to Table 8-5 and the following equations:

$$T_p = 0.7T_c \tag{8-1}$$

$$\text{Rev. } T_p = \frac{T_0}{(T_0/T_p)\,\text{Rev.}} \tag{8-2}$$

$$q_p = \frac{484A}{\text{Rev. } T_p} \tag{8-3}$$

$$t = (t/T_p)(\text{Rev. } T_p) \tag{8-4}$$

$$q = (q_c/q_p)(Q\,q_p) \tag{8-5}$$

where A = drainage area, sq mi
q = synthesized hydrograph rate, cfs
q_c = hydrograph rate for Q = 1 in.
q_p = hydrograph peak rate for Q = 1 in.
Q = synthesized hydrograph runoff, in.
Rev. T_p = revised time to peak, hr
t = time, hr at which synthesized hydrograph rate is computed

TABLE 8-4 Hydrograph Families and T_0/T_p Ratios for which Dimensionless Hydrograph Ratios Are Given in Table 8-5 (After Ref. 5)

Hydrograph family	T_0/T_p 1	1.5	2	3	4	6	10	16	25	36	50	75
1.	*	*	*	*	*	*	*	*	*	*	*	*
2.	*	*	*	*	*	*	*	*	*	*	*	*
3.	*	*	*	*	*	*	*	*	*	*	*	*
4.	*	*	*	*	*	*	*	*	*	*	*	
5.	*	*	*	*	*	*	*	*	*	*	*	

Asterisks signify that dimensionless hydrograph tabulations are given in Table 8-5.

T_c = time of concentration, hr
$(T_0/T_p)_{rev}$ = revised ratio from Table 8-4
T_p = time to peak, hr for synthesized hydrograph

Five families of design hydrographs are listed, Table 8-5, for different ratios of storm duration, T_0, to time to peak, T_p. A summary of these synthesized hydrographs is shown in Table 8-4.

An example of this procedure for determining the design storm hydrograph for a watershed with a time of concentration less than 6 hr is shown as Example 8-1. Determination of the design storm hydrograph for a watershed with a time of concentration greater than 6 hr is given in Example 8-2.

Example 8-1 Determine the emergency spillway hydrograph for a class (b) structure with a drainage area of 1.86 sq mi, time of concentration = 1.25 hr, runoff curve number = 82, which is geographically located to provide a design rainfall depth of 9.4 in.

1. *Determine areal factor.* No adjustment necessary since the area is less than 10 sq mi.
2. *Duration adjustment.* No adjustment necessary for $T < 6$ hr.
3. *Direct runoff Q.* Enter Fig. 8-4 and find Q = 7.21 in. for P = 9.4 in. and CN = 82.
4. *Hydrograph family.* Enter Fig. 8-6 with CN = 82 and P = 9.4 and read hydrograph family No. 2.

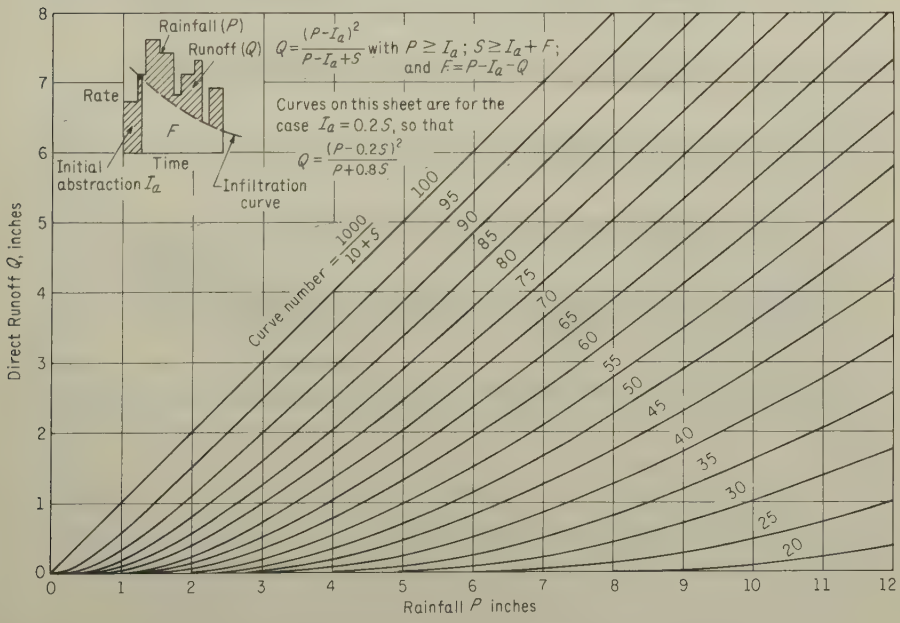


FIG. 8-4. Solution of direct runoff equation. (After Ref. 5.)

TABLE 8-5 Hydrograph Family No. 1 (After Ref. 5)

Line no.	$T_0/T_p = 1$		$T_0/T_p = 1.5$		$T_0/T_p = 2$		$T_0/T_p = 3$		$T_0/T_p = 4$		$T_0/T_p = 6$	
	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.28	.029	.32	.012	.29	.007	.35	.005	.35	.003	.44	.003
3	.56	.150	.64	.118	.58	.035	.70	.027	.70	.015	.98	.018
4	.84	.472	.96	.377	.87	.164	1.05	.101	1.05	.049	1.32	.041
5	1.12	.798	1.28	.711	1.16	.432	1.40	.302	1.40	.122	1.76	.084
6	1.40	.901	1.60	.815	1.45	.669	1.75	.563	1.75	.298	2.20	.176
7	1.68	.776	1.92	.719	1.74	.740	2.10	.650	2.10	.528	2.64	.386
8	1.96	.568	2.24	.526	2.03	.680	2.45	.576	2.45	.585	3.08	.497
9	2.24	.389	2.56	.352	2.32	.561	2.80	.460	2.80	.518	3.52	.430
10	2.52	.258	2.88	.225	2.61	.441	3.51	.374	3.15	.413	3.96	.335
11	2.80	.173	3.20	.143	2.90	.319	3.60	.290	3.50	.334	4.40	.258
12	3.08	.115	3.52	.090	3.19	.212	3.85	.201	3.85	.273	4.84	.202
13	3.36	.078	3.84	.057	3.48	.140	4.20	.127	4.20	.231	5.28	.164
14	3.64	.052	4.16	.037	3.77	.094	4.55	.078	4.55	.189	5.72	.139
15	3.92	.036	4.48	.024	4.06	.063	4.90	.047	4.90	.128	6.16	.124
16	4.20	.024	4.80	.015	4.35	.042	5.25	.028	5.25	.080	6.60	.100
17	4.48	.016	5.12	.008	4.64	.028	5.60	.016	5.60	.047	7.04	.060
18	4.76	.009	5.44	.004	4.93	.017	5.95	.009	5.95	.028	7.48	.033
19	5.04	.005	5.76	.002	5.22	.011	6.30	.005	6.30	.017	7.92	.018
20	5.32	.002	6.08	.001	5.51	.007	6.65	.003	6.65	.010	8.36	.009
21	5.60	.001	6.40	0	5.80	.004	7.00	.002	7.00	.006	8.80	.005
22	5.88	0			6.09	.002	7.35	.001	7.35	.004	9.24	.003
23					6.38	.001	7.70	0	7.70	.003	9.68	.002
24					6.67	0			8.05	.002	10.12	.001
25									8.40	.001	10.56	0
26									8.75	0		

Line no.	$T_0/T_p = 10$ t/T_p	q_c/q_p	$T_0/T_p = 16$ t/T_p	q_c/q_p	$T_0/T_p = 25$ t/T_p	q_c/q_p	$T_0/T_p = 36$ t/T_p	q_c/q_p	$T_0/T_p = 50$ t/T_p	q_c/q_p	$T_0/T_p = 75$ t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.56	.002	.66	.001	1.22	.002	1.70	.002	2.00	.0019	3.00	.0017
3	1.12	.013	1.32	.006	2.44	.009	3.40	.008	4.00	.0052	6.00	.0039
4	1.68	.027	1.98	.015	3.66	.018	5.10	.014	6.00	.0085	9.00	.0054
5	2.24	.047	2.64	.027	4.88	.027	6.80	.020	8.00	.0118	12.00	.0084
6	2.80	.071	3.30	.037	6.10	.036	8.50	.026	10.00	.0151	15.00	.0106
7	3.36	.115	3.96	.047	7.32	.046	10.20	.033	12.00	.0192	18.00	.0137
8	3.92	.278	4.62	.062	8.54	.116	11.90	.077	14.00	.0259	21.00	.0197
9	4.48	.394	5.28	.092	9.76	.232	13.60	.177	16.00	.0578	24.00	.0516
10	5.04	.322	5.94	.223	10.98	.146	15.30	.101	18.00	.1330	27.00	.0900
11	5.60	.235	6.60	.309	12.20	.088	17.00	.058	20.00	.0941	30.00	.0593
12	6.16	.174	7.26	.243	13.42	.062	18.70	.044	22.00	.0506	33.00	.0321
13	6.72	.136	7.92	.171	14.64	.051	20.40	.036	24.00	.0357	36.00	.0226
14	7.28	.110	8.58	.124	15.86	.045	22.10	.030	26.00	.0297	39.00	.0188
15	7.84	.092	9.24	.097	17.08	.039	23.80	.027	28.00	.0254	42.00	.0161
16	8.40	.079	9.90	.081	18.30	.035	25.50	.024	30.00	.0219	45.00	.0142
17	8.96	.073	10.56	.070	19.52	.031	27.20	.022	32.00	.0192	48.00	.0125
18	9.52	.068	11.22	.061	20.74	.027	28.90	.020	34.00	.0172	51.00	.0112
19	10.08	0.65	11.88	.055	21.96	.025	30.60	.018	36.00	.0159	54.00	.0105
20	10.64	.053	12.54	.050	23.18	.025	32.30	.017	38.00	.0150	57.00	.0100
21	11.20	.027	13.20	.047	24.40	.025	34.00	.017	40.00	.0145	60.00	.0097
22	11.76	.012	13.86	.045	25.62	.020	35.70	.017	42.00	.0140	63.00	.0094
23	12.32	.006	14.52	.044	26.84	.005	37.40	.004	44.00	.0136	66.00	.0090
24	12.88	.003	15.18	.043	28.06	.002	39.10	.002	46.00	.0131	69.00	.0087
25	13.44	.002	15.84	.040	29.28	0	40.80	0	48.00	.0125	72.00	.0084
26	14.00	.001	16.50	.034					50.00	.0123	75.00	.0081
27	14.56	0	17.16	.020					52.00	.0016	78.00	.0002
28			17.82	.008					54.00	0	81.00	0
29			18.48	.004								
30			19.14	.002								
31			19.80	.001								
32			20.46	0								

TABLE 8-5 Hydrograph Family No. 2

Line no.	$T_0/T_p = 1$ t/T_p	q_c/q_p	$T_0/T_p = 1.5$ t/T_p	q_c/q_p	$T_0/T_p = 2$ t/T_p	q_c/q_p	$T_0/T_p = 3$ t/T_p	q_c/q_p	$T_0/T_p = 4$ t/T_p	q_c/q_p	$T_0/T_p = 6$ t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.28	.026	.22	.003	.28	.004	.32	.003	.32	.002	.34	.001
3	.56	.170	.44	.041	.56	.040	.64	.017	.64	.009	.68	.005
4	.84	.480	.66	.161	.84	.170	.96	.093	.96	.036	1.02	.015
5	1.12	.802	.88	.362	1.12	.428	1.28	.311	1.28	.129	1.36	.037
6	1.40	.885	1.10	.604	1.40	.645	1.60	.530	1.60	.332	1.70	.098
7	1.68	.770	1.32	.740	1.68	.715	1.92	.615	1.92	.501	2.04	.244
8	1.96	.550	1.54	.790	1.96	.677	2.24	.575	2.24	.550	2.38	.407
9	2.24	.380	1.76	.746	2.24	.574	2.56	.487	2.56	.500	2.72	.464
10	2.52	.257	1.98	.640	2.52	.472	2.88	.409	2.88	.422	3.06	.429
11	2.80	.166	2.20	.536	2.80	.369	3.20	.344	3.20	.358	3.40	.367
12	3.08	.113	2.42	.414	3.08	.247	3.52	.279	3.52	.302	3.74	.309
13	3.36	.078	2.64	.303	3.36	.168	3.84	.206	3.84	.274	4.08	.261
14	3.64	.052	2.86	.219	3.64	.113	4.16	.135	4.16	.230	4.42	.224
15	3.92	.034	3.08	.160	3.92	.075	4.48	.087	4.48	.195	4.76	.193
16	4.20	.023	3.30	.117	4.20	.050	4.80	.054	4.80	.147	5.10	.169
17	4.48	.015	3.52	.088	4.48	.034	5.12	.032	5.12	.099	5.44	.152
18	4.76	.009	3.74	.064	4.76	.021	5.44	.019	5.44	.061	5.78	.139
19	5.04	.004	3.96	.047	5.04	.014	5.76	.012	5.76	.037	6.12	.129
20	5.32	.002	4.18	.035	5.32	.008	6.08	.008	6.08	.023	6.46	.113
21	5.60	.001	4.40	.025	5.60	.004	6.40	.005	6.40	.013	6.80	.085
22	5.88	0	4.62	.018	5.88	.003	6.72	.003	6.72	.008	7.14	.055
23			4.84	.012	6.16	.002	7.04	.002	7.04	.005	7.48	.035
24			5.06	.007	6.44	.001	7.36	.001	7.36	.004	7.82	.020
25			5.28	.004	6.72	0	7.68	0	7.68	.003	8.16	.012
26			5.50	.003					8.00	.002	8.50	.008
27			5.72	.002					8.32	.001	8.84	.005
28			5.94	.001					8.64	0	9.18	.004
29			6.16	0							9.52	.003
30											9.86	.002
31											10.20	.001
32											10.54	0

Line no.	T_0/T_p = 10		T_0/T_p = 16		T_0/T_p = 25		T_0/T_p = 36		T_0/T_p = 50		T_0/T_p = 75	
	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.63	.002	.90	.002	1.30	.002	1.79	.002	2.50	.0018	3.00	.0012
3	1.26	.009	1.80	.007	2.60	.006	3.58	.006	5.00	.0047	6.00	.0027
4	1.89	.027	2.70	.020	3.90	.014	5.37	.012	7.50	.0087	9.00	.0044
5	2.52	.063	3.60	.037	5.20	.024	7.16	.019	10.00	.0145	12.00	.0067
6	3.15	.236	4.50	.148	6.50	.088	8.95	.057	12.50	.0615	15.00	.0108
7	.378	.364	5.40	.277	7.80	.210	10.74	.157	15.00	.1184	18.00	.0309
8	4.41	.307	6.30	.214	9.10	.146	12.53	.104	17.50	.0621	21.00	.0790
9	5.04	.226	7.20	.149	10.40	.097	14.32	.068	20.00	.0433	24.00	.0624
10	5.67	.172	8.10	.112	11.70	.072	16.11	.047	22.50	.0342	27.00	.0357
11					13.00	.057	17.90	.040	25.00	.0274	30.00	.0283
12					14.30	.049	19.69	.034	27.50	.0234	33.00	.0234
13	6.30	.136	9.00	.088	15.60	.044	21.48	.030	30.00	.0209	36.00	.0196
14	6.93	.113	9.90	.073	16.90	.039	23.27	.026	32.50	.0187	39.00	.0167
15	7.56	.097	10.80	.063	18.20	.035	25.06	.025	35.00	.0167	42.00	.0150
16	8.19	.085	11.70	.056	19.50	.033	26.85	.023	37.50	.0159	45.00	.0137
17	8.82	.078	12.60	.052	20.80	.031	28.64	.021	40.00	.0153	48.00	.0126
18	9.45	.074	13.50	.048	22.10	.029	30.43	.020	42.50	.0147	51.00	.0115
19	10.08	.069	14.40	.045	23.40	.028	32.22	.019	45.00	.0142	54.00	.0108
20	10.71	.053	15.30	.044	24.70	.027	34.01	.018	47.50	.0136	57.00	.0104
21	11.34	.025	16.20	.042	26.00	.014	35.80	.017	50.00	.0131	60.00	.0101
22	11.97	.009	17.10	.023	27.30	.004	37.59	.007	52.50	.0008	63.00	.0098
23	12.60	.004	18.00	.006	28.60	.001	39.38	.001	55.00	0	66.00	.0095
24	13.23	.002	18.90	.003	29.90	0	41.17	0			69.00	.0092
25	13.86	.001	19.80	.001		0		0			72.00	.0089
26	14.49	0	20.70	0							75.00	.0086
27											78.00	.0003
28											81.00	0

TABLE 8-5 Hydrograph Family No. 3

Line no.	$T_0/T_p = 1$		$T_0/T_p = 1.5$		$T_0/T_p = 2$		$T_0/T_p = 3$		$T_0/T_p = 4$		$T_0/T_p = 6$	
	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.26	.048	.29	.028	.30	.012	.34	.004	.36	.003	.42	.002
3	.52	.219	.58	.190	.60	.123	.68	.088	.72	.044	.84	.021
4	.78	.521	.87	.450	.90	.343	1.02	.289	1.08	.203	1.26	.138
5	1.04	.762	1.16	.656	1.20	.570	1.36	.489	1.44	.400	1.68	.320
6	1.30	.844	1.45	.734	1.50	.657	1.70	.543	1.80	.478	2.10	.390
7	1.56	.778	1.74	.685	1.80	.630	2.04	.507	2.16	.450	2.52	.363
8	1.82	.621	2.03	.585	2.10	.562	2.38	.445	2.52	.397	2.94	.314
9	2.08	.441	2.32	.445	2.40	.484	2.72	.385	2.88	.342	3.36	.270
10	2.34	.305	2.61	.350	2.70	.379	3.06	.340	3.24	.296	3.78	.232
11	2.60	.214	2.90	.199	3.00	.267	3.40	.294	3.60	.257	4.20	.199
12	2.86	.149	3.19	.132	3.30	.177	3.74	.223	3.96	.234	4.62	.174
13	3.12	.103	3.48	.089	3.60	.116	4.08	.149	4.32	.210	5.04	.155
14	3.38	.070	3.77	.057	3.90	.076	4.42	.096	4.68	.169	5.46	.144
15	3.64	.048	4.06	.038	4.20	.050	4.76	.056	5.04	.111	5.88	.137
16	3.90	.034	4.35	.025	4.50	.033	5.10	.033	5.40	.067	6.30	.127
17	4.16	.024	4.64	.015	4.80	.020	5.44	.019	5.76	.037	6.72	.101
18	4.42	.016	4.93	.008	5.10	.011	5.78	.013	6.12	.022	7.14	.063
19	4.68	.010	5.22	.005	5.40	.006	6.12	.008	6.48	.014	7.56	.033
20	4.94	.006	5.51	.003	5.70	.004	6.46	.004	6.84	.008	7.98	.018
21	5.20	.003	5.80	.002	6.00	.002	6.80	.003	7.20	.006	8.40	.010
22	5.46	.001	6.09	.001	6.30	.001	7.14	.002	7.56	.004	8.82	.005
23	5.72	0	6.38	0	6.60	0	7.48	.001	7.92	.002	9.24	.003
24							7.82	0	8.28	.001	9.66	.002
25									8.64	0	10.08	.001
26											10.50	
27											10.92	0

Line no.	T_0/T_p = 10 t/T_p	q_c/q_p	T_0/T_p = 16 t/T_p	q_c/q_p	T_0/T_p = 25 t/T_p	q_c/q_p	T_0/T_p = 36 t/T_p	q_c/q_p	T_0/T_p = 50 t/T_p	q_c/q_p	T_0/T_p = 75 t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.54	.001	.90	.002	1.23	.002	1.62	.002	2.25	.0008	3.25	.0009
3	1.08	.008	1.80	.016	2.46	.009	3.24	.006	4.50	.0070	6.50	.0057
4	1.62	.069	2.70	.122	3.69	.073	4.86	.047	6.75	.0474	9.75	.0289
5	2.16	.231	3.60	.230	4.92	.173	6.48	.130	9.00	.0972	13.00	.0667
6	2.70	.303	4.50	.185	6.15	.132	8.10	.097	11.25	.0642	16.25	.0445
7	3.24	.269	5.40	.139	7.38	.096	9.72	.069	13.50	.0460	19.50	.0317
8	3.78	.223	6.30	.113	8.61	.076	11.34	.052	15.75	.0375	22.75	.0257
9	4.32	.188	7.20	.094	9.84	.064	12.96	.045	18.00	.0322	26.00	.0219
10	4.86	.159	8.10	.081	11.07	.055	14.58	.041	20.25	.0285	29.25	.0195
11	5.40	.139	9.00	.072	12.30	.050	16.20	.037	22.50	.0258	32.50	.0176
12	5.94	.122	9.90	.064	13.53	.046	17.82	.034	24.75	.0239	35.75	.0160
13	6.48	.108	10.80	.057	14.76	.042	19.44	.031	27.00	.0219	39.00	.0147
14	7.02	.097	11.70	.053	15.99	.038	21.06	.028	29.25	.0201	42.25	.0136
15	7.56	.089	12.60	.050	17.22	.035	22.68	.025	31.50	.0185	45.50	.0127
16	8.10	.081	13.50	.049	18.45	.033	24.30	.024	33.75	.0173	48.75	.0118
17	8.64	.078	14.40	.048	19.68	.032	25.92	.024	36.00	.0165	52.00	.0113
18	9.18	.077	15.30	.047	20.91	.031	27.54	.024	38.25	.0162	55.25	.0109
19	9.72	.077	16.20	.046	22.14	.031	29.16	.024	40.50	.0159	58.50	.0107
20	10.26	.075	17.10	.024	23.37	.031	30.78	.023	42.75	.0156	61.75	.0105
21	10.80	.055	18.00	.006	24.60	.031	32.40	.023	45.00	.0153	65.00	.0103
22	11.34	.030	18.90	.004	25.83	.025	34.02	.023	47.25	.0150	68.25	.0101
23	11.88	.012	19.80	.002	27.06	.004	35.64	.023	49.50	.0147	71.50	.0099
24	12.42	.006	20.70	0	28.29	.001	37.26	.007	51.75	.0028	74.75	.0097
25	12.96	.004			29.52	0	38.88	.003	54.00	0	78.00	.0003
26	13.50	.002					40.50	0			81.25	0
27	14.04	.001										
28	14.58	0										

TABLE 8-5 Hydrograph Family No. 4

Line no.	$T_0/T_p = 1$		$T_0/T_p = 1.5$		$T_0/T_p = 2$		$T_0/T_p = 3$		$T_0/T_p = 4$		$T_0/T_p = 6$	
	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.28	.051	.28	.038	.32	.031	.28	.018	.40	.023	.40	.014
3	.56	.220	.56	.166	.64	.173	.56	.086	.80	.143	.80	.088
4	.84	.490	.84	.360	.96	.360	.84	.200	1.20	.272	1.20	.191
5	1.12	.738	1.12	.551	1.28	.494	1.12	.311	1.60	.326	1.60	.244
6	1.40	.830	1.40	.651	1.60	.555	1.40	.386	2.00	.340	2.00	.250
7	1.68	.751	1.68	.686	1.92	.567	1.68	.415	2.40	.337	2.40	.246
8	1.96	.573	1.96	.650	2.24	.555	1.96	.422	2.80	.323	2.80	.240
9	2.24	.392	2.24	.543	2.56	.490	2.24	.417	3.20	.306	3.20	.233
10	2.52	.259	2.52	.392	2.88	.370	2.52	.402	3.60	.293	3.60	.223
11	2.80	.174	2.80	.267	3.20	.242	2.80	.394	4.00	.286	4.00	.212
12	3.08	.118	3.08	.180	3.52	.150	3.08	.387	4.40	.266	4.40	.202
13	3.36	.079	3.36	.120	3.84	.098	3.36	.363	4.80	.197	4.80	.194
14	3.64	.053	3.64	.081	4.16	.063	3.64	.316	5.20	.122	5.20	.189
15	3.92	.036	3.92	.055	4.48	.038	3.92	.236	5.60	.067	5.60	.187
16	4.20	.025	4.20	.036	4.80	.024	4.20	.164	6.00	.036	6.00	.185
17	4.48	.017	4.48	.024	5.12	.013	4.48	.108	6.40	.021	6.40	.175
18	4.76	.011	4.76	.015	5.44	.008	4.76	.073	6.80	.013	6.80	.131
19	5.04	.006	5.04	.009	5.76	.004	5.04	.047	7.20	.008	7.20	.080
20	5.32	.003	5.32	.005	6.08	.002	5.32	.030	7.60	.005	7.60	.046
21	5.60	.001	5.60	.003	6.40	.001	5.60	.020	8.00	.002	8.00	.027
22	5.88	0	5.88	.001	6.72	0	5.88	.013	8.40	.001	8.40	.016
23			6.16	0			6.16	.008	8.80	0	8.80	.009
24							6.44	.005			9.20	.005
25							6.72	.003			9.60	.003
26							7.00	.002			10.00	.002
27							7.28	.001			10.40	.001
28							7.56				10.80	0
29							7.84	0				

Line no.	$T_0/T_p = 10$ t/T_p	q_c/q_p	$T_0/T_p = 16$ t/T_p	q_c/q_p	$T_0/T_p = 25$ t/T_p	q_c/q_p	$T_0/T_p = 36$ t/T_p	q_c/q_p	$T_0/T_p = 50$ t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0
2	.50	.015	.62	.015	1.02	.025	1.50	.0306	2.00	.0277
3	1.00	.079	1.24	.064	2.04	.070	3.00	.0575	4.00	.0464
4	1.50	.151	1.86	.112	3.06	.092	4.50	.0672	6.00	.0435
5	2.00	.177	2.48	.128	4.08	.082	6.00	.0492	8.00	.0378
6	2.50	.170	3.10	.119	5.10	.068	7.50	.0433	10.00	.0335
7	3.00	.159	3.72	.105	6.12	.062	9.00	.0418	12.00	.0307
8	3.50	.152	4.34	.097	7.14	.059	10.50	.0408	14.00	.0291
9	4.00	.146	4.96	.094	8.16	.056	12.00	.0400	16.00	.0282
10	4.50	.141	5.58	.091	9.18	.055	13.50	.0391	18.00	.0274
11	5.00	.136	6.20	.089	10.20	.054	15.00	.0382	20.00	.0266
12	5.50	.131	6.82	.087	11.22	.053	16.50	.0371	22.00	.0258
13	6.00	.126	7.44	.085	12.24	.052	18.00	.0358	24.00	.0250
14	6.50	.121	8.06	.082	13.26	.050	19.50	.0341	26.00	.0242
15	7.00	.166	8.68	.079	14.28	.049	21.00	.0319	28.00	.0234
16	7.50	.112	9.30	.076	15.30	.047	22.50	.0308	30.00	.0230
17	8.00	.112	9.92	.074	16.32	.046	24.00	.0306	32.00	.0229
18	8.50	.111	10.54	.072	17.34	.045	25.50	.0306	34.00	.0227
19	9.00	.111	11.16	.071	18.36	.044	27.00	.0306	36.00	.0226
20	9.50	.110	11.78	.070	19.38	.044	28.50	.0306	38.00	.0225
21	10.00	.110	12.40	.069	20.40	.044	30.00	.0306	40.00	.0224
22	10.50	.100	13.02	.069	21.42	.044	31.50	.0306	42.00	.0222
23	11.00	.065	13.64	.069	22.44	.044	33.00	.0306	44.00	.0221
24	11.50	.033	14.26	.069	23.46	.044	34.50	.0306	46.00	.0219
25	12.00	.025	14.88	.069	24.48	.044	36.00	.0306	48.00	.0219
26	12.50	.007	15.50	.069	25.50	.039	37.50	.0085	50.00	.0217
27	13.00	.004	16.12	.068	26.52	.012	39.00	.0009	52.00	.0029
28	13.50	.002	16.74	.053	27.54	.004	40.50	0	54.00	0
29	14.00	.001	17.36	.023	28.56	.001				
30	14.50	0	17.98	.009	29.58	0				
31									18.60	.004
32									19.22	.002
33									19.84	.001
34									20.46	0

TABLE 8-5 Hydrograph Family No. 5

Line no.	$T_0/T_p = 1$		$T_0/T_p = 1.5$		$T_0/T_p = 2$		$T_0/T_p = 3$		$T_0/T_p = 4$		$T_0/T_p = 6$	
	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p	t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.26	.021	.25	.013	.25	.010	.34	.010	.36	.010	.52	.015
3	.52	.106	.50	.065	.50	.048	.68	.068	.72	.053	1.04	.070
4	.78	.289	.75	.173	.75	.127	1.02	.150	1.08	.124	1.56	.130
5	1.04	.530	1.00	.306	1.00	.227	1.36	.229	1.44	.181	2.08	.159
6	1.30	.740	1.25	.434	1.25	.318	1.70	.283	1.80	.220	2.60	.172
7	1.56	.848	1.50	.562	1.50	.389	2.04	.315	2.16	.243	3.12	.178
8	1.82	.767	1.75	.680	1.75	.448	2.38	.339	2.52	.256	.364	.182
9	2.08	.590	2.00	.737	2.00	.523	2.72	.378	2.88	.263	4.16	.183
10	2.34	.406	2.25	.673	2.25	.609	3.06	.459	3.24	.273	4.68	.184
11	2.60	.279	2.50	.530	2.50	.642	3.40	.509	3.60	.308	5.20	.218
12	2.86	.193	3.75	.381	2.75	.576	3.74	.446	3.96	.380	5.72	.285
13	3.12	.134	3.00	.262	3.00	.450	4.08	.310	4.32	.427	6.24	.324
14	3.38	.092	3.25	.185	3.25	.322	4.42	.190	4.68	.377	6.76	.267
15	3.64	.065	3.50	.129	3.50	.222	4.76	.117	5.04	.260	7.28	.133
16	3.90	.044	3.75	.090	3.75	.156	5.10	.069	5.40	.155	7.80	.064
17	4.16	.030	4.00	.063	4.00	.109	5.44	.040	5.76	.094	8.32	.029
18	4.42	.021	4.25	.045	4.25	.075	5.78	.025	6.12	.055	8.84	.016
19	4.68	.015	4.50	.031	4.50	.053	6.12	.016	6.48	.032	9.36	.007
20	4.94	.009	4.75	.022	4.75	.037	6.46	.009	6.84	.019	9.88	.003
21	5.20	.005	5.00	.014	5.00	.025	6.80	.005	7.20	.012	10.40	.001
22	5.46	.002	5.25	.009	5.25	.017	7.14	.003	7.56	.007	10.92	0
23	5.72	0	5.50	.005	5.50	.011	7.48	.001	7.92	.004		0
24			5.75	.003	5.75	.007	7.82	0	8.28	.002		
25			6.00	.001	6.00	.004			8.64	0		
26			6.25	0	6.25	.002						
27					6.50	.001						
28					6.75	0						

Line no.	$T_0/T_p = 10$ t/T_p	q_c/q_p	$T_0/T_p = 16$ t/T_p	q_c/q_p	$T_0/T_p = 25$ t/T_p	q_c/q_p	$T_0/T_p = 36$ t/T_p	q_c/q_p	$T_0/T_p = 50$ t/T_p	q_c/q_p
1	0	0	0	0	0	0	0	0	0	0
2	.67	.013	.80	.008	1.25	.015	1.50	.0195	2.00	.0167
3	1.34	.061	1.60	.046	2.50	.039	3.00	.0275	4.00	.0204
4	2.01	.091	2.40	.060	3.75	.043	4.50	.0294	6.00	.0214
5	2.68	.102	3.20	.065	5.00	0.44	6.00	.0300	8.00	.0216
6	3.35	.107	4.00	.067	6.25	.044	7.50	.0301	10.00	.0216
7	4.02	.110	4.80	.067	7.50	.044	9.00	.0301	12.00	.0216
8	4.69	.111	5.60	.068	8.75	.044	10.50	.0301	14.00	.0216
9	5.36	.111	6.40	.068	10.00	.044	12.00	.0301	16.00	.0216
10	6.03	.112	7.20	.068	11.25	.044	13.50	.0301	18.00	.0216
11	6.70	.112	8.00	.068	12.50	.044	15.00	.0301	20.00	.0216
12	7.37	.112	8.80	.068	13.75	.044	16.50	.0301	22.00	.0216
13	8.04	.116	9.60	.068	15.00	.044	18.00	.0301	24.00	.0216
14	8.71	.160	10.40	.068	16.25	.044	19.00	.0301	26.00	.0216
15	9.38	.198	11.20	.068	17.50	.044	21.00	.0301	28.00	.0216
16	10.05	.212	12.00	.068	18.75	.045	22.50	.0301	30.00	.0216
17	10.72	.168	12.80	.086	20.00	.067	24.00	.0311	32.00	.0217
18	11.39	.074	13.60	.121	21.25	.083	25.50	.0364	34.00	.0243
19	12.06	.027	14.40	.133	22.50	.087	27.00	.0425	36.00	.0287
20	12.73	.010	15.20	.136	23.75	.087	28.50	.0480	38.00	.0329
21	13.40	.005	16.00	.137	25.00	.088	30.00	.0525	40.00	.0363
22	14.07	.002	16.80	.098	26.25	.035	31.50	.0561	42.00	.0391
23	14.74	0	17.60	.033	27.50	.006	33.00	.0584	44.00	.0411
24			18.40	.012	28.75	.002	34.50	.0598	46.00	.0423
25			19.20	.004	30.00	0	36.00	.0603	48.00	.0430
26			20.00	.001			37.50	.0167	50.00	.0433
27			20.80	0			39.00	.0018	52.00	.0058
28							40.50	0	54.00	.0002
29									56.00	0

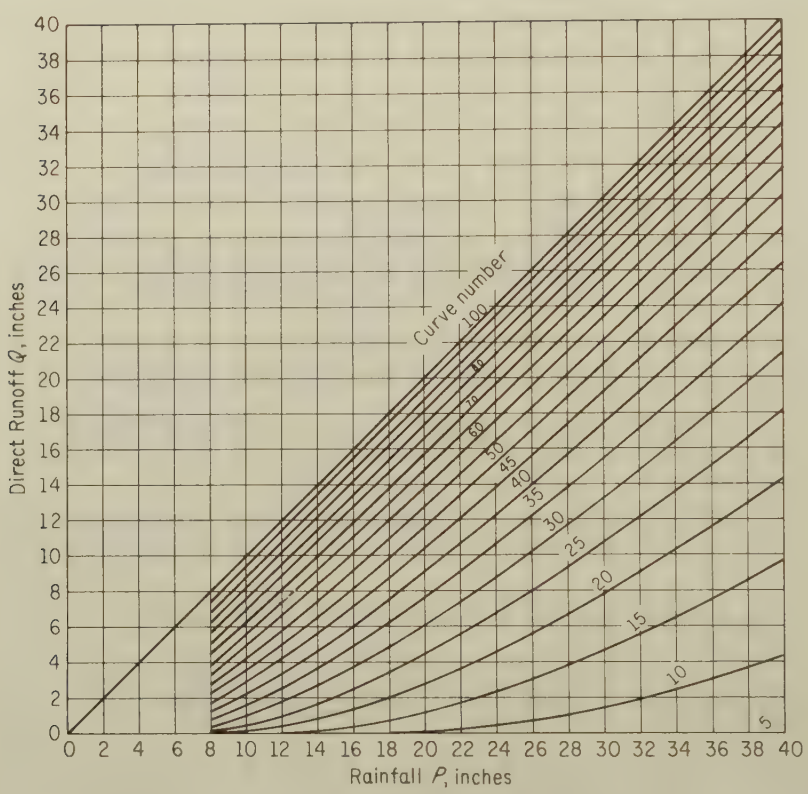


FIG. 8-5. Solution of direct runoff equation. (After Ref. 5.)

5. *Duration of rainfall excess:* Enter Fig. 8-7 with $P = 9.4$ in. at $CN = 82$ and read $T_0 = 5.37$ hr.
6. *Compute* T_p; $T_p = 0.7\ T_c = 0.88$ hr.
7. T_0/T_p *ratio.* $5.37/0.88 = 6.10$.
8. *Revised* T_0/T_p *based on available curves from Table 8-8:* $(T_0/T_p)_{rev} = 6$.
9. *Rev* T_p*:* Rev $T_p = 5.37/6 = 0.895$ hr.
10. q_p*:* $q_p = 484(A) \div$ Rev $T_p = 1006$ cfs.
11. $Q\,q_p$*:* $(Q)\,(q_p) = (7.21)\,(1006) = 7{,}250$ cfs.
12. From selected hydrograph family in Table 8-5, multiply each value of the t/T_p column by Rev T_p to obtain t in hours.
13. From the selected hydrograph family in Table 8-5, multiply each value of the q_c/q_p column by $(Q)\,(q_p)$ to obtain values of q in cfs.

Solution for Example 8-1

Line no.	t, hr	q, cfs	Line no.	t, hr	q, cfs
1	0	0	21	6.09	616
2	0.30	7	22	6.39	399

(*Continued on next page*)

Line no.	t, hr	q, cfs	Line no.	t, hr	q, cfs
3	0.61	36	23	6.69	254
4	0.91	109	24	7.00	145
5	1.22	268	25	7.30	87
6	1.52	710	26	7.61	58
7	1.82	1769	27	7.91	36
8	2.13	2951	28	8.22	29
9	2.43	3364	29	8.52	22
10	2.74	3110	30	8.82	14
11	3.04	2661	31	9.13	7
12	3.35	2240	32	9.43	0
13	3.65	1892	33		
14	3.96	1624	34		
15	4.26	1399	35		
16	4.56	1225	36		
17	4.87	1102	37		
18	5.17	1008	38		
19	5.48	935	39		
20	5.78	819	40		

Solution for Example 8-2

Line no.	t, hr	q, cfs	Line no.	t, hr	q, cfs
1	0	0	21	43.78	0
2	2.19	482	22		
3	4.38	4745	23		
4	6.57	15160	24		
5	8.76	28591	25		
6	10.94	32773	26		
7	13.13	28912	27		
8	15.32	21152	28		
9	17.51	14155	29		
10	19.70	9048	30		
11	21.89	5750	31		
12	24.08	3619	32		
13	26.26	2292	33		
14	28.45	1488	34		
15	30.64	965	35		
16	32.83	603	36		
17	35.02	322	37		
18	37.21	161	38		
19	39.40	80	39		
20	41.59	40	40		

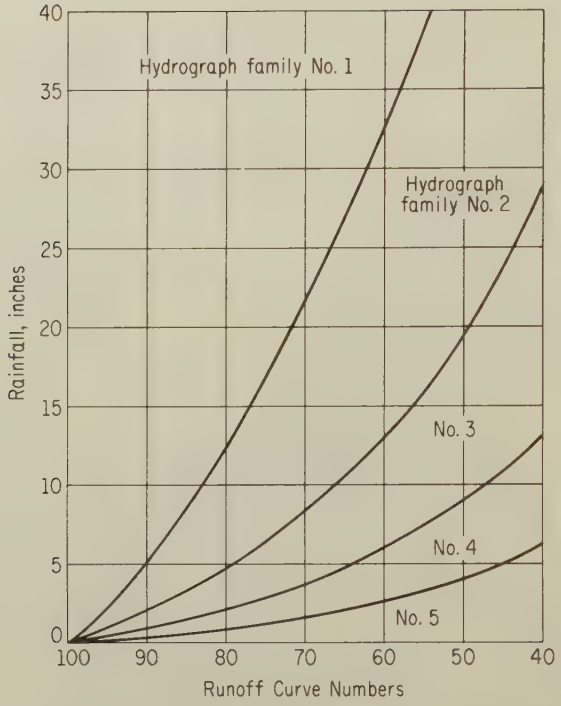


FIG. 8-6. Selecting hydrograph family. (After Ref. 5.)

Example 8-2 Determine the freeboard design hydrograph for a class (c) structure with a drainage area = 23.0 sq mi, time of concentration = 10.8 hr, *CN* of 77, which is geographically located to produce a $P = 25.5$ in.

1. *Determine the areal factor.* Use the appropriate curve of Fig. 8-8. As an example the humid and subhumid climate curve factor is 0.93. P adjusted = $25.5 \times 0.93 = 23.72$ in.

TABLE 8-6 Rainfall Prior to Excess Rainfall (After Ref. 5)

CN	*P**, in.	*CN*	*P**, in.	*CN*	*P**, in.	*CN*	*P**, in.	*CN*	*P**, in.
100	0	86	0.33	72	0.79	58	1.45	44	2.54
99	.02	85	.35	71	.82	57	1.51	43	2.64
98	.04	84	.38	70	.86	56	1.57	42	2.76
97	.06	83	.41	69	.90	55	1.64	41	2.88
96	.08	82	.44	68	.94	54	1.70	40	3.00
95	.11	81	.47	67	.98	53	1.77	39	3.12
94	.13	80	.50	66	1.03	52	1.85	38	3.26
93	.15	79	.53	65	1.08	51	1.92	37	3.40
92	.17	78	.56	64	1.12	50	2.00	36	3.56
91	.20	77	.60	63	1.17	49	2.08	35	3.72
90	.22	76	.63	62	1.23	48	2.16	34	3.88
89	.25	75	.67	61	1.28	47	2.26	33	4.06
88	.27	74	.70	60	1.33	46	2.34	32	4.24
87	.30	73	.74	59	1.39	45	2.44	31	4.44

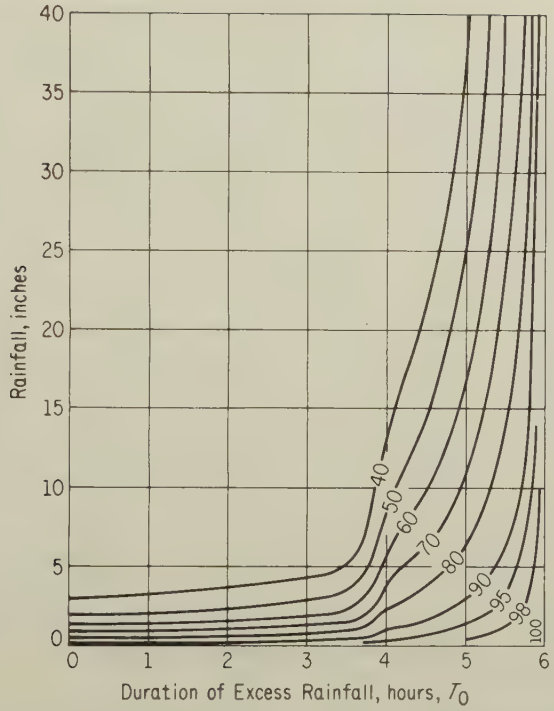


FIG. 8-7. Duration of excess rainfall. (After Ref. 5.)

2. *Duration adjustment.* The duration is made equal to the time of concentration by adjusting the rainfall amount. Enter Fig. 8-9 to find the adjustment factor of 1.18. P adjusted = 23.72 × 1.18 = 27.99 in. or 28 in.
3. *Direct runoff Q.* Enter Fig. 8-5 and find Q = 24.7 in. for P = 28 in. and CN = 77.
4. *Hydrograph Family.* Enter Fig. 8-6 with CN = 77 and P = 28 and read hydrograph family No. 1.
5. *Duration of rainfall excess.* Enter Table 8-6 with CN = 77 and find P^*,

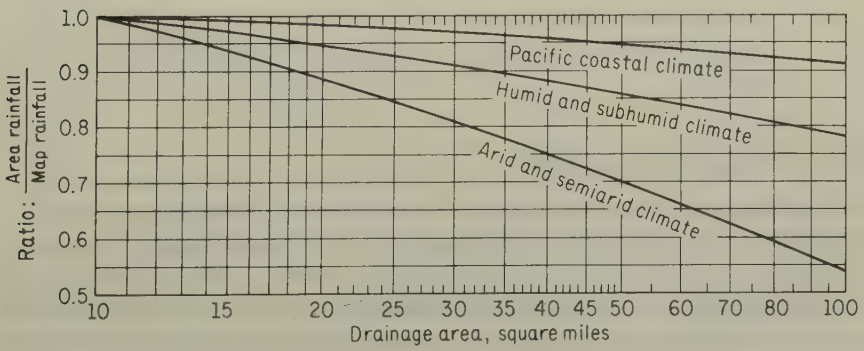

Ratio: Area rainfall / Map rainfall
Pacific coastal climate
Humid and subhumid climate
Arid and semiarid climate
Drainage area, square miles

FIG. 8-8. Rainfall ratios for 10 to 100 sq mi. (After Ref. 5.)

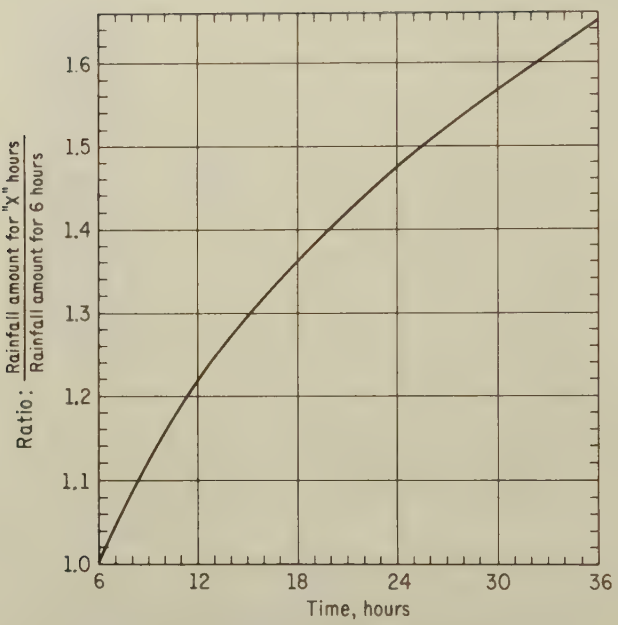


FIG. 8-9. Relative increase in rainfall amount for storm durations over six hours. (After Ref. 5.)

the rainfall prior to P, is 0.60 in. Enter Table 8-7 with the ratio P^*/P = 0.60/28.0 = 0.0214 and read a time ratio of 0.950. T_0 = (time ratio) × (storm duration) = 0.950(10.8) = 10.26 hr.

6. Compute $T_p \cdot T_p = 0.7\,T_c$ = 7.56 hr.
7. T_0/T_p ratio. 10.26/7.56 = 1.357.
8. *Revised T_0/T_p based on available curves from Table 8-4.* $(T_0/T_p)_{Rev}$ = 1.5
9. *Rev T_p*; Rev T_p = 10.26/1.5 = 6.84 hr.
10. $q_p \cdot q_p$ = 484(23.0)/6.84 = 1,628 cfs.
11. $Qq_p \cdot (Q)(q_p)$ = (24.7)(1,628) = 40,212 cfs.
12. From selected hydrograph family in Table 8-5, multiply each value of the t/T_p column by Rev T_p to obtain t in hours.
13. From the selected hydrograph family in Table 8-5, multiply each value of the q_c/q_p column by $(Q)(q_p)$ to obtain values of q in cfs. (See p. 289 for solution.)

8-5 Critical Storm Method

The length of storm used in the SCS method is of 6-hr duration or greater. Often the type of minor structure being designed cannot be economically justified on the basis of this length of storm. For many minor structures, a design flood hydrograph is based upon a storm duration equal to the time of concentration of the watershed. This procedure

TABLE 8-7 Rainfall and Time Ratios for Determining T_0 When Storm Duration is Greater than Six Hours (After Ref. 5)

Rainfall ratio	Time ratio	Rainfall ratio	Time ratio	Rainfall ratio	Time ratio	Rainfall ratio	Time ratio
0	1.00	0.070	0.852	0.140	0.746	0.210	0.684
.002	.995	.072	.848	.142	.744	.212	.682
.004	.990	.074	.844	.144	.742	.214	.680
.006	.985	.076	.841	.146	.740	.216	.679
.008	.981	.078	.837	.148	.739	.218	.677
.010	.976	.080	.833	.150	.737	.220	.675
.012	.971	.082	.830	.152	.735	.222	.673
.014	.967	.084	.827	.154	.733	.224	.672
.016	.962	.086	.824	.156	.732	.226	.670
.018	.957	.088	.821	.158	.730	.228	.668
.020	.952	.090	.818	.160	.728	.230	.667
.022	.948	.092	.815	.162	.726	.232	.666
.024	.943	.094	.812	.164	.724	.234	.666
.026	.938	.096	.809	.166	.723	.236	.665
.228	.933	.098	.806	.168	.721	.238	.665
.030	.929	.100	.803	.170	.719	.240	.664
.032	.924	.102	.800	.172	.717		
.034	.919	.104	.797	.174	.716	(Change in	
.036	.915	.106	.794	.176	.714	tabulation	
.038	.911	.108	.791	.178	.712	increment)	
.040	.908	.110	.788	.180	.710	.250	.662
.042	.904	.112	.785	.182	.709	.300	.651
.044	.900	.114	.782	.184	.707	.350	.640
.046	.896	.116	.779	.186	.705	.400	.628
.048	.893	.118	.776	.188	.703	.450	.617
.050	.889	.120	.773	.190	.702	.500	.606
.052	.885	.122	.770	.192	.700	.550	.595
.054	.882	.124	.767	.194	.698	.600	.583
.056	.878	.126	.764	.196	.696	.650	.542
.058	.874	.128	.761	.198	.695	.700	.500
.060	.870	.130	.758	.200	.693	.750	.447
.062	.867	.132	.755	.202	.691	.800	.386
.064	.863	.134	.751	.304	.689	.850	.310
.066	.859	.136	.749	.206	.687	.900	.220
.068	.856	.138	.747	.208	.686	.950	.116

utilizes the synthetic unit hydrographs of Chapter 3 and a critical storm pattern produced by rearranging the rainfall excess pattern into the most critical sequence.

The two rainfall-excess patterns normally investigated are first-quartile and second-quartile storms. Additional information on the most probable storm pattern over areas up to 400 sq mi has been provided by Huff. Short-duration storms—those less than 12 hr—are more frequently significant in design of minor structures. Their patterns are shown in Figs.

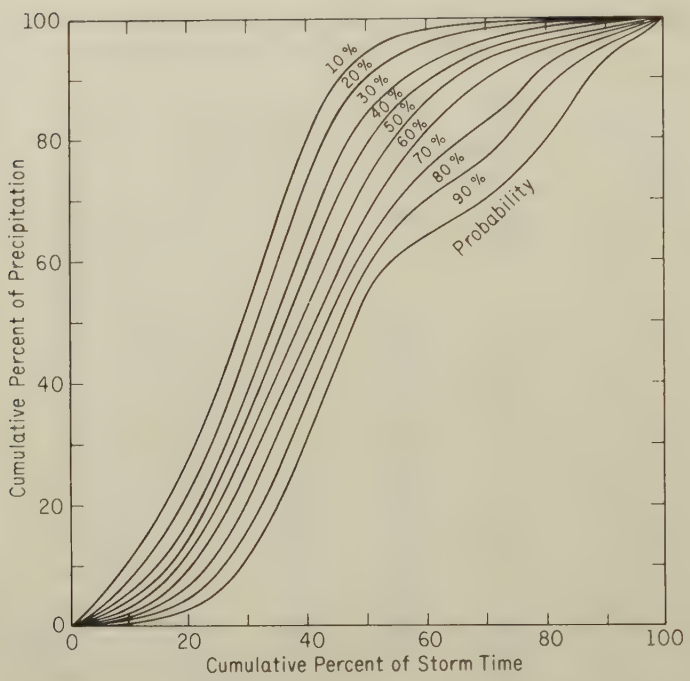


FIG. 8-10. Time distribution of second-quartile storms. (After Ref. 6.)

8-10 and 8-11. The two types are elliptically shaped and make up 66 percent of the total number of storms registered on small watersheds in the midwest. Each has an almost equal chance of occurrence.

Curves for each probability level drawn in Figs. 8-10 and 8-11 can be used to design for various levels of severity. A 90 percent level is the distribution occurring in 10 percent or less of the storms. Storm characteristics are revealed by Figs. 8-10 and 8-11. Eighty percent of total rainfall occurs in the first 20 percent of storm time for the 10 percent level in the first-quartile storm. Passage of an intense, prefrontal squall line, typical of thunderstorms, will produce this particular rainfall distribution. On the other hand, the 90 percent level is more indicative of a steady rain or a series of rain showers.

Figure 8-12 shows the 10, 50, and 90 percent histograms for first-quartile storms. Using these storm distributions permits construction of rainfall hyetographs for the design rainfall excess.

Rainfall excess can be obtained from the rainfall atlas[7] for a selected design frequency and duration. Direct runoff resulting from this precipitation is estimated from Figs. 8-4 and 8-5. The antecedent condition II for

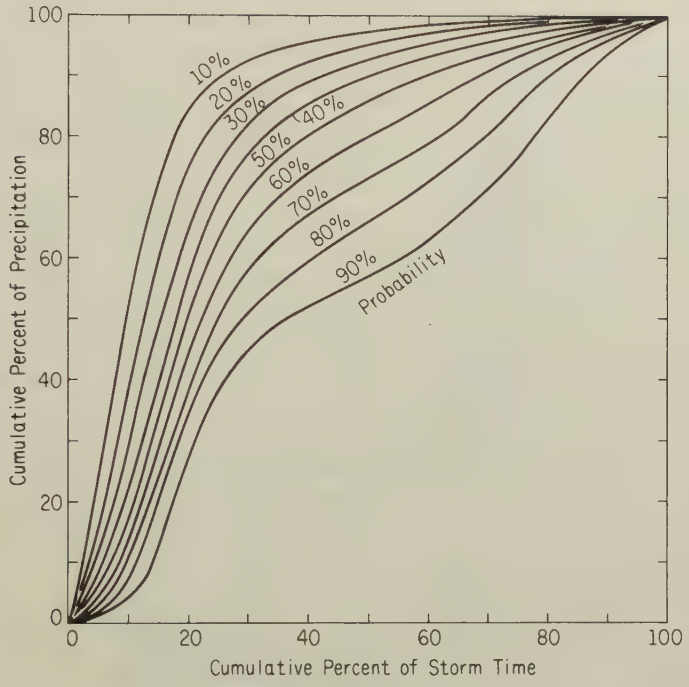


FIG. 8-11. Time distribution of first-quartile storms. (After Ref. 6.)

selection of *CN* is typical of Midwestern small watersheds when the design storm precipitation is greater than 1 in. Since precipitation figures are point values, adjustment must be made to the direct runoff in accounting for smaller rainfall depths over larger areas. The ratio of mean rainfall over the basin to maximum point rainfall is shown in Table 8-8. Ratios for 50-, 100-, and 200-sq mi values are equal to those in Table 8-8 multiplied by 0.91, 0.94, and 0.97 respectively. For uniform rainfall the 95 percent ratios of the table are recommended. With extreme variability the 5 percent ratio applies. The 50 percent ratios approximate average conditions.

Steps in constructing the critical storm method hydrograph are

1. Determine time of concentration of watershed.
2. Choose design frequency.
3. Obtain storm depth based upon selected frequency and duration.
4. Compute direct runoff.
5. Synthesize unit hydrograph for watershed.
6. Select design storm distribution as first-quartile or second-quartile storm.
7. Pick probability level of storm pattern.

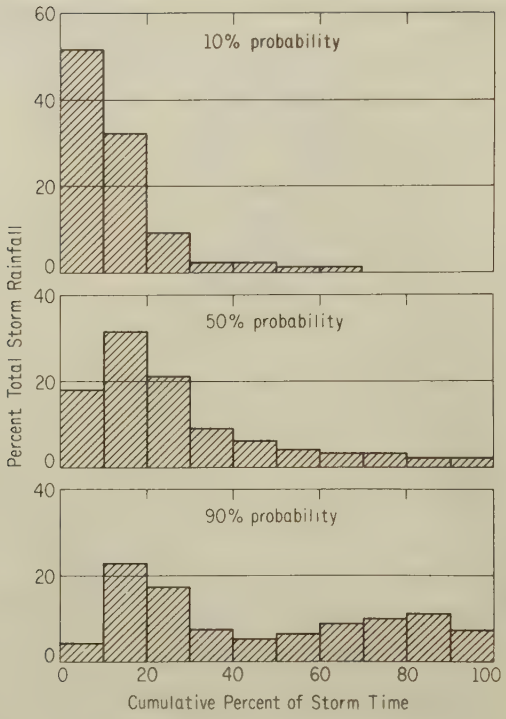


FIG. 8-12. Selected histograms for first-quartile storms. (After Ref. 6.)

8. Apply derived rainfall excess pattern to synthetic unit hydrograph to get storm hydrograph.

8-6 Flood-Frequency Method

Thus far the methods used to determine a storm discharge hydrograph have been based upon a rainfall frequency. Frequency of the storm runoff derived by this method does not necessarily remain the same as that of the rainfall due to assumptions regarding antecedent conditions and the estimate of abstractions from the initial precipitation. A method of dealing with the runoff directly is called the *flood-frequency method*. Unfortunately this method does not provide a hydrograph shape, but gives only a peak discharge of known frequency.

Total direct runoff in this method must be approximated for most cases, accomplished by using Ref. 7 and Figs. 8-4 and 8-5. Storm-volume–frequency curves can also be developed for regional areas to aid in finding the direct runoff to be expected for a selected design frequency. Shape of the hydrograph can be sketched to produce the design peak discharge

TABLE 8-8 Ratio of Maximum to Mean Rainfall on 400 Sq Mi (After Ref. 6)

Rainfall period, hr	Mean rainfall, in.							
	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
				5% Probability Level Ratios				
0.5	5.20	3.00	2.18	1.70	1.41	1.30	1.26	1.22
1	5.50	3.21	2.29	1.80	1.48	1.35	1.30	1.25
2	5.80	3.38	2.44	1.90	1.55	1.41	1.33	1.28
3	6.05	3.54	2.53	1.99	1.61	1.46	1.36	1.30
6		3.77	2.69	2.12	1.72	1.52	1.43	1.35
12		4.01	2.86	2.25	1.83	1.60	1.50	1.40
18		4.14	2.96	2.33	1.90	1.65	1.54	1.43
24		4.27	3.05	2.40	1.96	1.69	1.57	1.45
48		4.55	3.25	2.55	2.08	1.77	1.63	1.50
				50% Probability Level Ratios				
0.5	2.66	2.02	1.57	1.32	1.22	1.16	1.14	1.12
1	3.03	2.15	1.65	1.39	1.27	1.20	1.18	1.16
2	3.46	2.29	1.75	1.46	1.32	1.24	1.21	1.19
3	3.77	2.42	1.85	1.52	1.38	1.28	1.23	1.22
6		2.59	1.98	1.63	1.43	1.33	1.28	1.26
12		2.78	2.12	1.75	1.50	1.39	1.32	1.30
18		2.89	2.20	1.81	1.57	1.43	1.35	1.32
24		3.00	2.28	1.87	1.60	1.47	1.38	1.33
48		3.17	2.44	1.99	1.68	1.53	1.46	1.38
				95% Probability Level Ratios				
0.5	2.38	1.53	1.28	1.18	1.16	1.13	1.11	1.10
1	2.75	1.72	1.38	1.23	1.20	1.17	1.15	1.14
2	3.15	1.90	1.47	1.28	1.24	1.20	1.18	1.16
3	3.46	2.02	1.53	1.33	1.27	1.22	1.20	1.18
6		2.24	1.67	1.43	1.31	1.27	1.24	1.21
12		2.50	1.78	1.50	1.38	1.31	1.28	1.25
18		2.67	1.89	1.53	1.41	1.33	1.30	1.27
24		2.77	1.92	1.58	1.43	1.35	1.32	1.29
48		3.07	2.07	1.64	1.47	1.40	1.36	1.33

and direct runoff volume. This procedure is trial-and-error but is accomplished quickly. For sketching purposes, the time base of the hydrograph is generally a 2.75 multiple of the period to peak (estimated as 0.85 of concentration time) for an advanced hydrograph. An advanced-storm hydrograph is sketched from this information. Depending on the structure's purpose, a delayed-storm hydrograph should be investigated to examine its effect upon storage and spillway design. The delayed-storm hydrograph can be drawn readily by reversing dimensions given for the advanced pattern.

8-7 Method of Airport Design

Airport drainage is required to dispose of surface water and minimize interruption of traffic into and out of the area. The total drainage system

has several functions.[8] (1) Collect and carry off surface water, (2) remove excess groundwater, (3) lower the water table, and (4) protect all slopes from erosion. Only the problem of collecting and removing surface water will be discussed here.

Design of a surface-water disposal system relies upon the *rational method*, although some efforts are being made to evaluate these by means of the spatially varied unsteady flow equations of Chapter 6. Drainage calculations are usually based upon the collected rainfall data[7] and a 1-ft contour topographic map of the proposed finished site. The entire system of drainage is outlined on this map with proper identification of subareas, main and lateral storm drains, direction of flow, gradients, inlets, and surface channels. Final design is attained by calculating the most reasonable cost to provide satisfactory drainage.

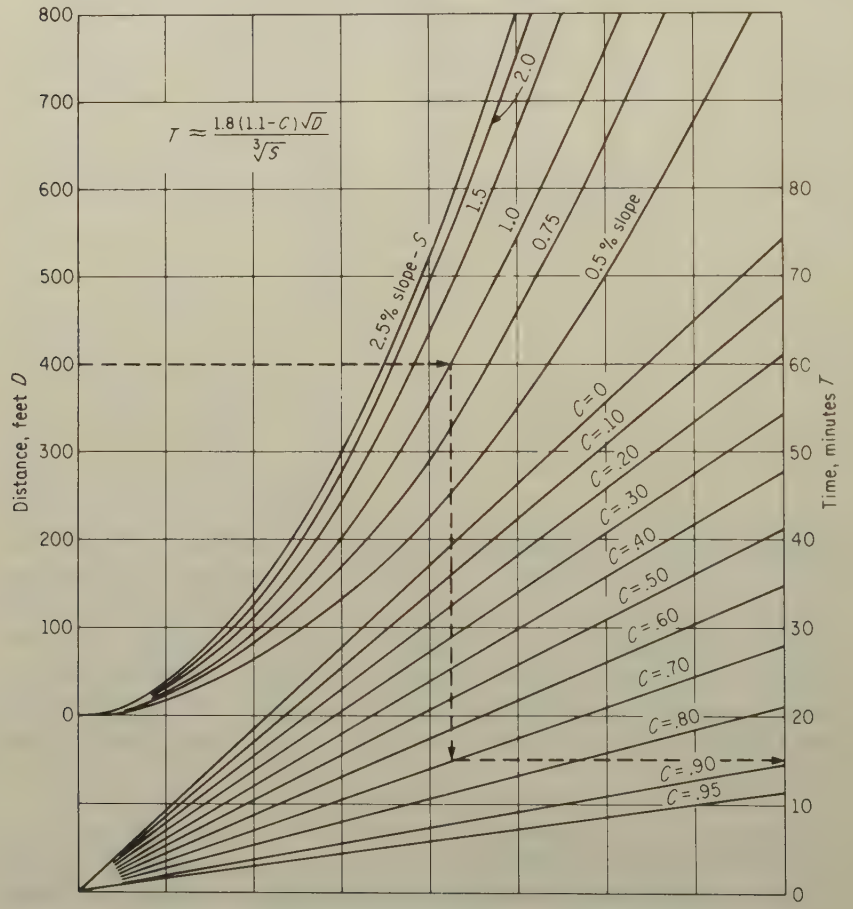


FIG. 8-13. Surface flow time curves. (After Ref. 8.)

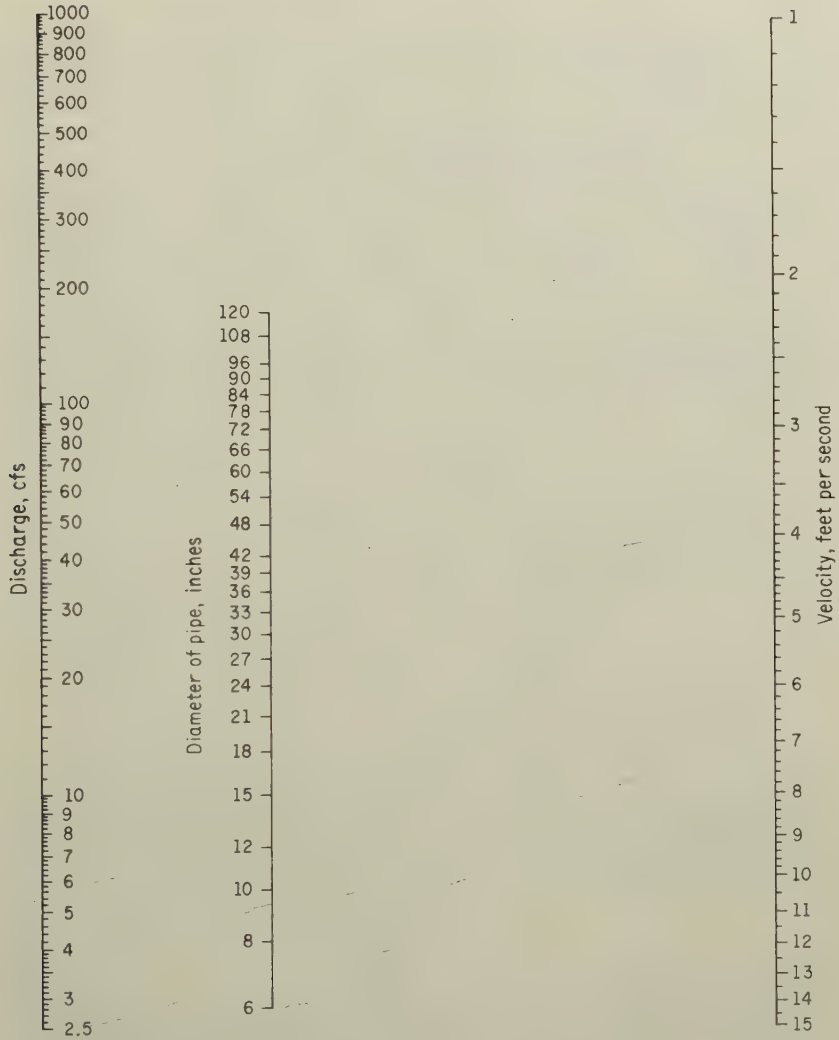


FIG. 8-14. Nomograph for determining velocity of circular full-flowing pipe. (After Ref. 8.)

A step-by-step procedure to design a portion of the surface-drainage facilities of an airport is outlined in Example 8-3 as a straightforward application of the rational method. Each subarea size is outlined and a weighted C adopted based upon $C = 0.90$ for pavement and $C = 0.30$ for the turf areas. Time of concentration in the system is composed of inlet time, and duration of travel in the conduit. Figures 8-13 and 8-14 provide estimates for these times based upon field research by the Corps of Engineers.[8] Design of the drainage system should be adequate to insure that

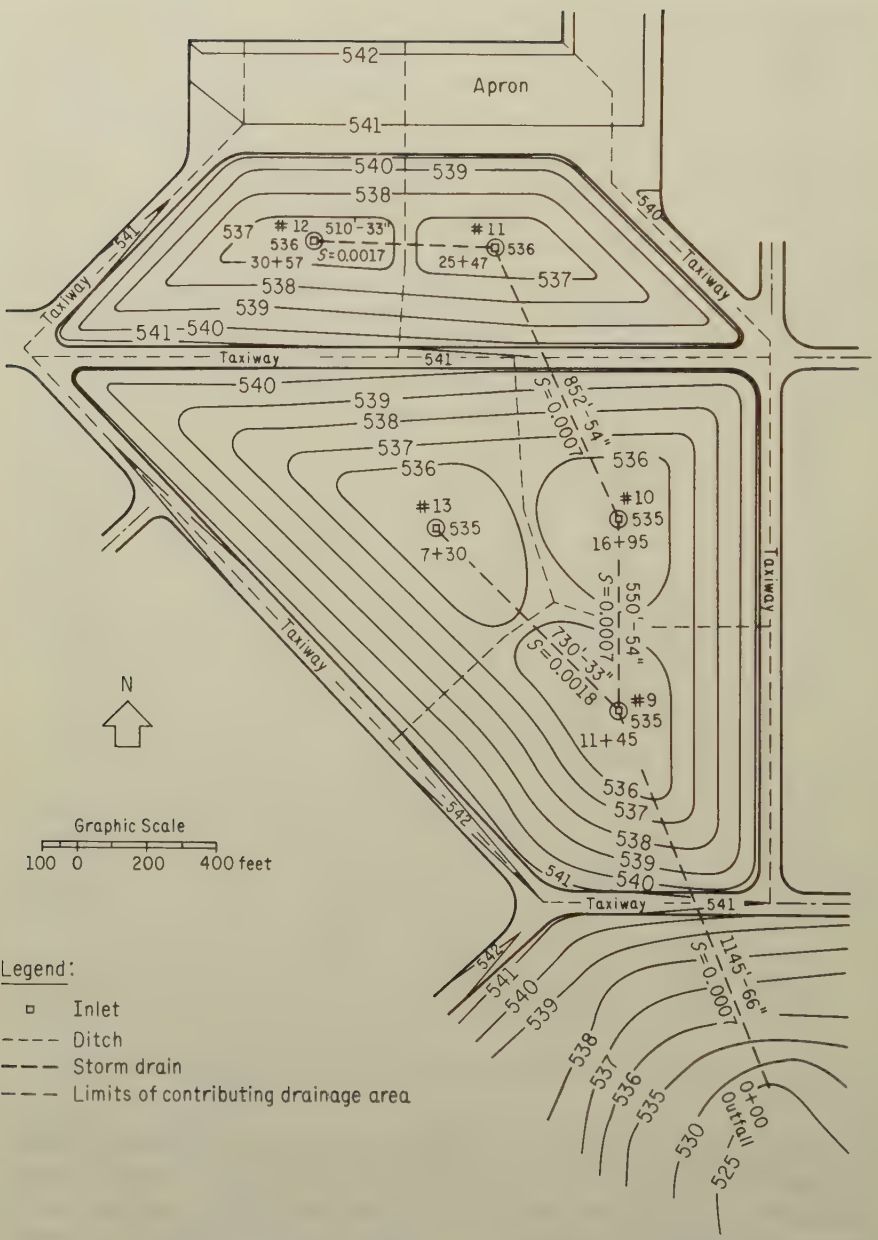


FIG. 8-15. Portion of airport showing drainage design. (After Ref. 8.)

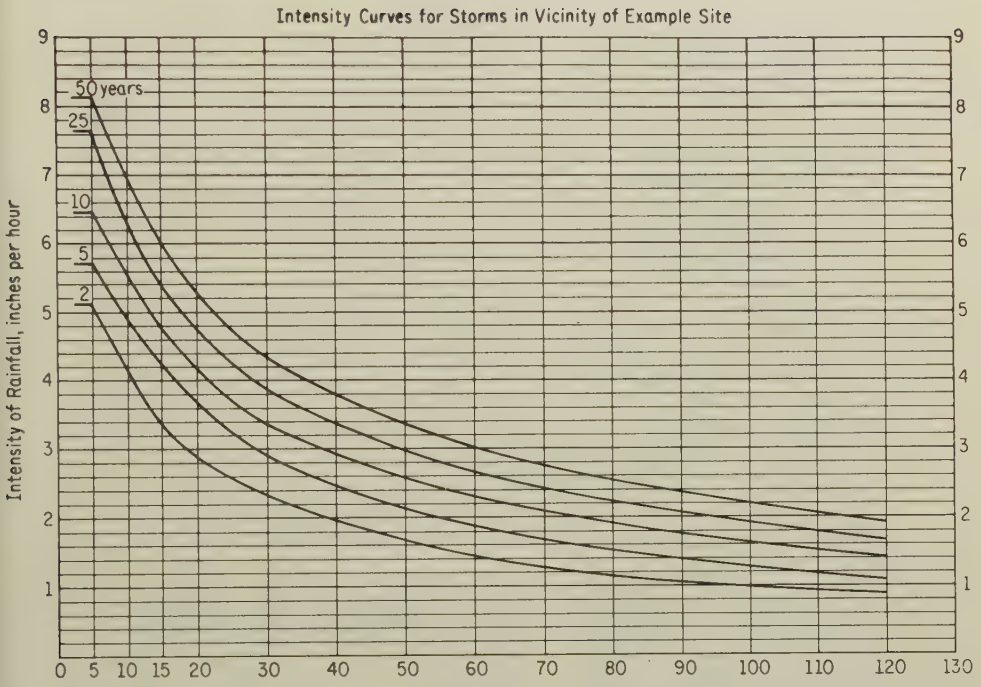

Intensity Curves for Storms in Vicinity of Example Site
Intensity of Rainfall, inches per hour
50 years
25
10
5
2
0 5 10 15 20 30 40 50 60 70 80 90 100 110 120 130
1 2 3 4 5 6 7 8 9

FIG. 8-16. Duration of rainfall (minutes). (After Ref. 8.)

ponding will not be excessive in areas adjacent to the runways. The general criteria is that these areas should be at least 75 ft from bordering pavement. To prevent saturation of nearby ground, rough calculations to insure adequate ponding volumes for the design rainfall are desirable. If critical places occur, routing techniques such as those described in Chapter 6 facilitate calculations of ponding depth from the known storage and outflow characteristics of the system.

Example 8-3 Prepare a surface drainage design for the portion of an airfield shown in Fig. 8-15 for a 5-yr frequency rainfall as drawn in the intensity-duration frequency curve, Fig. 8-16.

Solution:

Computation of subarea sizes, values of weighted C, and inlet times are listed in Table 8-9. Calculation of final pipe sizes necessary to drain the system will vary with slope, and type of pipe selected. Slope of the pipe is usually controlled by the outlet elevation which must be maintained to allow the system to drain freely. Designs in this example are based upon use of a concrete pipe with $n = 0.015$. A nomograph for solution of the discharge as a function of size and slope is provided in Fig. 8-17. Minimum velocity allowed in the pipe is 2.5 fps to prevent excessive settling of sediment. The final drainage system design is shown in Table 8-10.

Column	Comment
1	Inlet being investigated
2	Line segment
3	Length of line
4	Inlet time
5	Flow time in line and is obtained by dividing length of line by velocity of drain
6	Time of concentration
7	Weighted value of C
8	Rainfall intensity based on time of concentration as duration and 5 yr as frequency of design
9	Acreage of subarea immediately tributary to inlet
10	$Q = CIA$
11	Accumulated runoff which must be carried by next line being computed
12	Velocity of flow through line A/A or nomograph Fig. 8-14
13	Pipe size from Fig. 8-17
14	Slope of line
15	Pipe capacity which must be larger than estimated flow
16	Invert elevation of the pipeline
17	Pertinent remarks relative to design

TABLE 8-9 Design Data for Drainage Example in Table 8-10 (After Ref. 8)

Inlet no.	Tributary area to inlets, acres					Distance remote point to inlet, ft			Time for overland flow, min		
	Pavement	Turf	Both	Subtotal	C*	Pavement	Turf	Total	Pavement	Turf	Total
12	4.78	9.91	14.69	14.69	.49	100	790	890	4	37	41
11	5.48	9.24	14.72	29.41	.53	90	750	840	4	36	40
10	1.02	10.95	11.97	41.38	.35	65	565	630	3.5	31.3	34.8
13	1.99	19.51	21.50	21.50	.35	110	1140	1250	4.3	44.3	48.6
9	1.46	14.59	16.05	78.93	.35	85	612	697	3.9	32.4	36.3
Totals	14.73	64.20	78.93								

*Weighted C based on $C = 0.9$, pavement; $C = 0.3$, turf.

8-8 Urban Storm-Drain Systems

Problems encountered in hydrologic design of urban storm-drain systems are more complex than those associated with airfield projects. The ideal approach is to (a) determine the flow to each inlet area, (b) attenuate the peak flow from each subarea as it moves down the pipe, and (c) sum the attenuated peaks to get the total discharge at the design point. Calculation of individual hydrographs to the inlet of each section is influenced by infiltration capacity of the pervious areas, overland flow delays, depression storage, detention in gutters, house drains, catch basins and the storm-sewer system, and interception in extensively landscaped locations.

Two major items normally accounted for in urban storm-drain design are

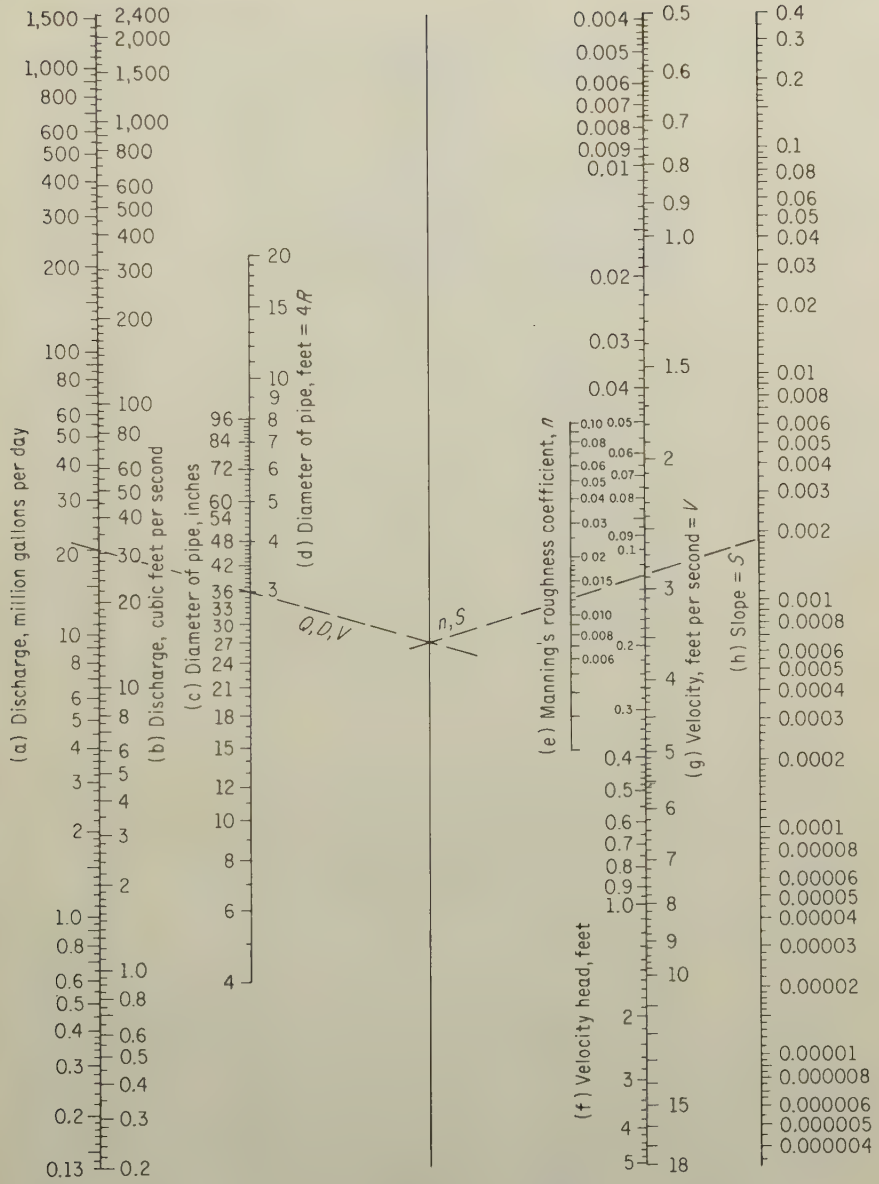

(a) Discharge, million gallons per day
(b) Discharge, cubic feet per second
(c) Diameter of pipe, inches
(d) Diameter of pipe, feet = 4R
(e) Manning's roughness coefficient, n
(f) Velocity head, feet
(g) Velocity, feet per second = V
(h) Slope = S
Q, D, V
n, S

FIG. 8-17. Flow in pipes (Manning's formula). (After Ref. 9.)

TABLE 8-10 Drainage System Design Data (After Ref. 8)

Inlet	Line segment	Length of segment, ft	Inlet time, min	Flow time, min	Time of concentration, min	Runoff coefficient C	Rainfall intensity I, in./hr	Tributary area A, acres	Runoff Q, cfs	Accumulated runoff, cfs	Velocity of drain, ft/sec*	Size of pipe, in.†	Slope of pipe, ft/ft	Capacity of pipe, cfs	Invert elevation	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
12	12–11	510	41	2.7	41	0.49	2.40	14.69	17.28	17.28	3.18	33	.0017	18.90	530.65	(n = 0.015)
11	11–10	852	40	5.0	43.7	0.53	2.31	14.72	18.02	35.30	2.84	54	.0007	45.00	528.03	See note below
10	10–9	550	34.8	3.3	48.7	0.35	2.15	11.97	9.01	44.31	2.84	54	.0007	45.00	527.44	See note below
13	13–9	730	48.6	3.7	48.6	0.35	2.16	21.50	16.25	16.25	3.27	33	.0018	19.40	530.11	
9	9–Out	1145	36.3	5.9	52.3	0.35	2.03	16.05	11.40	71.96	3.24	66	.0007	77.00	526.05	
Out															525.25	

*Minimum velocity is 2.5 fps.

†Minimum pipe size is 12 in. diameter for maintenance purposes.

NOTE: Time of concentration for Inlet No. 11 is 43.7 min (41 + 2.7 = 43.7) which is the most time remote point for this inlet. Likewise time of concentration for Inlet No. 10 is 48.8 min (41 + 2.7 + 5.0 = 48.7).

1. *Infiltration.* Ability of the soil to infiltrate water depends upon many characteristics of the soil as noted in Chapter 2. The following range of values are typical of various bare soils after 1 hr of continuous rainfall.[9]

Soil group	Infiltration, in./hr
High (sandy, open-structured)	0.50 to 1.00
Intermediate (loam)	0.10 to 0.50
Low (clay, close-structured)	0.01 to 0.10

The influence of grass cover increase these values 3 to 7.5 times.

2. *Retention.* Usually assumed to be 0.10 in. for pervious surfaces such as lawns and normal urban pervious surfaces.

Techniques employed to design urban storm-drain systems vary but normally are based upon the rational method or a *routing method.* The Road Research Laboratory Method of Chapter 6 is one approach of the routing method but utilizes only runoff from the impervious areas. As a result, a less densely populated subdivision or urban area is not accurately analyzed. Two design methods will be presented; however an individual situation frequently requires modifications of the procedures to fit a particular problem.

Rational Method

Most urban storm-drain systems are designed by the rational method, since it is easy to apply. Considerable judgment is involved when using this method because time of concentration affects the value of C, and many factors affecting C are difficult to evaluate. A common procedure is to select a C and assume it remains constant throughout the storm. Typical values of C from Table 8-11 are applicable for storms of 5- to 10-yr frequencies.

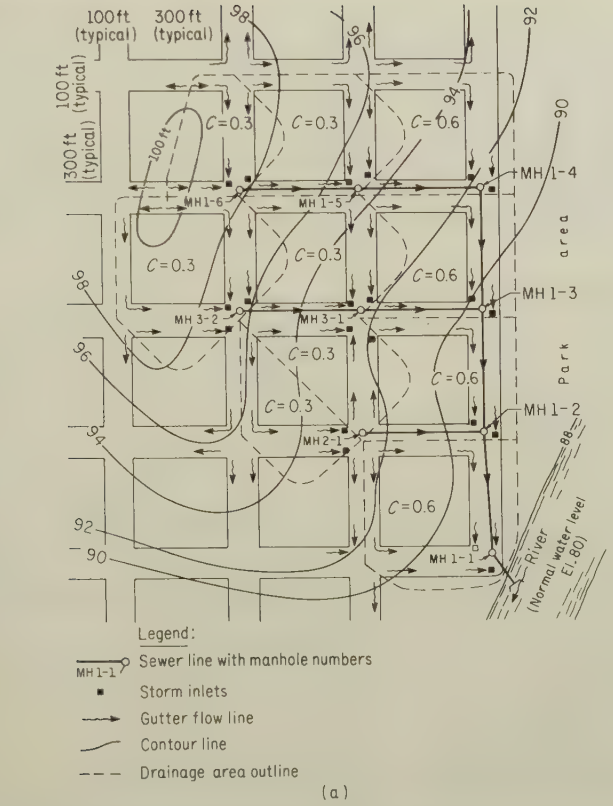
The time of concentration used in the rational method is extremely difficult to establish. The usual assumption accepts it as the interval required for water to flow from the most remote part of the watershed to a designated point. In an urban area it is composed of an inlet time and period of flow in the storm sewer. Calculation of flow time in a storm sewer can be readily estimated knowing the type of pipe, slope, size, and discharge. Generally the pipe is assumed to flow full for this calculation (Fig. 8-14). Estimation of inlet time is commonly based on judgment although curves such as Fig. 8-13 assist in the selection. Reported values vary from 5 to 30 min. Densely developed areas with impervious tracts immediately adjacent to the inlet often are assigned inlet periods of 5 min. Designers assign a minimum value of 10 to 20 min for upper parts of the

TABLE 8-11 Typical C Values for 5-yr to 10-yr Frequency Design

Description of area	Runoff coefficients
Business	
Downtown areas	0.70–0.95
Neighborhood areas	0.50–0.70
Residential	
Single-family areas	0.30–0.50
Multiunits, detached	0.40–0.60
Multiunits, attached	0.60–0.75
Residential (suburban)	0.25–0.40
Apartment dwelling areas	0.50–0.70
Industrial	
Light areas	0.50–0.80
Heavy areas	0.60–0.90
Parks, cemeteries	0.10–0.25
Playgrounds	0.20–0.35
Railroad yard areas	0.20–0.40
Unimproved areas	0.10–0.30
Streets	
Asphaltic	0.70–0.95
Concrete	0.80–0.95
Brick	0.70–0.85
Drives and walks	0.75–0.85
Roofs	0.75–0.95
Lawns; Sandy Soil:	
Flat, 2%	0.05–0.10
Average, 2 to 7%	0.10–0.15
Steep, 7%	0.15–0.20
Lawns; Heavy Soil:	
Flat, 2%	0.13–0.17
Average, 2 to 7%	0.18–0.22
Steep, 7%	0.25–0.35

system, since short rainfalls do not normally warrant consideration in design.

Time of concentration is not started at the beginning of rainfall. Knowledge of the rainfall storm pattern is necessary to accurately set it, inasmuch as this interval is a measure of the average intensity of rainfall during the storm for a given pattern. Runoff obtained by the rational formula is a function of mean rainfall rate during the time required for the flow to travel from the most remote part of the watershed. Most designers applying this method do not use time of concentration in its strictest sense. Rather the sum of inlet plus travel time in the storm sewer is taken as the time of concentration. Caution is required in applying this method. Peak discharge is not the summation of the individual discharges, because peaks from subareas occur at different times. The value of C must be rechecked for each area under consideration. The intensity



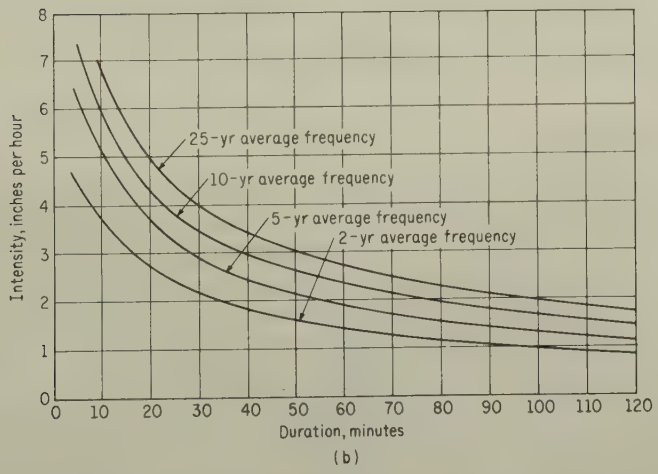


FIG. 8-18. Sample storm drainage problem: (a) typical storm-sewer design plan; and (b) intensity-duration rainfall curves for Davenport, Iowa. (After Ref. 9.)

I is that for the time of concentration of the area being designed and decreases when proceeding downstream in the urban watershed. A constant value for I is not suitable for all areas.

Example 8-4 Based on the storm-sewer arrangement of Fig. 8-18 (a) determine the outfall discharge. Assume $C = 0.3$ for residential areas and $C = 0.6$ for business tracts. Use a 5-yr frequency rainfall from Fig. 8-18b and assume a minimum 20-min inlet time.

Solution:
Final design is tabulated in Table 8-12.

Column	Comment
1	Line being investigated
2, 3	Inlet or manhole being investigated
4	Length of line
5	Subarea for inlet
6	Accumulated subareas
7	Value of the concentration time for the area draining into the inlet
8	Travel time in the pipe line
9	Weighted C for the area being drained
10	Rainfall intensity based on time of concentration and a 5-yr frequency curve
11	Runoff $Q = CIA$
12	Accumulated runoff which must be carried by line
13	Slope of line
14	Size of pipe
15	Pipe capacity
16	Velocity of full pipe
17	Actual velocity of pipe

Additional columns can be provided to list elevations of manhole inverts, sewer inverts, and ground elevations. This information is helpful in checking designs and for subsequent use in drawing final design plans.

Routing Method

Hydrologic and hydraulic routing techniques employing high-speed digital computers are spreading. Larger cities are developing methods to assist in designing urban storm-drain systems. Typically, the approach is to assume an average set of conditions which apply to a typical block in the urban area, establish the degree of imperviousness, estimate depression storage, and route the flow in a storm sewer to obtain a discharge hydrograph based upon a design storm typical of the area.

Keifer utilized this procedure to prepare design curves for the city of Chicago (Fig. 8-19). A 4.01-in. average-intensity 5-yr frequency rainfall was used to develop a rainfall hyetograph of 3-hr duration with a peak intensity of 8.1 in./hr at the $\frac{3}{8}$ point of the period. Infiltration losses were assumed according to earlier work by Horner and Jens as shown

TABLE 8-12 Typical Storm-Sewer Computations for Rational Method for System (After Ref. 9)

Line	Manhole no.		Length, ft	Area, acres		Flow time, min		Average runoff coefficient	Rainfall, in. per hr	Runoff, cfs per acre	Total runoff, cfs	Slope of sewer, percent	Diameter, in.	Capacity, full	Velocity, full	Design flow				Manhole losses, ft	Manhole invert drop, ft	Fall in sewer, ft	Sewer invert		Ground elevation	
	From	To		Increment	Total	To upper end	In section									Velocity, fps	Velocity head, ft	Depth of flow, in.	Total energy, ft				Upper end	Lower end	Upper end	Lower end
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
1	1–6	1–5	400	2.64	2.64	20.0	1.4	0.3	3.7	1.11	2.93	0.85	12	3.3	4.0	4.6		9			...	3.40	93.00	89.60	98.4	94.9
1	1–5	1–4	400	3.61	6.25	21.4	1.2	0.3	3.6	1.08	6.75	0.75	18	9.2	5.1	5.6		11			0.40	3.00	89.20	86.20	94.9	91.8
1	1–4	1–3	400	3.88	10.13	22.6	1.2	0.42	3.4	1.43	14.5	0.45	24	15.2	4.8	5.6		18			0.40	1.80	85.80	84.00	91.8	89.7
3	3–2	3–1	400	5.55	5.55	20	1.1	0.3	3.7	1.11	6.16	1.00	15	6.4	5.1	5.9		12			...	4.00	91.00	87.00	96.2	92.3
3	3–1	1–3	400	6.43	11.98	21.1	1.1	0.3	3.6	1.08	12.92	0.60	24	17.5	5.5	6.1		15			0.60	2.40	86.40	84.00	92.3	89.7
1	1–3	1–2	400	3.92	26.03	23.8	1.1	0.39	3.3	1.29	33.6	0.30	36	37.0	5.1	5.9		26			0.80	1.20	83.20	82.00	89.7	89.5
2	2–1	1–2	400	2.52	2.52	20	1.4	0.3	3.7	1.11	2.80	0.90	12	3.2	4.1	4.7		9			...	3.60	87.50	83.90	92.7	89.5
1	1–2	1–1	400	3.86	32.41	24.9	1.1	0.41	3.2	1.31	42.5	0.24	42	50.0	5.2	5.9		29			0.40	0.96	81.60	80.64	89.5	88.5
1	1–1	Out-fall	125	5.44	37.85	26.0	...	0.44	3.2	1.41	53.2	0.30	42	56.0	5.7	6.6		33			0.10	0.38	80.54	80.16	88.5	...

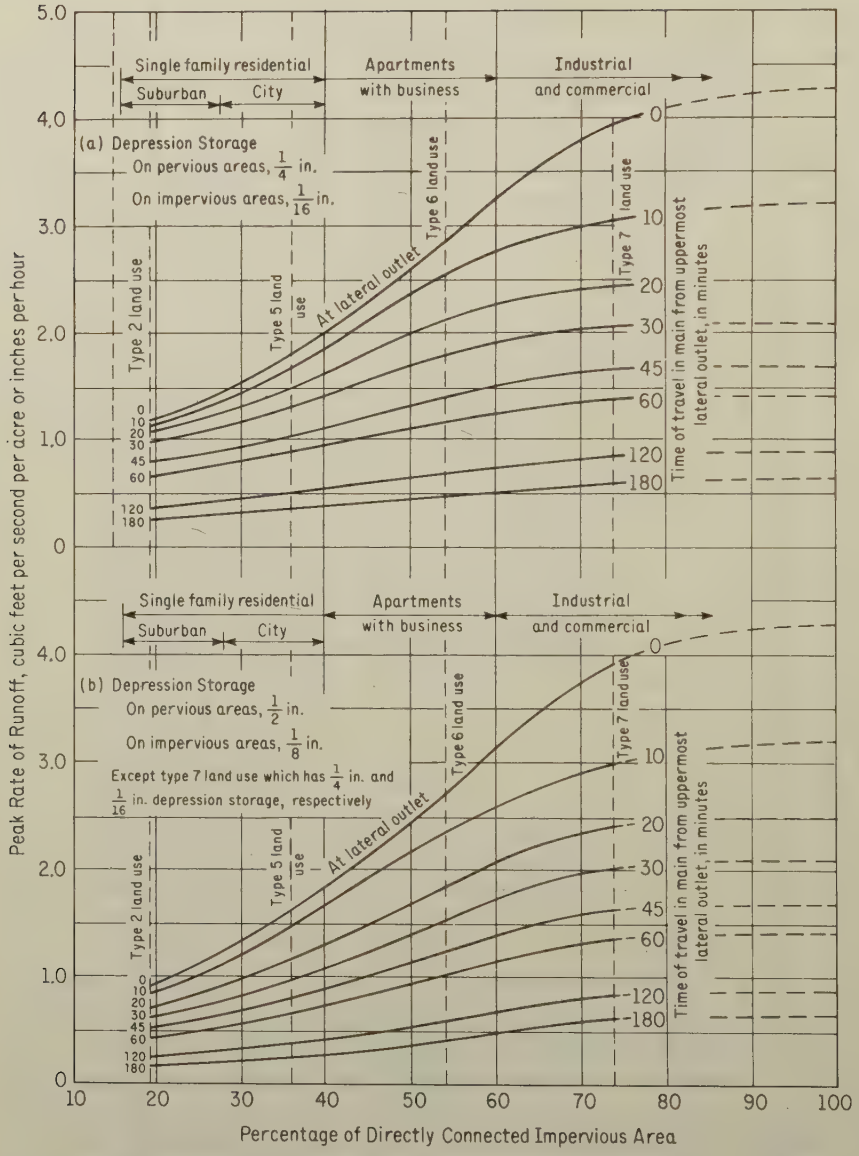

Single family residential
Suburban
City
Apartments with business
Industrial and commercial
(a) Depression Storage
On pervious areas, 1/4 in.
On impervious areas, 1/16 in.
Type 2 land use
Type 5 land use
At lateral outlet
Type 6 land use
Type 7 land use
Time of travel in main from uppermost lateral outlet, in minutes
(b) Depression Storage
On pervious areas, 1/2 in.
On impervious areas, 1/8 in.
Except type 7 land use which has 1/4 in. and 1/16 in. depression storage, respectively
Peak Rate of Runoff, cubic feet per second per acre or inches per hour
Percentage of Directly Connected Impervious Area

FIG. 8-19. Runoff rates in sewer system vs. travel time and percentage of directly connected impervious areas—average ground slope = 0.01. (After Ref. 9.)

in Fig. 8-20. Runoff was presumed to occur when the mass curve of rainfall exceeded that of infiltration. Depression storage was then subtracted from the rainfall excess before overland flow equations were used to compute the discharge from elemental strips in the typical block. Routing these strip inflows through street gutters to catch basins or in-

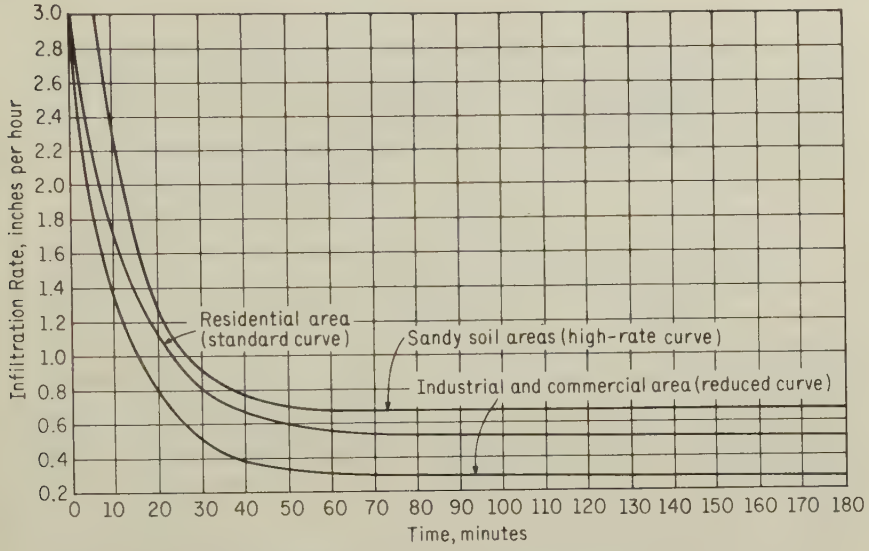


FIG. 8-20. Standard infiltration-capacity curves for pervious surface. (After Ref. 9.)

lets was done by hydrologic routing procedures. Additional input came from roof and street runoff. Design charts giving the peak discharge to be expected under various conditions of imperviousness, depression storage, and slope of land are available.[10] A sample is shown in Fig. 8-19 and an example problem presented in Example 8-5.

Example 8-5 A layout of a proposed storm sewer system is shown in Fig. 8-21. Determine peak discharge at the outfall.

Solution:

A tabular arrangement of the calculations is shown in Table 8-13.

Data for cols. 1 to 4 are taken from Fig. 8-21. Computations then proceed as follows:

Cols. 8-14 show the size of areas and derivations of the percentage of imperviousness for areas tributary to the street intersections indicated in cols. 1 and 2. Multiplying the acreage of each subdrainage tract by its percentage of imperviousness, adding the products, and then dividing the cumulative total by its area, the percentage of imperviousness of the cumulative area is derived and listed in col. 14.

Taking the value of imperviousness from col. 14 and time of travel in col. 7, the unit rate of flow shown in col. 15 is obtained from Fig. 8-19b.

Total flow is then computed by multiplying the unit rate by the cumulative area to the point in question.

Using total flow from col. 16 and slope of hydraulic gradient from col. 4, the velocity of flow and time of travel can be picked from conventional hydraulic charts in the customary manner.

TABLE 8-13 Computation of Runoff by Hydrograph Method (After Ref. 9)

	Location		Length of reach, ft	Slope of hydraulic gradient, ft/ft	Velocity in reach, ft/sec	Time of travel		Area of addition			Imperviousness				Peak flows	
	In	At				In reach, min	To min	No.	A, acres	ΣA acres	I_p, %	$A \times I_p$, acres	ΣAI_p, acres	I_p for ΣA, %	Unit rate, cfs per acre	Total cfs
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1		Cornelia Avenue					0	(1)	20	20	38.6	772	772	38.6	1.82	36
2	Tenth Street		330	0.002	4.5	1.2										
3		Blackhawk					1.2	(2)	20	40	34.5	690	1,462	36.5	1.69	68
4	Tenth Street		330	0.0017	4.9	1.1										
5		Albany Avenue					2.3	(3)	20	60	36.0	720	2,182	36.4	1.67	100
6	Tenth Street		330	0.0014	5.0	1.1										
7		Main Street					3.4	(4)	20	80	45.5	910	3,092	38.6	1.75	140
8	Main Street		2,640	0.0012	5.2	8.5										
9		Sixth Street						(5)	20		42.8	856				
10		Sixth Street					11.9	Br.[a] A	60	160	35.2	2,112	6,060	37.9	1.51	242
11	Main Street		2,640	0.001	5.5	8.0										
12		Second Street						(6)	23		50.1	1,152				
13		Second Street					19.9	Br.[a] B	34	217	37.2	1,268	8,480	39.0	1.30	282
14	Main Street		1,650	0.0009	5.6	4.9										
15		River					24.8									282

[a]Branch.

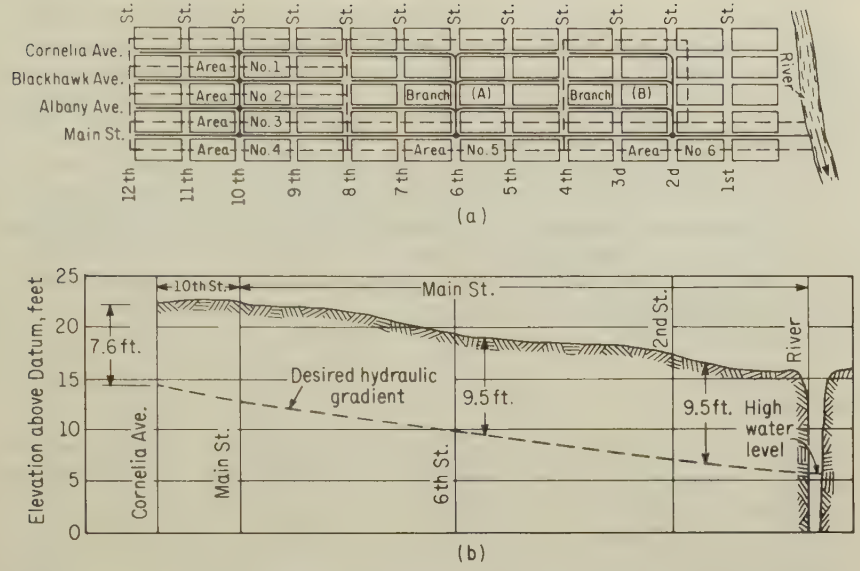


FIG. 8-21. Layout of proposed sewer system by hydrograph method: (a) plan; and (b) profile. (After Ref. 9.)

Time of flow shown in col. 6 is obtained by dividing length of reach by velocity of flow in col. 5; the total is then recorded at each junction point, col. 7.

Where additional branches enter the main sewer as at the corner of Sixth and Main Street, separate drainage areas are individually listed in lines 9 and 10, and the summation of all areas tributary to that point used to calculate flow rates and time of travel below such a junction. For example the value of 160 acres for col. 10, line 10, is found by adding the 80 acres tabulated in col. 10, line 7, to the 20 acres from col. 9, lines 9 and 10. Similarly, the entry in col. 13, line 10, is the summation of the figures in col. 13, line 7, and col. 12, lines 9 and 10.

References

1. "Generalized estimates of Maximum Probable Precipitation over the United States East of the 105th Meridian," U.S. Weather Bureau Hydrometeorol. Report No. 23, June 1947.

2. "Generalized Estimates of Probable Maximum Precipitation for the United States West of the 105th Meridian for Areas to 400 Square Miles and Durations to 24 Hours," U.S. Weather Bureau Tech. Paper 38, 1960.

3. "Design of Small Dams," Bureau of Reclamation, United States Department of the Interior. Washington, D. C.: Government Printing Office, 1965.

4. Ogrosky, H. O., "Hydrology of Spillway Design: Small Structures—Limited Data," *A.S.C.E., J. Hydr. Div.*, Vol. 90, No. HY3 (May 1964), pp. 295–310.

5. Hydrology, Suppl. A to Sec. 4, *Engineering Handbook*. Washington, D. C.: U.S. Department of Agriculture, Soil Conservation Service, 1968.

6. Huff, F. A., "Time Distribution of Rainfall in Heavy Storms," *Water Resources Research*, Vol. 3, No. 4 (1967), pp. 1007–1019.

7. "Rainfall Frequency Atlas of the United States for Durations from 30 Minutes to 24 Hours and Return Periods from 1 to 100 Years," U.S. Weather Bureau Tech. Paper 40, 1963.

8. "Airport Drainage," Federal Aviation Agency, Advisory Circular, AC No.-AC 150/5320-5A. Washington, D. C.: Government Printing Office, 1966.

9. "Design and Construction of Sanitary Storm Sewers," *A.S.C.E. Manuals and Reports on Engineering Practice*, No. 37.

10. Tholin, A. L., and C. J. Keifer, "Hydrology of Urban Runoff," *Trans. A.S.C.E.*, Paper No. 3061, Vol. 125 (1960).

Problems

8-1. Given the following data, route a storm hydrograph through a reservoir and plot the resulting outflow hydrograph for the Deer Run watershed at Salt Lick. Show all work in a neat and logical order.

(a) Area = 3.75 sq mi
(b) Length = 5.80 mi
(c) Elevation of the bottom of the spillway = 1160.0 ft
(d) Spillway width B = 10.0 ft
(e) Spillway Coefficient C = 3.0

Elev., ft	Dist., ft	Area $\times 10^{-6}$ ft^2 (of respective contours)
1110	0	0.0
1120	1,400	0.85
1140	4,600	3.75
1160	8,000	10.80
1180	11,500	25.00
1200	14,700	
1220	17,400	
1240	20,000	
1260	22,600	
1280	24,800	
1300	26,600	
1320	28,000	
1340	29,200	
1360	30,000	
1376	30,624	

Required:

(a) A unit hydrograph for this watershed by Gray's method.
(b) The storm (inflow) hydrograph resulting from a storm that produced 6.0 in. of rainfall excess during the first 45-min. period, and 4.0 in. during the last 45-min. interval
(c) Route the storm hydrograph through the reservoir. Assume that at the start of rainfall, elevation of the water surface in the reservoir is 1158.0 ft.

(d) Elevation for top of the dam. Discuss.

(e) Spillway design adequacy. Discuss.

8-2. Repeat Prob. 8-1 for a type (b) classified structure in an area selected by the instructor.

8-3. Repeat Prob. 8-2 for a type (a) classified structure. Calculate the increase cost for it assuming \$125/cu yd for concrete and 70¢/cu yd for fill dirt.

8-4. Rework Example 8-1 for a 5-sq mi drainage area, time of concentration 3.8 hr, and runoff number of 73 in your geographic area.

8-5. Use information from Prob. 8-4 and redo Example 8-2.

8-6. Calculate a design-storm hyetograph for a 100-yr frequency level which has a 6-hr duration. Assume the storm occurred near Springfield, Illinois, and for the information in Sec. 8-5 compute the rainfall excess distribution likely to occur only 10 percent of the time.

8-7. Rework Example 8-4 based on a $C = 0.2$ and $C = 0.4$. Compare and discuss the effect of C on discharge at the outfall.

9

MAJOR STRUCTURE DESIGN

Hydrologic design aspects of a major structure are considerably more complex than the design of a small dam, crossroad culvert, or urban drainage system. A design-storm hydrograph for a large dam still is required but its use is not restricted to sizing the spillway. The design-storm hydrograph is routed to determine the combination of discharge capacity of spillways and outlets in conjunction with reservoir storage. Economic selection of spillway size from the various combinations dictates the final design and is a function of security to downstream property, project economy, agency policy and construction standards, and reservoir operational requirements.

9-1 Design Philosophy

Critical in the design of a huge structure such as a dam is the possibility of failure. Sudden release of large volumes of impounded water can create damages far greater than those experienced prior to construction. Also, initial heights of retarded water behind the dam, disregarding total volume of stored water, can produce destructive flood waves considerable distance downstream. Based upon these two criteria the Task Force on Spillway Design Floods[1] recommended classifying dams as listed in Table 9-1. Type of construction has not been included in this grouping although it governs the extent of failure resulting from overtopping.

Hydrologic data necessary to design large dams stem from the need for a more intensive examination of rainfall, storm pattern (both temporal and spatial), and runoff records. Drainage areas contributing to the runoff can vary from 1,000 to as many as 200,000 sq mi. A sample of reservoir sizes in scattered locations throughout the United States is given in Table 9-2.

TABLE 9-1 Classification of Dams (After Ref. 1)

Category	Impoundment danger potential		Failure damage[a] potential		Spillway design flood
	Storage, acre-ft[b]	Height, ft	Loss of life	Damage	
(1)	(2)	(3)	(4)	(5)	(6)
Major; failure cannot be tolerated	> 50,000	> 60	Considerable	Excessive or as matter of policy	Probable maximum; most severe flood considered reasonably possible on the basin.
Intermediate	1,000 to 50,000	40 to 100	Possible but small	Within financial capability of owner	Standard project; based on most severe storm or meteorological conditions considered reasonably characteristic of the specific region.
Minor	< 1,000	< 50	None	Of same magnitude as cost of dam	Frequency basis; 50–100-yr recurrence interval.

[a] Based on consideration of height of dam above tailwater, storage volume, and length of damage reach, present and future potential population and economic development of the flood plain.
[b] Storage at design spillway pool level.

TABLE 9-2 A Few of the Large Dams in the United States

Name	Drainage area, sq mi
Lake of the Ozarks, Mo.	14,000
Lake Brownwood, Tex.	1,544
Great Salt Plains Reservoir, Okla.	31,566
Tongue Reservoir, Wyo.	1,740
Canton Reservoir, Okla.	12,483
Conchas Reservoir, N. M.	7,350
Lake Mead (Ariz.–Nev.)	168,000

The drainage areas contributing to large dams require stricter analysis of an area's hydrology than is necessary in designing minor structures. Possibility of a uniformly intense rainfall over the entire basin is an unrealistic assumption for large watersheds. The influence of temporal and spatial variations of rainfall should thus be considered. For major dams, the estimated "worst possible" rainfall values are generally converted to a design discharge hydrograph which is then used in reservoir routing calculations to proportion reservoir and spillway size, surcharge storage, and any additional outlets needed to maintain power requirements or sus-

tained downstream flow for navigation, irrigation, or water supply. The basic concern in hydrologic design of a large dam is to preserve downstream interests and have a realistic estimate for the design storm hydrograph.

9-2 Downstream Damages

The term *danger reach* has been used[1] to define the distance downstream from the dam site within which release of impounded water from a sudden failure would cause excessive damage over and above that expected from the same storm had the dam not existed. Calculation of the peak discharge to be expected from a breached dam can be computed by[2]

$$Q_{max} = 0.29\sqrt{g}\,K^{0.28}\,W_b D_b^{1.5} \tag{9-1}$$

where Q_{max} = peak discharge at the dam, cfs
g = acceleration of gravity
W_b = width of breach, ft
D_b = depth of water above breach, ft
$K = \dfrac{W_d}{W_b}\dfrac{Y_0}{D_b} = 1$ for complete breach
W_d = width of dam, ft
Y_0 = depth of water before breach, ft

Channel routing techniques shown in Chapter 6 can be used to define the shape of the flood wave as it moves downstream. Frequently the shape of the hydrograph at the dam will have to be estimated, but volume of the flood wave is known from the reservoir storage at pool level, and the peak discharge can be obtained from Eq. 9-1. Assumed hydrograph shapes in the form of triangles, and additional field measurements of downstream valley cross sections, will allow rapid calculation of the movement of a downstream flood wave.

9-3 Design Floods

In major structure design, three general terms are employed to designate design floods: (a) probable maximum flood, (b) standard project flood, and (c) frequency-based flood. The concept of *flood* in this section is described by an entire discharge hydrograph which is generally synthesized from rainfall estimates called "storm" values. Then, corresponding to the 3 designations, storm values—i.e., depth of rainfall, are referred to in terms of (a) probable maximum precipitation, (b) standard project storm, and (c) frequency-based storm. Design of the various reservoir system components is commonly based upon one of these representative terms.

Probable Maximum Flood

The probable maximum flood is defined as the most severe flood considered reasonably possible to occur. It is customarily obtained by using unit hydrographs and rainfall estimates prepared by the Environmental Science Service Administration[3] (see Fig. 9-1). In areas of sparse data it

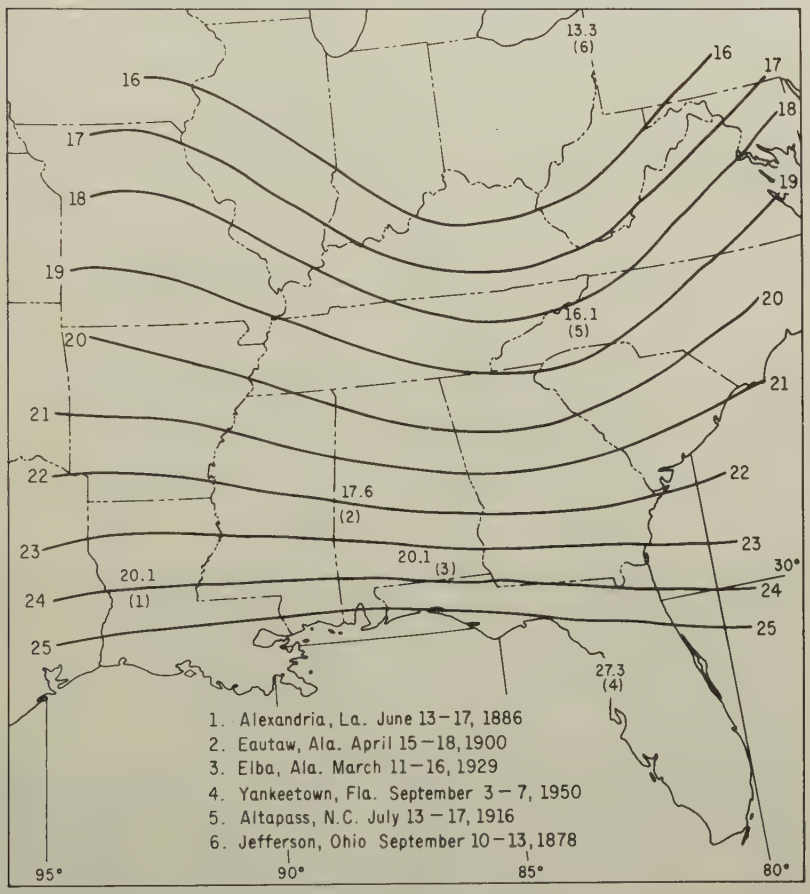


FIG. 9-1. 24-hr 2,000-sq-mi PMP (in.). (After Ref. 3.)

can be gotten by drawing envelope curves of the maximum floods of record in the region under study, see Fig. 9-2. In cases where estimates of PMP have not been made, volumes of rainfall to be expected can also be approximated from envelope curves of the world-record rainfalls depicted in Fig. 9-3.

A proposed method to estimate PMP advocated by Hershfield[4] sug-

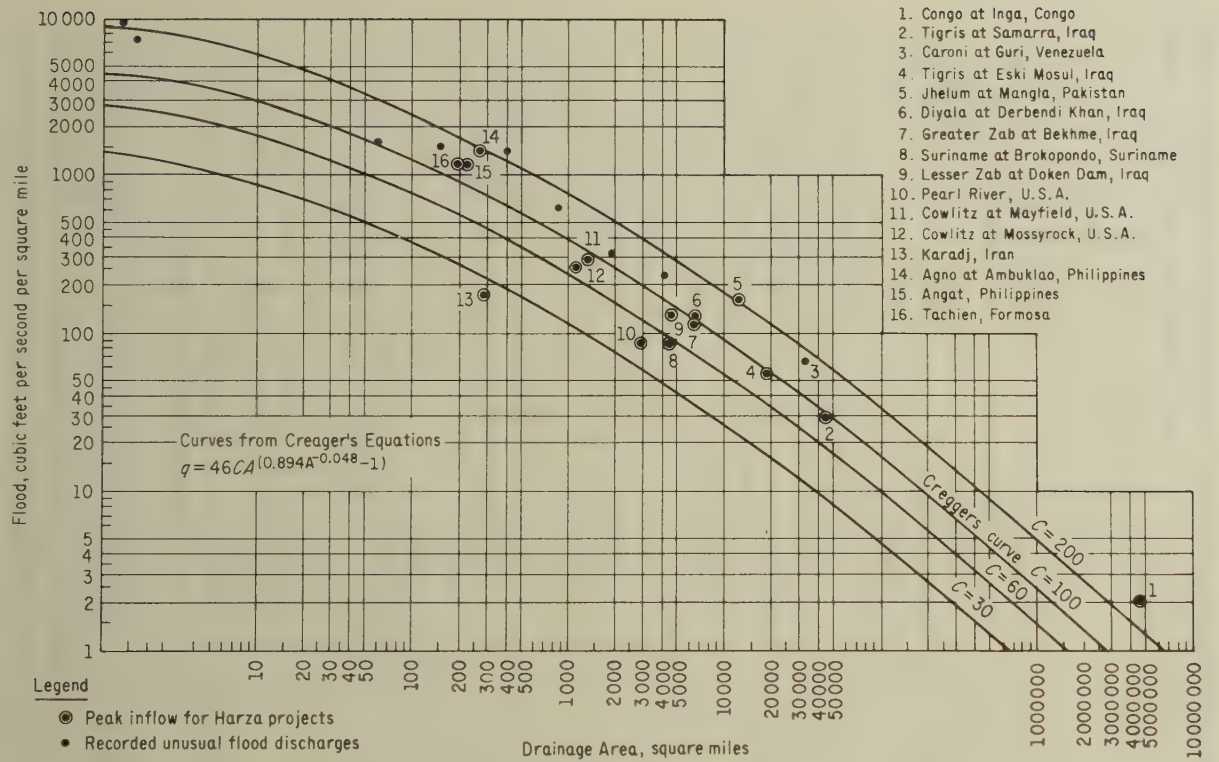
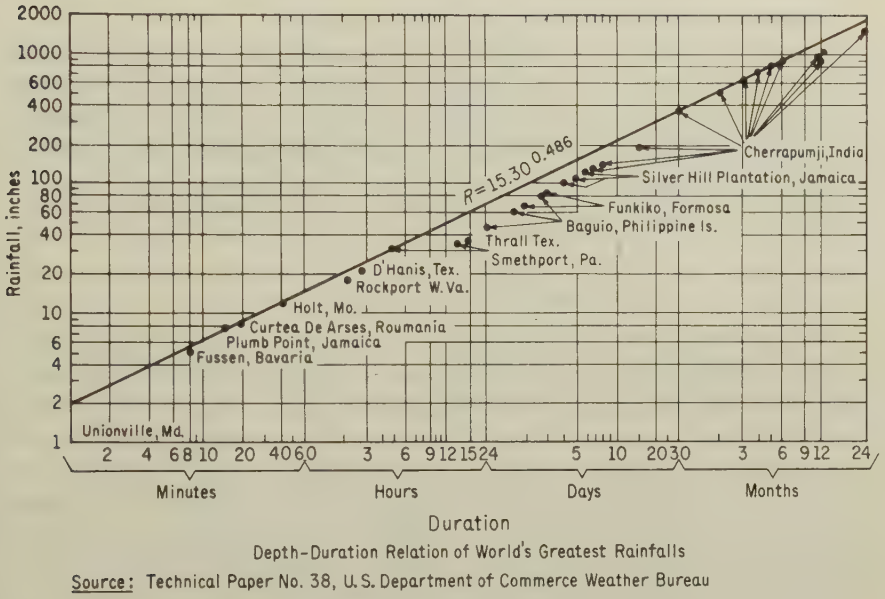

1. Congo at Inga, Congo
2. Tigris at Samarra, Iraq
3. Caroni at Guri, Venezuela
4. Tigris at Eski Mosul, Iraq
5. Jhelum at Mangla, Pakistan
6. Diyala at Derbendi Khan, Iraq
7. Greater Zab at Bekhme, Iraq
8. Suriname at Brokopondo, Suriname
9. Lesser Zab at Doken Dam, Iraq
10. Pearl River, U.S.A.
11. Cowlitz at Mayfield, U.S.A.
12. Cowlitz at Mossyrock, U.S.A.
13. Karadj, Iran
14. Agno at Ambuklao, Philippines
15. Angat, Philippines
16. Tachien, Formosa
Flood, cubic feet per second per square mile
Drainage Area, square miles
Curves from Creager's Equations
$q = 46CA^{(0.894A^{-0.048}-1)}$
Creager's curve
C=200
C=100
C=60
C=30
Legend
Peak inflow for Harza projects
Recorded unusual flood discharges
Note
Curves taken from "Hydroelectric Handbook"
by Creager and Justin
(John Wiley & Sons Inc. 1950)

FIG. 9-2. Creager envelope curves.



Depth-Duration Relation of World's Greatest Rainfalls

Source: Technical Paper No. 38, U.S. Department of Commerce Weather Bureau

FIG. 9-3. World's greatest rainfalls. (After Ref. 3.)

gests that the 24-hr PMP at a point be computed by the equation

$$\text{PMP}_{24} = \bar{P} + KS_n \tag{9-2}$$

where PMP_{24} = 24-hr probable maximum precipitation
$\bar{P}$ = mean of the 24-hr annual maximums over period of record
S_n = standard deviation of the 24-hr annual maximums
K = a constant equal to 15

Adjustments to the value of $\bar{P}$ and S_n for record length are noted by Hershfield. However, for appraisal purposes these adjustments probably will not significantly alter results more than 5 to 10 percent.

Standard Project Flood

This storm differs from a PMP estimate since it should result from a storm of record which causes the most severe rainfall depth-area-duration relationship. Appropriate allowances should be made for inclusion of snow melt in calculating design-storm hydrographs from the standard-project storm. Generally the standard project storm rainfall amounts to approximately 50 percent of the PMP. Records of the four or five largest storms should be critically examined to find a suitable composite for use in calculating the SPF. Where these data are not available, a reasonable percentage of the PMP can be substituted.

Frequency-Based Flood

Frequency curves can be plotted for streams where lengthy records are available. Most often, however, location of the dam is not the gaging site, but on the same stream routing techniques can effectively transfer the flood peaks. Regionalized flood frequency data may also be employed to advantage. Customarily, frequency-based floods are not a part of the design criteria for major structures.

9-4 Obtaining Storm Rainfall Estimates

The procedure described in Chapter 8 dealt with rainfall durations of approximately 6 hr or less. These durations are satisfactory for small watersheds but the length of storms on large areas require storm-depth values for periods up to ten days. Such values are available[5] for frequencies of from two to ten days for locations within the United States. Similar data are also available for other selected areas outside of the United States. Generally however, design criteria for large dams require estimates of storm values which do not have frequency levels assigned

Typical storm figures required are the Probable Maximum Precipitation, PMP, or the Standard Project Storm, SPS. Values of PMP for most large drainage basins have been tabulated. Examples of extreme rainfall occurrences are listed in Tables 9-3 and 9-4. Numbers in the former table are for selected point events exceeding 10 in. in 6 hr whereas those in Table 9-4 are for maximum average rainfall depths over an area of 2,000 sq mi.

Values of PMP for drainage areas of 2,000 sq mi are shown in Fig. 9-1. These values reflect a decreasing rainfall depth with increasing area. The depth-area-duration relationship can be taken from numerous ESSA publications* and demonstrates the preceding statement that there is increasing depth with duration (Fig. 9-3). Figures 9-1 and 9-4 are extremely useful in determining PMP for large watersheds.

The Standard Project Storm is another rainfall depth value which is crucial in the design of large dams. This storm value is usually obtained from a survey of severe storms in the general vicinity of the drainage basin (Tables 9-3 and 9-4). The storm selected as the SPS may be oriented to produce the maximum amount of runoff for the SPF. Severe storms can be transposed into the study area from meteorologically "similar" areas. The purpose of this process is to increase the number of severe storm experiences for the study watershed.

Transposition limits are based upon climatological similarity, synoptic weather patterns and a knowledge of atmospheric processes. A number

*See Refs. 3 and 4 as examples of available PMP information.

TABLE 9-3 U.S. Rainfall Occurrences Equaling or Exceeding 10 In. in 6 Hr (After Ref. 3)*

Date	Amount,† in.	Location	Date	Amount, in.	Location
Jun 13–17, 1886	11.5	Alexandria, La.	May 30–31, 1938	10.0	Sharon Springs, Kans.
Jun 23–27, 1891	10.4	Larabee, Ia.	Jul 19–25, 1938	11.5	Eldorado, Texas
Jun 4–7, 1896	12.0	Greenley, Nebr.	Aug 12–15, 1938	10.9	Koll, La.
Jul 26–29, 1897	13.0	Jewell, Md.	May 25, 1939	8.2	Lebanon, Va.
Jun 12–13, 1907	6.2 (3 hr)	Fort Meade, S.D.	Jun 19–20, 1939	18.8	Synder, Texas
Jul 18–23, 1909	10.5	Ironwood, Mich.	Jul 4–5, 1939	18.6 (3 hr)	Kimpson P.O., Ky.
Jul 18–23, 1909	10.5	Beaulieu, Mont.	Jul 4–5, 1939	20.0	Simpson P.O., Ky.
Aug 28–31, 1911	14.9	St. George, Ga.	Aug 21, 1939	9.5	Baldwin, Me.
Aug 31–Sep 1, 1914	12.6	Cooper, Mich.	Jun 3–4, 1940	13.0	Grant Township, Nebr.
Aug 1–3, 1915	12.9	St. Petersburg, Fla.	Jun 28–30, 1940	11.0	Engle, Texas
Sep 28–30, 1915	10.1	Franklinton, La.	Sep 1, 1940	20.1	Ewan, N.J.
Jul 5–10, 1916	15.9	Bonifay, Fla.	Sep 2–6, 1940	18.4	Hallett, Okla.
Jun 2–6, 1921	10.4	Pueblo, Colo.	May 22, 1941	6.5 (3 hr)	Plainville, Ill.
Jun 17–21, 1921	10.5	Springbrook, Mont.	Oct 17–22, 1941	12.9	Trenton, Fla.
Sep 8–10, 1921	22.4	Thrall (Taylor), Texas	Apr 14–17, 1942	13.1	Green Acres City, Fla.
Jul 9–12, 1922	10.8	Grant City, Mo.	Jul 17–18, 1942	24.7	Smethport, Pa.
Oct 4–11, 1924	13.6	New Smyrna, Fla.	May 12–20, 1943	15.9	Near Mounds, Okla.
Sep 11–16, 1926	13.4	Neosho Falls, Kans.	Jun 5–7, 1943	14.2	Silver Lake, Texas
Sep 17–19, 1926	18.4	Boyden, Ia.	Jul 27–29, 1943	10.7	Devers, Texas
Apr 12–16, 1927	13.8	Jeff-Plaq Drain. Dist., La.	Aug 4–5, 1943	11.1	Glenville, W. Va.
Mar 11–16, 1929	14.0	Elba, Ala.	Jun 10–13, 1944	13.4	Stanton, Nebr.
May 25–30, 1929	11.3	Henly, Texas	Jul 9, 1945	9.1 (4 hr)	Easton, Pa.
Jun 30–Jul 2, 1932	13.3	State Fish Hatchery, Texas	Aug 26–29, 1945	10.1	Hockley, Texas
Aug 30–Sep 5, 1932	10.0	Fairfield, Texas	Aug 12–15, 1946	10.6	Cole Camp, Mo.
Apr 3–4, 1934	17.3	Cheyenne, Okla.	Sep 26–27, 1946	15.8	San Antonio, Texas
May 2–7, 1935	10.6	Melville, La.	Jun 18–23, 1947	11.5	Holt, Mo.
May 16–20, 1935	13.8	Simmesport, La.	Aug 27–28, 1947	13.8	Wickes, Ark.
May 30–31, 1935	20.6	NE of Colorado Springs, Colo.	Aug 24–27, 1947	10.9	Dallas, Texas
Jun 27–Jul 4, 1936	14.0	Bebe, Texas	Jun 23–24, 1948	13.2	Del Rio, Texas
Sep 14–18, 1936	16.0	Broome, Texas	Sep 3–7, 1950	16.0	Yankeetown, Fla.

*A few cases of storms less than 6-hr duration are included.
†6 hour duration on 10 sq mi.

TABLE 9-4 Maximum Observed and Moisture-Maximized Storm Rainfall for 24 Hr Over 2,000 Sq Mi (After Ref. 3)

Date	Location	Observed amount, in.
9/10–13/1878	Jefferson, Ohio	10.4
6/13–17/1886	Alexandria, La.	17.3
6/27–7/1/1899	Hearne, Texas	19.0
4/15–18/1900	Eautaw, Ala.	10.8
10/7–11/1903	Cortland, N.Y.	10.2
8/28–31/1911	St. George, Ga.	11.3
3/24–28/1914	Merryville, La.	10.1
9/28–30/1915	Franklinton, La.	11.4
7/5–10/1916	Bonifay, Fla.	14.6
7/13–17/1916	Altapass, N.C.	13.3
9/8–10/1921	Thrall, Texas	20.6
9/13–17/1924	Beaufort, N.C.	10.7
10/4–11/1924	New Smyrna, Fla.	11.9
4/12–16/1927	Louisiana	13.3
6/1–5/1928	Thomasville, Ala.	10.9
9/16–19/1928	Darlington, S.C.	10.3
3/11–16/1929	Elba, Ala.	15.0
9/23–28/1929	Washington, Ga.	12.1
6/30–7/2/1932	State Fish Hatchery, Texas	16.9
8/30–9/5/1932	Fairfield, Texas	12.8
7/22–27/1933	Logansport, La.	13.0
12/5–8/1935	Satsuma, Texas	11.9
6/27–7/4/1936	Bebe, Texas	12.2
9/14–18/1936	Broome, Texas	11.6
8/6–9/1940	Miller Is., La.	16.7
9/2–6/1940	Hallett, Okla.	10.7
10/17–22/1941	Trenton, Fla.	15.2
7/17–18/1942	Smethport, Pa.	10.2
9/3–7/1950	Yankeetown, Fla.	24.8
6/23–28/1954	Pierce, Texas	14.7

of severe storms are selected from the storms of record and those transposed. The depth-area-duration curves for these are developed and presented in graphical form similar to Fig. 9-4, usually for a particular point location within the basin (referred to as an index point). This position is used as a reference location for future calculations regarding the SPS.

The typical isohyetal pattern for SPS storms has been established as generally eliptical in shape as shown in Fig. 9-5. Variations in the rainfall depth found in the study of a standard project storm will diverge from a maximum at the storm center to a value considerably less than average depth at the edges of the watershed boundaries. This variation can be incorporated in the design procedure for establishing depth values of the SPS.

As an example, a procedure utilizing a ratio of isohyetal value for vari-

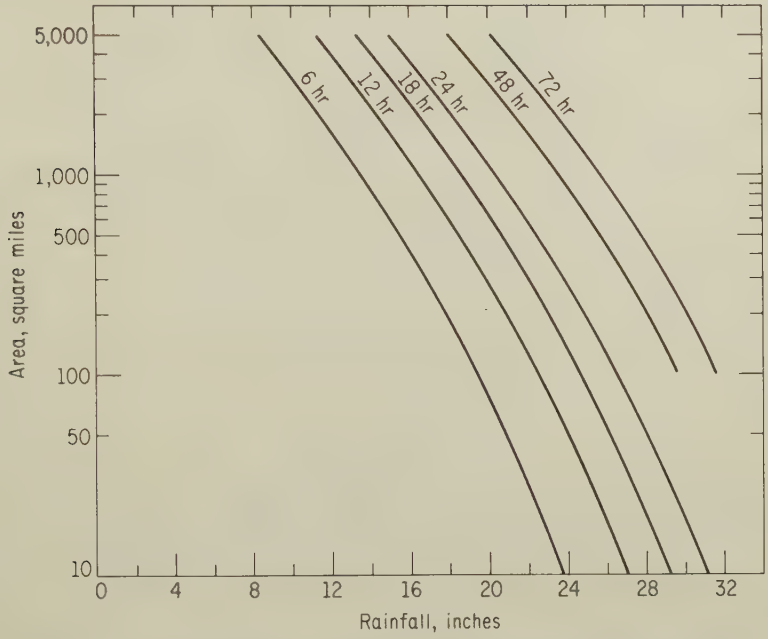


FIG. 9-4. Depth-duration-area curves for PMP at Knoxville. (After Ref. 3.)

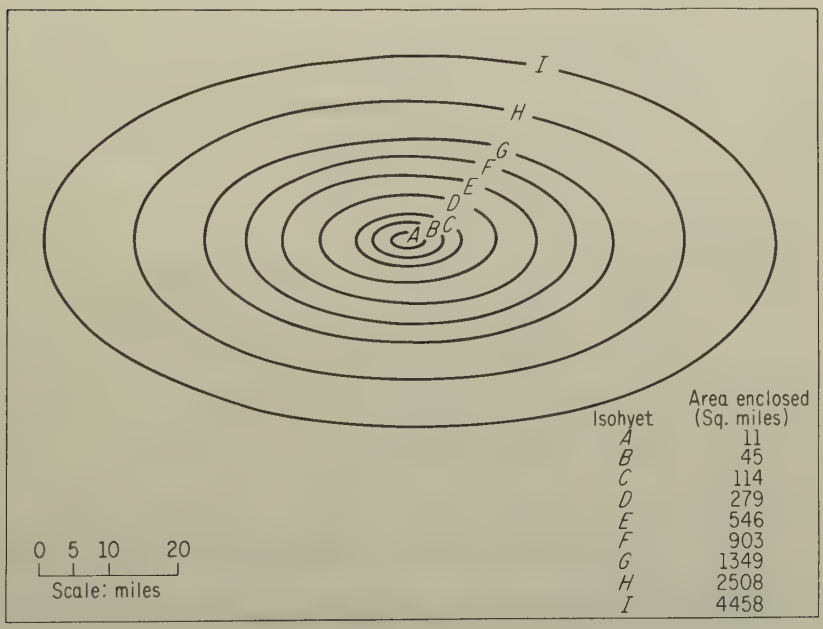


FIG. 9-5. Generalized pattern storm. (After Ref. 3.)

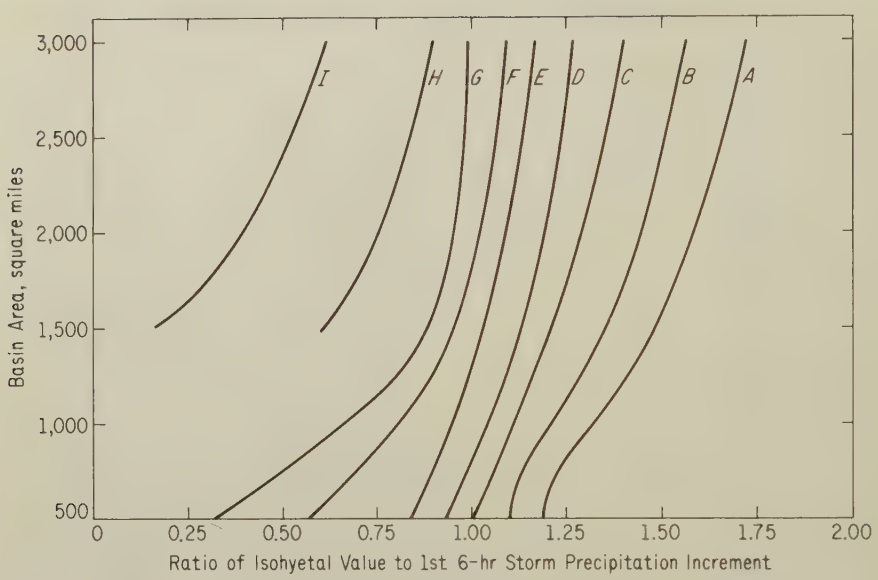


FIG. 9-6. Nomogram for isohyet values in pattern storms, 1st 6-hr increment. (After Ref. 3.)

ous storm increments is used in designing TVA projects.[3] The set of curves in Figs. 9-6 through 9-8 allows subdivision of the SPS into 6-hr increments of rainfall.

The following procedure is utilized to get a temporal change of the storm isohyets. The area contained by each summing of the product of successive isohyets, multiplied by the average value of the encompassing isohyets, and dividing the aggregate by the total watershed area results in the average rainfall for that time period within the storm.

1. Determine SPS values for 6-, 12-, 18-, 24-, 48-, and 72-hr rainfall depths for size of the study area.
2. Adjust these values by the ratio PMP_{ss}/PMP_{ip}, where

   PMP_{ss} = 2,000 sq mi PMP at the study site

   PMP_{ip} = 2,000 sq mi PMP at the index site
3. Reconstruct the adjusted depth-area-duration curve and tabulate successive 6-hr storm depth values by subtraction.
4. Place standard isohyetal pattern in position to maximize average storm value.
5. Obtain isohyetal values for 1st 6-hr storm duration by applying ratios of Fig. 9-6 to depth values of step 3; similarly for 2d and 3d 6-hr increments.
6. The remaining 6-hr periods are assumed to be uniform throughout the study area and equal to the values obtained in step 3.

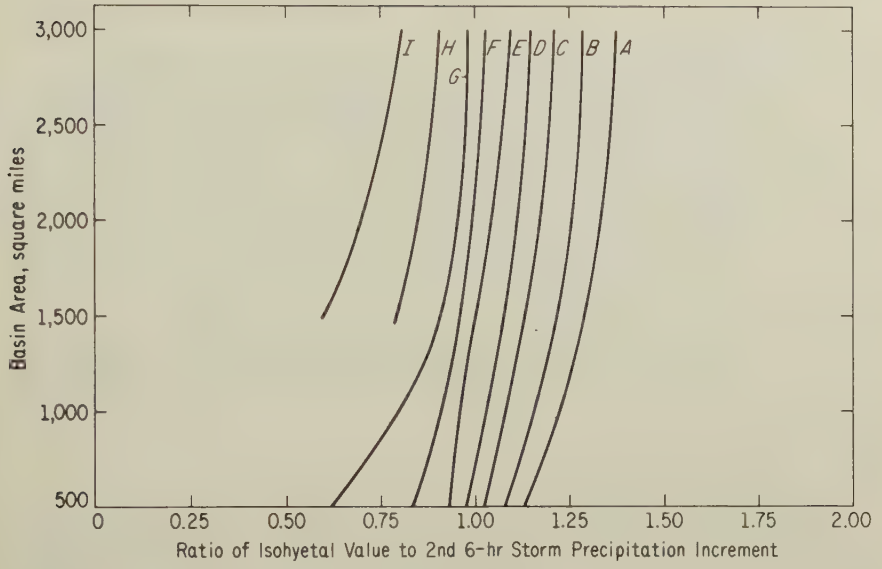


FIG. 9-7. Nomogram for isohyet values in pattern storms, 2d 6-hr increment. (After Ref. 3.)

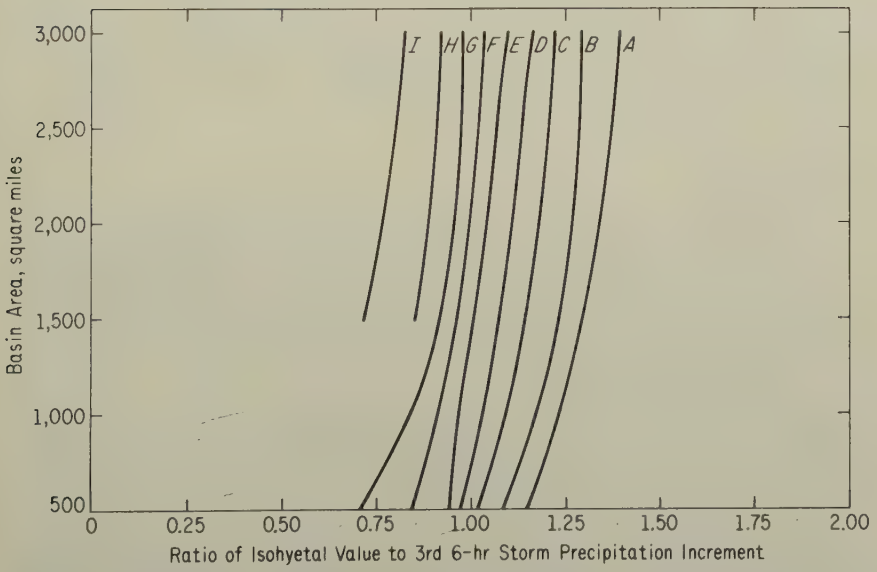


FIG. 9-8. Nomogram for isohyet values in pattern storms, 3d 6-hr increment. (After Ref. 3.)

7. Calculate average storm rainfall over the study area for each 6-hr time increment.
8. Arrange the 6-hr increments according to the following criteria.
   (a) Group the four heaviest 6-hr increments of the 72-hr storm in a 24-hr sequence, the middle four, and then the smallest four increments in 24-hr successions.
   (b) Arrange the three 24-hr sequences with the second highest 24-hr period next to the highest and the third at either end. Any possible combination of the three 24-hr periods is acceptable providing that the lighest 24-hr period is in the middle.
   (c) The 6-hr increments are to be arranged within a 24-hr period such that the highest two and the highest three 6-hr increments adjoin.

9-5 Allocating Storage

Most information and techniques presented thus far have been directed toward the flood protection aspect of small and large structures. Needless to say, a major structure is designed for more than just flood protection: they are multipurpose and may provide storage for irrigation, power, water supply, navigation, and low-flow augmentation. Proper allocation of storage to these various uses requires an understanding of the entire streamflow history in terms of frequency of occurrence of low flows, average monthly, seasonal, and yearly flows, as well as the PMP and SPF floods. Material in Chapter 10 is presented to provide a hydrologist with the tools to develop complete streamflow histories to achieve the design of Major Structures.

References

1. Snyder, F. F., "Hydrology of Spillway Design: Large Structures—Adequate Data," *A.S.C.E., J. Hydr. Div.*, Vol. 90, No. HY3 (May 1964), pp. 239–259.

2. "Floods Resulting from Suddenly Breached Dams," Misc. Paper 2-374 U.S. Army Corps of Engineering Waterways Experiment Station, Conditions of Minimum Resistance, Rept. No. 1, February 1960; Conditions of High Resistance, Report No. 2, November 1961.

3. "Probable Maximum TVA Precipitation for Tennessee River Basins up to 3,000 Square Miles in Area and Durations to 72 Hours," U.S. Dept. of Commerce, ESSA, Weather Bureau Hydometeorological Report No. 45, 1969.

4. Hershfield, D. M., "Estimating The Probable Maximum Precipitation," *A.S.C.E., J. Hydr. Div.*, Vol. 87, No. HY5 (1961), pp. 99–116.

5. "Two- to Ten-Day Precipitation for Return Periods of 2 to 100 Years in the Contiguous United States," U.S. Department of Commerce, Technical Paper No. 49, 1964.

Problems

9-1. Calculate the peak discharge for a dam failure in your area. State your assumptions relative to the breach.

9-2. Using data from NOAA publications calculate the 24-hr PMP for St. Louis, Missouri. Compare your calculations with Fig. 9-1.

9-3. Use Fig. 9-6 through 9-8 to distribute a 48-hr SPS storm for a 2,000-sq mi watershed near Little Rock, Arkansas.

10

HYDROLOGIC SIMULATION

The problems of decision making in both the design and operation of large scale systems of flood-control reservoirs, canals, aqueducts, and water-supply operations, have resulted in a need for mathematical approaches such as simulation to investigate the total project. This method is simply the mathematical description of a series of events which can be recreated for a selected time base. For example, calculating additional records of daily, monthly or seasonal streamflow based on the previous historical streamflow statistics; computing the discharge hydrograph resulting from a known or hypothetical storm; or generating additional years of rainfall records.

Simulation studies have commonly arisen from requirements for streamflow or rainfall input to reservoir studies. Many locations throughout the world are constantly reevaluating their water supply demands. Normally only very scanty or nonexistent records exist; thus simulation fills in missing records, extends others to a reasonable length, and estimates data for ungauged sites. Depending on the simulation model structure, a variety of data can be produced ranging from individual storm rainfall to average annual precipitation or mean yearly streamflow.

The general trend in simulation has resulted from applications of this technique to extending streamflow records at short-term stations. This simulated streamflow may be only for selected storm events or can, in certain cases provide a long-period time trace of simulated streamflow. The simulation approach has also been effectively used to generate hourly, daily, and yearly rainfall records. A primary disadvantage in proceeding by increments of days or shorter periods, however, lies in the large amount of computation time required for large-scale simulation problem. If streamflow or rainfall occur during each and every period, it is fre-

quently necessary to simulate each individual period of time, particularly during critical periods necessary in flood forecasting or reservoir routing problems.

Developing techniques used to extend records, either rainfall or streamflow, fit mainly under the method classified as "stochastic simulation." This procedure relies on the statistical properties of an existing record or regional estimates of these parameters.

Stochastic simulation can be further categorized as either "event" or "sequential" simulation. As examples, for intermittent streams, or hourly or daily rainfall, there are time periods when either the streamflow is zero or a storm did not occur. In such cases an "event' ' simulation is advantageous and employs a statistical description of the time between events, and another for the duration of the event based on fitting known statistical distributions to the properties of these two times. Actual calculation of the event occurs only during its time, and jumps to the next one by means of known statistical properties of the time between them.

Recently methods of modeling hydrologic behavior of a watershed have become popular. This technique is generally referred to as *deterministic simulation.* Basically this approach describes the behavior of the hydrologic cycle in terms of mathematical relationships outlining the interactions of various phases of the hydrologic cycle. Frequently the models are structured to simulate a streamflow value, hourly or daily, from given rainfalls within the watershed boundaries. The model is "verified" or "calibrated" by comparing results of the simulation with existing records. Once the model is adjusted to fit the known period of data, additional periods of streamflow can be generated. Numerous models exist for specific applications. A few will be examined in this text to indicate their structuring. For more complete treatment of individual models the student should refer to the cited references.

10-1 EPA Model—Urban Runoff

A continuing interest by the Environmental Protection Agency has resulted in recent models or deterministic simulation studies of the behavior of urban storm water. One phase of this investigation or submodel, has resulted in computer simulation of the overland flow phase and gutter routing of storm rainfall in an urban area.[1] This model is designed to simulate the runoff of a drainage basin for any predescribed rainfall pattern. The total watershed is broken into a finite number of smaller units or subcatchments which can be readily described by their hydraulic or geometric properties. A flow chart for the process is shown in Fig. 10-1.

Necessary inputs in the model are surface area, width of subcatchment, ground slope, Manning's roughness coefficient, infiltration rate, and de-

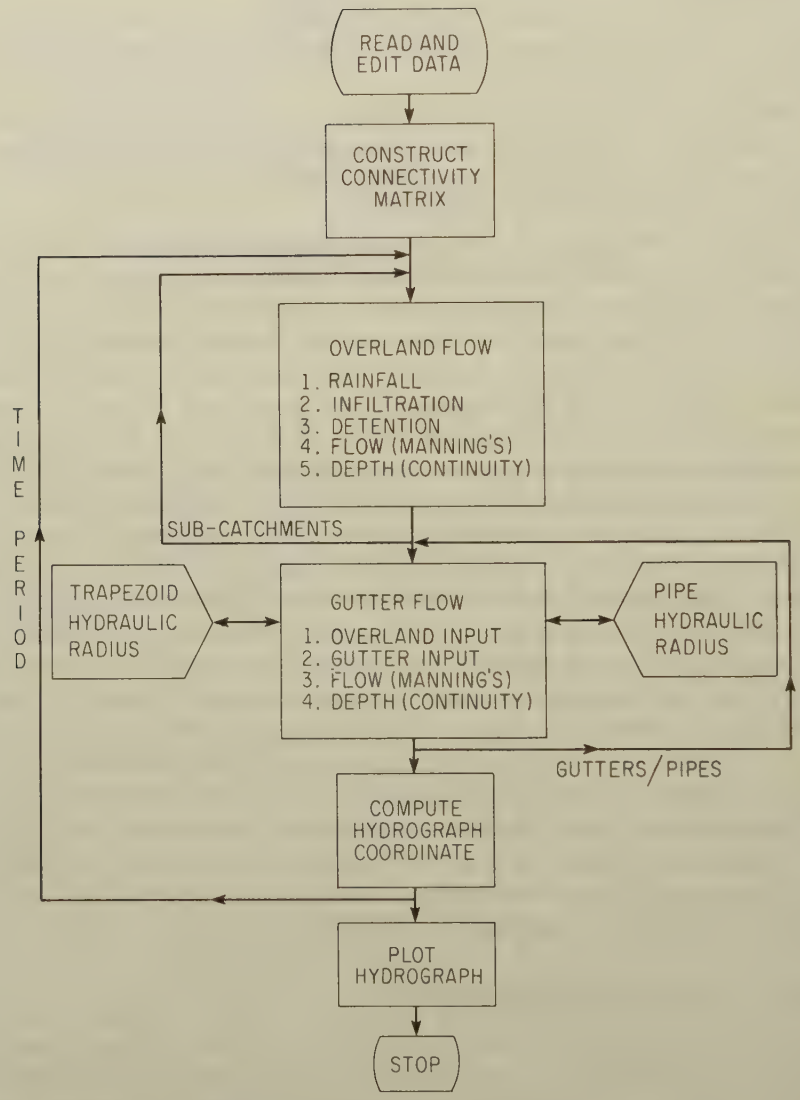


FIG. 10-1. Flow chart for hydrographic computation. (After Ref. 1.)

tention depth. Gutter descriptions are length, Manning's roughness coefficient, invert slope, diameter for pipes, and cross-sectional dimensions of the gutter. A step-by-step process accounts for all inflow, infiltration losses, and flow from upstream subcatchment areas, providing a calculated discharge hydrograph at the drainage basin outlet. The following description of the simulation process will aid in understanding the logic of the model.[1]

1. Rainfall is added to the subcatchment according to the specified hyetograph:

$$D_1 = D_t + R_t \Delta t \tag{10-1}$$

where D_1 is the water depth after rainfall, D_t the water depth of the subcatchment at time t, and R_t is the intensity of rainfall in time interval Δt.

2. Infiltration I_t is computed by Horton's exponential function, $I_t = f_0 + (f_1 - f_0)e^{-\alpha t}$, and subtracted from water depth existing on the subcatchment

$$D_2 = D_1 - I_t \Delta t \tag{10-2}$$

where f_0, f_1, and α are coefficients in Horton's equations and D_2 is the intermediate water depth after accounting for infiltration.

3. If the resulting water depth of subcatchment D_2 is larger than specified detention depth D_d, an outflow rate is computed using Manning's equation.

$$V = \frac{1.49}{n}(D_2 - D_d)^{2/3} S^{1/2} \tag{10-3}$$

and

$$Q_w = VW(D_2 - D_d) \tag{10-4}$$

where V is the velocity, n is Manning's coefficient, S is the ground slope, W the width, and Q_w the outflow rate.

4. The continuity equation is solved to determine water depth of the subcatchments resulting from rainfall, infiltration, and outflow. Thus

$$D_{t+\Delta t} = D_2 - \frac{Q_w}{A} \Delta t \tag{10-5}$$

where A is the surface area of the subcatchment.

5. Steps 1 to 4 are repeated until computations for all subcatchments are completed.

6. Inflow (Q_{in}) to a gutter is computed as a summation of outflow from tributary subcatchments ($Q_{w,i}$) and flow rate of immediate upstream gutters ($Q_{g,i}$)

$$Q_{\text{in}} = \Sigma Q_{w,i} + \Sigma Q_{g,i} \tag{10-6}$$

7. The inflow is added to raise the existing water depth of the gutter according to its geometry. Thus

$$Y_1 = Y_t + \frac{Q_{\text{in}}}{A_s} \Delta t \tag{10-7}$$

where Y_1 and Y_t are water depth of the gutter and A_s is the mean water surface area between Y_1 and Y_t.

8. Outflow is calculated for the gutter using Manning's equation:

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{10-8}$$

and

$$Q_g = VA_c \tag{10-9}$$

where R is the hydraulic radius, S_i the invert slope, and A_c the cross-sectional area at Y_1.

9. The continuity equation is solved to determine water depth of the gutter resulting from the inflow and outflow. Thus

$$Y_{t+\Delta t} = Y_1 + (Q_{\text{in}} - Q_g) \frac{\Delta t}{A_s} \tag{10-10}$$

10. Steps 6 to 9 are repeated until all gutters are finished.
11. Flows reaching the point concerned are added to produce a hydrograph coordinate along the time axis.
12. The processes from 1 to 11 are repeated in succeeding time periods until the complete hydrograph is computed.

The model is presently functioning to provide storm hydrographs for areas having lateral sewers up to 30 in. in diameter.

10-2 API Model

This model is structured to give a deterministic simulation of a continuous streamflow hydrograph. It has been tested on watersheds of 68 and 817 sq mi^2, but must be calibrated to each watershed to obtain a reliable method of simulating the streamflow. A flow diagram showing the structure is given in Fig. 10-2. Four basic components describe the interrelationships pertaining to this model of streamflow in a river: a unit hydrograph; an API rainfall-runoff connection; a relation for groundwater recession; and a relation for computing the groundwater flow hydrograph as a function of the direct runoff hydrograph. This model generates both groundwater flow and direct runoff discharge from precipitation values.

10-3 Stanford Watershed Model IV

This model has had more application in recent years than other deterministic ones, primarily because of its versatility in design. Figure 10-3 is a flow chart showing structure of the model. Again, this model should be calibrated to each watershed from existing records of rainfall and streamflow. The only input parameters required are rainfall and potential evapotranspiration in addition to physical descriptions of the watershed and hydraulic properties.

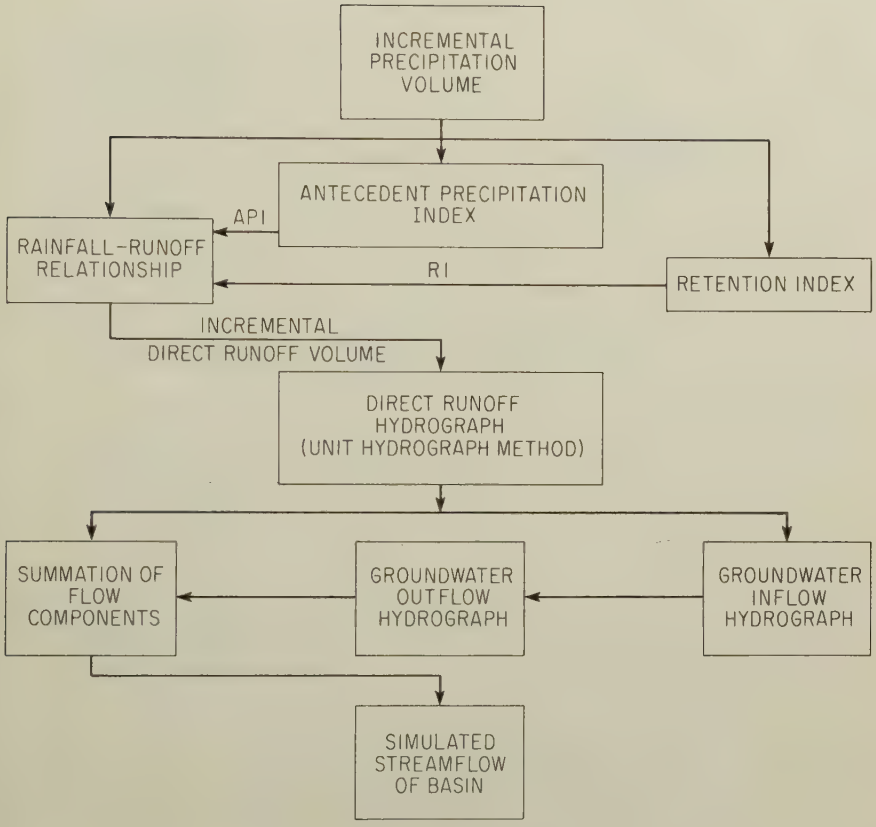


FIG. 10-2. Schematic diagram of *API* type hydrologic model. (After Ref. 2.)

The sequence of calculations diagrammed in Fig. 10-3 can be carried out for a number of watersheds or subwatersheds from given values of precipitation at any number of sites, thus modeling the hydrologic cycle. Use of three zones of moisture regulate soil-moisture profiles and groundwater conditions. The rapid response encountered in smaller watersheds is accounted for in the upper zone, while both upper and lower zones control such factors as overland flow, infiltration and groundwater storage. The lower zone is responsible for longer-term infiltration and groundwater storage which is later released as base flow to the stream. Total stream flow is a combination of overland flow, groundwater flow, and interflow.

Complete description of the model is beyond the scope of this text. Applications have been made to both urban and rural watersheds and results are dependent upon the care exercised in calibrating the model.

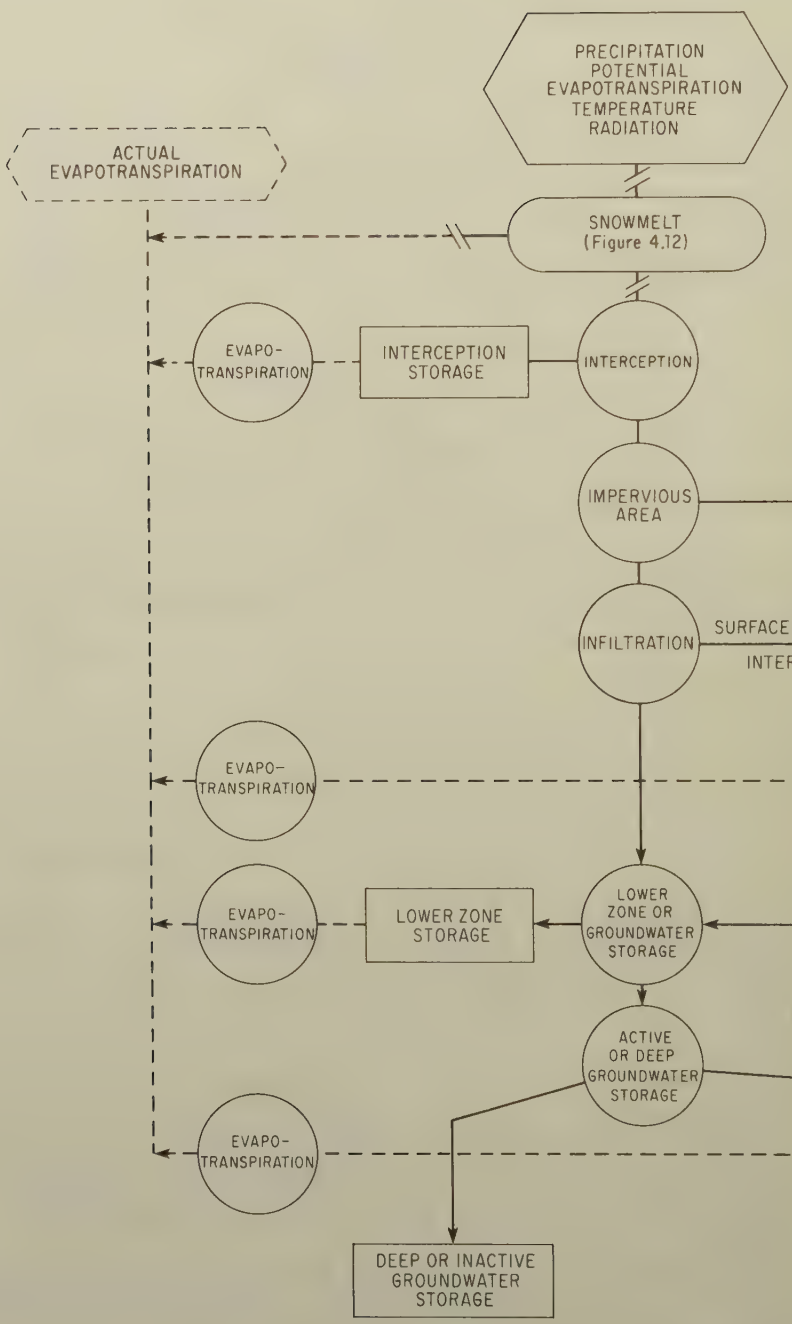
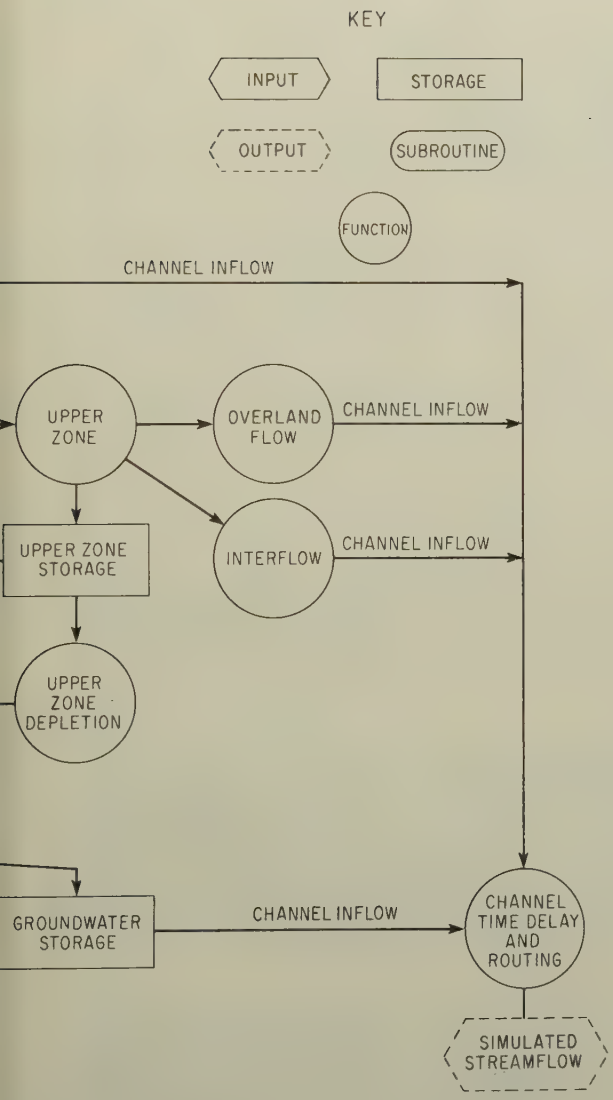


FIG. 10-3. Stanford watersh



odel IV flow chart. (After Ref. 3.)

10-4 Sequential Simulation

General formulation of a sequential simulation model is based on development of a recursion formula. One of the first types[4] of recursion formulas used to simulate or extend the period of record of monthly streamflow had the lag-one Markov chain form:

$$Q_{i+1} = \bar{Q}_{j+1} + b_j(Q_i - \bar{Q}_j) + e \tag{10-11}$$

where Q_{i+1} = generated streamflow in month $i + 1$
$\bar{Q}_{j+1}$ = mean of observed flows in month $j + 1$
Q_i = generated flow in month i
$\bar{Q}_j$ = mean of observed flows in month j
b_j = regression coefficient for relation of month $j + 1$ to month j
e = simulation error due to unexplained variance

Now since e represents the error in the relationship, rewriting Eq. 10-11,

$$Q_{i+1} = \bar{Q}_{j+1} + b_j(Q_i - \bar{Q}_j) + t\sigma_{j+1}(1 - r_j^2)^{1/2} \tag{10-12}$$

where t = a random number selected from a normal distribution having a zero mean and a unit variance
σ_{j+1} = standard deviation of observed flows for calendar month $j + 1$
r_j = correlation coefficients describing the relation of flows for month $j + 1$ to flows for month j

The flow model requires some adjustment to handle monthly or other seasonal simulations since serial correlations usually are stronger in these cases. The process is referred to as a *circular random walk* and requires a double indexing subscript. Thus Eq. 10-12 is rewritten as

$$Q_{i,j} = \bar{Q}_j + b_j(Q_{i-1,j-1} - \bar{Q}_{j-1}) + t_i\sigma_j(1 - r_j^2)^{1/2} \tag{10-12a}$$

where j is the number of seasonal periods in the year and the other terms have been defined except for $b_j = r_j(\sigma_{j+1}/\sigma_j)$ which must be employed since $\bar{Q}_{j+1} \neq \bar{Q}_j$. This model has been used extensively in streamflow analysis although the generation procedure results in random occurrence of some negative flows. However, it makes little difference what is done to them; they can be set to zero or generated again.

Example 10-1 Table 10-1 lists the parameters describing quarterly flows in million gallons per day for the Patapsco River. Applying the information in conjunction with a table of random numbers create a 2-yr record of quarterly flows. The solution is tabulated in Table 10-2. A flow chart for the process is sketched in Fig. 10-4.

TABLE 10-1 Quarterly Streamflow Parameters, Patapsco River

j	b_j	σ_j	r_j	$(1 - r_j^2)^{1/2}$	$\bar{Q}_j$
1	.66	73	.57	.82	68
2	.43	85	.41	.91	137
3	.14	90	.43	.90	183
4	.91	29	.36	.93	107

TABLE 10-2 Tabular Generation of Synthetic Quarterly Streamflows

i (1)	j (2)	$\bar{Q}_j$ (3)	$\bar{Q}_{j-1}$ (4)	$Q_{i-1,j-1}$ (5)	k (6)	t_i (7)	$b_j(Q_{i-1,j-1} - \bar{Q}_{j-1})$ (8)	$t_i\sigma_j(1 - r_j^2)^{1/2}$ (9)	$Q_{i,j}$ (10) [(3) + (8) + (9)]
1	1	68	107	107	0.5374	.094	0.0	5.6	73.6
2	2	137	68	73.6	0.6338	.342	2.4	26.5	165.9
3	3	183	137	165.9	0.3530	−.377	4.0	−30.6	156.4
4	4	107	183	156.4	0.6343	.343	−24.2	9.3	92.1
5	1	68	107	92.1	0.0263	−1.94	−9.8	−116.4	0.0
6	2	137	68	0.0	0.6455	.373	−29.2	28.9	136.7
7	3	183	137	136.7	0.8507	1.04	−0.04	84.5	267.5
8	4	107	183	267.5	0.3485	.39	76.8	10.6	194.5

Col. 1. Period index, $i = 1$ to 8 quarters for a 2-yr run.
Col. 2. Quarterly index, $j = 1$ to 4, two cycles.
Col. 3. Mean quarterly flow.
Col. 4. Preceding mean quarterly flow.
Col. 5. The generated flow for the preceding quarter. Note first entry is same as col. 4, i.e., generation begins using the mean flow from the preceding quarter.
Col. 6. Uniform random digits from a random number table. Could be produced simply by drawing randomly from a card deck, with replacement.
Col. 7. Random normal deviates from Appendix C, Table I. If col. 6 is less than 0.5, find z for $(0.5 - k)$ and set $t = -z$; if $k > 0.5$, find z for $(k - 0.5)$ and set $t = z$.
Col. 8. Regression component of Eq. 10-12a.
Col. 9. Random component of Eq. 10-12a.
Col. 10. Generated flow.

Normal Distributions

The example model is applicable to simulating flows at a single site where the population of flows is normally distributed for each calendar month. The equation as stated preserves the mean, the variance, and the correlation between successive flows of the historic monthly streamflow sequence. Simulation procedure begins by assuming an initial value of Q_i, computing Q_{i+1}, and repeating by replacing Q_i by Q_{i+1}. In extending a record the initial assumption of Q_i is the last recorded monthly streamflow.

Log-Normal Distributions

Monthly flows frequently do not appear to be from a normally distributed population. Experience has shown, however, that annual flows are generally normal and monthly flows are log normal or Pearson III.

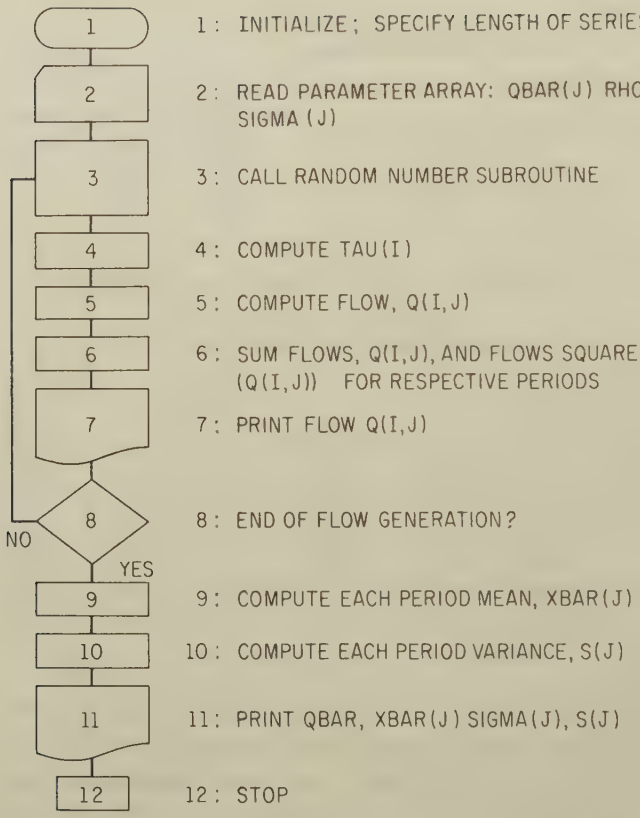


FIG. 10-4. Flow chart for computing synthetic streamflow.

A recursion equation for log-normal simulation presented by Matalas[5] preserves the mean, standard deviation, skewness and lag-one correlation coefficient. Using $Y = \log (Q - a)$, the recursion Eq. 10-12 appears as

$$(Y_{i+1} - \mu_y) = r(Y_i - \mu_y) + t\sigma_y(1 - r_y^2)^{1/2} \qquad (10\text{-}13)$$

where Y_{i+1} = generated log-normal flow in month $i + 1$
μ_y = mean of the log-normal flow in month
r = regression coefficient of the month $i + 1$ on month i
σ_y = standard deviation of observed flows in month i
t = normally distributed random number with zero mean and unit variance.

Values for a, μ_y, σ_y, and r_y must be found in the following manner to insure preserving a synthetic sequence that will resemble the historic sequence in terms of μ_x, σ_x, γ_x, and r_x. Relationships for μ_x, σ_x^2, γ_x, and

r_x are given as

$$\mu_x = a + \exp(1/2\sigma_y^2 + \mu_y) \tag{10-14}$$

$$\sigma_x^2 = \exp(2(\sigma_y^2 + \mu_y)) - \exp(\sigma_y^2 + 2\mu_y) \tag{10-15}$$

$$r_x = (\exp(\sigma_y^2 r_y) - 1)/(\exp(\sigma_y^2) - 1) \tag{10-16}$$

$$\gamma_x = \frac{\exp(3\sigma_y^2) - 3\exp(\sigma_y^2) + 2}{(\exp(\sigma_y^2) - 1)^{3/2}} \tag{10-17}$$

Since values of μ_x, σ_x, γ_x, r_x are known, Eqs. 10-14 through 10-17 can be solved for μ_y, σ_y, r_y and a. Equation 10-13 may then be utilized to step forward in monthly increments finding values of Y_{i+1}. To get the synthetic sequence of streamflow the value of a is added to the antilog of each value of Y_{i+1}.

Pearson III Distributions

The Pearson III distribution for simulating daily streamflows in reservoir studies has been advocated by Beard,[6] and is recommended when simulating daily flows for critical flood months. The reason for this is obvious—there would be an immense number of calculations to simulate daily flows for any lengthy period. The basic recursion formula is

$$Q_{i+2} = b_1 Q_{i+1} + b_2 Q_i + X_r(1 - r^2) \tag{10-18}$$

where Q_{i+2}, Q_{i+1}, Q_i = normal standardized variates for successive days
X_r = random standardized variate
b_1, b_2 = regression coefficients
r = correlation coefficient for regression equation

This simulation model is a second-order Markov chain. Normal standardized variates for successive days are found by classifying streamflows into months, determining the logarithmic monthly mean, standard deviation, and skew coefficient.* Logarithmic daily flows are next standardized by subtracting the mean monthly logarithm from each daily value and dividing by the monthly standard deviation. This standardized variate is transformed to normal by the Pearson type III approximation.

$$t = \frac{6}{g}\left[\left(\frac{g}{2}k + 1\right)^{1/3} - 1\right] + \frac{g}{6} \tag{10-19}$$

where t = normal standard deviate
k = Pearson Type III standard deviate
g = skew coefficient

*In order to avoid zero flows, 0.01 times the monthly mean is added to each daily flow before taking the logarithm.

A typical simulation with this technique requires calculation of the following statistics from existing streamflow record prior to simulation.

1. Logarithmic mean, serial-correlation coefficient, and skew coefficient of daily flows for each month of the record period.
2. Correlation coefficients between logarithmic flows of successive days.
3. A regression coefficient of the standard deviation for the daily-flow logarithms within each month of record to the logarithm of the monthly total flow.

Given the calculated statistics, noted simulation of daily flows could be structured in the following manner:

1. Generate standardized variates from Eq. 10-18.
2. Use logarithm of the monthly mean flow as an initial estimate of the mean of the logarithmic daily flows for the month.
3. Calculate the standard deviation of flow logarithms by previously determined regression equation.
4. Apply the inverse of Eq. 10-19.

$$k = \frac{2}{g}\left[\frac{g}{6}\left(t - \frac{g}{6}\right) + 1\right]^3 - \frac{2}{g}$$

   to transform the standardized variates to flows, multiplying by the appropriate standard deviation and adding the mean.
5. Add differences in the total monthly flow generated and given monthly flow to the given monthly flow and repeat simulation.
6. Multiply daily results of the second pass by the ratio of given monthly total to generated monthly total to the generated monthly total.

Each simulation technique commonly requires some modifications when applied to individual problems. Methods outlined thus far can be utilized as guides in establishing a procedure to follow in constructing a sequential simulation program. Only simulation of flows for a given station have been presented with serial correlation, skewness, means, and standard deviations maintained. When generating streamflow sequences for regional areas, preservation of cross-correlation—that is, between stations, becomes a significant factor.

10-5 Regional Simulation

Regression Approach

Problems in planning habitually require simulation of streamflow or rainfall at ungauged sites. This introduces some difficulty since most simulation techniques rely heavily upon some length of record at the site

to secure a representative sample and basic statistics necessary for the simulation procedure. A regression technique[7] can be used to generate these statistics from regional data. The process resorts to a multiple regression relating monthly or annual average flows to the physiographic and climatological factors of the region. A typical regression for streamflow synthesis could take the form

$$\ln Y = \ln a + b_1 \ln A + b_2 \ln S + b_3 \ln S_t + b_4 \ln P + b_5 \ln S_n + b_6 \ln F \qquad (10\text{-}20)$$

where Y = monthly or annual mean or standard deviation
A = drainage area, sq mi
S = slope of main channel, ft/mi
S_t = surface storage, percent
P = mean annual precipitation, in.
S_n = annual snowfall, in.
F = forested area, percent

Additional regional statistics for simulating rainfall, can also be developed through a multiple-regression approach taking the following form for average monthly or annual precipitation.

$$P = b_0 + b_1 X + b_2 X^2 + b_3 XY + b_4 XZ + b_5 Y + b_6 Y^2 + b_7 YZ + b_8 Z + b_9 Z^2 + b_{10} X^3 + b_{11} X^2 Y + b_{12} X^2 Z + b_{13} XY^2 + b_{14} XZ^2 + b_{15} Y^3 + b_{16} XYZ + b_{17} Y^2 Z + b_{18} YZ^2 + b_{19} Z^3 \qquad (10\text{-}21)$$

where X = measure of latitude, degrees
Y = measure of longitude, degrees
Z = measure of elevation, ft
P = monthly or annual precipitation, in.

Choosing a regression approach to establish a regional value for the standard deviation of the monthly or annual precipitation generally possesses more variability. One way to overcome this problem is to add another independent variable to Eq. 10-21. Addition of a term for the mean will generally increase the reliability of the prediction equation which is similar to Eq. 10-21.

$$S_p = b_0 + b_1 X + \cdots + b_{20} P \qquad (10\text{-}22)$$

where S_p = standard deviation of monthly or annual precipitation
P = mean monthly or annual precipitation

Care should be taken in this type of regional approach, since skewness of the values has not been preserved. The intial data probably can be transformed to eliminate or reduce it to a negligible amount. Typical transformations are logarithms, square roots, cube roots, and/or simply normalizing the data.

Multivariate Generating

The regression approach does not explicitly retain the serial and cross-correlations of the synthetic streamflow or rainfall sequence. Its primary advantage is in generating data for ungauged sites while preserving regional characteristics. A method to maintain serial and cross-correlation has been developed[5] to produce additional records at recording sites. The technique presented applies only to existing sites. One problem of the following method is the necessity of having equal-length records at every site. Initial data must be augmented in some way to make the base record length the same for all stations.

A general scheme for augmenting data, whether precipitation or streamflow, can take the form:[8]

$$\ln P_{2i} = a \ln P_{1i-1} + b \ln P_{2i-1} + c \ln P_{1i} + d\epsilon_i \qquad (10\text{-}23)$$

where P = standardized value of daily, monthly, or yearly rainfall or streamflow at sites 1 and 2

a, b, c = least-squares regression coefficients obtained from existing station data

d = unexplained variance

ϵ = normally distributed random number

This technique does preserve the serial and cross-correlation between stations. Once all the data have been augmented to a common period, the multivariate generating scheme will synthetically generate additional years of record.

For a typical region under consideration, all stations are double subscripted to designate both the time period involved and the station index number. For example, the notation X_{i+1}^{P} is the $i+1$ time period for station P where $P = 1, \ldots, m$ represents the index number associated with the m stations in the region.

Although various methods are available for creating multivariate sequences, one of the most useful is the lag-one scheme. This system has been proposed in the literature by Matalas[5] and retains means, standard deviation, lag-one serial correlations and lag-zero cross correlations. For example, $X^{(p)}$, $p = 1, \ldots, m$ denotes the rainfall in the pth station in basin, a matrix representation of the lag-one:

$$X_{i+1} = AX_i + B\epsilon_{i+1} \qquad (10\text{-}24)$$

where

$$\begin{bmatrix} X_{i+1}^{(1)} - \mu_x^{(1)} \\ X_{i+1}^{(2)} - \mu_x^{(2)} \\ \vdots \\ X_{i+1}^{(p)} - \mu_x^{(p)} \end{bmatrix} = A \begin{bmatrix} X_i^{(1)} - \mu_x^{(1)} \\ X_i^{(2)} - \mu_x^{(2)} \\ \vdots \\ X_i^{(p)} - \mu_x^{(p)} \end{bmatrix} + B\epsilon_{i+1}$$

The expected value of ϵ_{i+1} equals zero. Thus by postmultiplying Eq. 10-24 by X_i^T, the expected value of the matrix products is

$$M_1 = AM_0 \tag{10-25}$$

where

$$M_0 = E[X_i X_i^T]$$

$$M_1 = E[X_{i+1} X_i^T]$$

M_0 is now a $m \times m$ matrix whose diagonal elements are the variances $\hat{\sigma}_x^{2(p)}$, $p = 1, \ldots, m$ and whose off-diagonal elements are covariances $\hat{\rho}_x^{(p)(q)}(0)\hat{\sigma}_x^{(p)}\hat{\sigma}_x^{(q)}$, $p, q = 1, \ldots, m$ where $p \neq q$. M_1 is an $(m \times m)$ matrix having diagonal elements $\hat{\rho}_x^{(p)}(1)\sigma_x^{2(p)}$, $p = 1, \ldots, m$ and off-diagonal elements* $\hat{\rho}_x^{(p)(q)}(1)\sigma_x^{(p)}\sigma_x^{(q)}$. Then postmultiply Eq. 10-24 by X_{i+1}^T,

$$M_0 = AM_1^T + BB^T \tag{10-26}$$

and

$$A = M_1 M_0^{-1}$$

$$BB^T = M_0 - M_1 M_0^{-1} M_1^T$$

Setting

$$BB^T = C \tag{10-27}$$

where BB^T is an $m \times m$ diagonal matrix whose elements are the eigenvalues of $M_0 - M_1 M_0^{-1} M_1^T$. A rapid procedure to find the elements of matrix B is offered. Since

$$BB^T = C$$

$$[b_{ij}][b_{ij}^1] = [C_{ij}]$$

with

$$C_{ij} = \sum_{k=1}^{n} b_{ij} b_{kj}^1$$

but

$$b_{kj}^1 = b_{jk}$$

$$\therefore C_{ij} = \sum_{k=1}^{n} b_{ij} b_{jk}$$

and

$$C_{ij} = C_{ji}$$

Assume B a lower triangular matrix:

$$C_{11} = \sum_{k=1}^{n} b_{1k} b_{1k} = b_{11}^2$$

$$\therefore b_{11} = \sqrt{C_{11}}$$

*$\hat{\rho}_x^{(p)(q)}(1)$ is a lag-one cross correlation.

and

$$C_{12} = \sum_{k=1}^{n} b_{1k} b_{2k}$$

$$C_{12} = b_{11} b_{21}$$

$$\therefore b_{21} = \frac{C_{12}}{\sqrt{C_{11}}}$$

$$C_{11} = b_{11}^2 \quad \text{and} \quad b_{11} = \sqrt{C_{11}}$$

$$C_{12} = b_{11} b_{21} \quad \text{and} \quad b_{21} = \frac{C_{12}}{b_{11}}$$

$$\vdots \qquad \vdots$$

$$b_{n1} \frac{C_{n1}}{b_{11}}$$

Likewise

$$C_{22} = b_{21} b_{21} + b_{22} b_{22} \quad \text{and} \quad b_{22} = \sqrt{C_{22} - b_{21}^2}$$

$$C_{23} = b_{21} b_{31} + b_{22} b_{32} \quad \text{and} \quad b_{32} = \frac{C_{23} - b_{21} b_{31}}{b_{22}}$$

$$\vdots$$

$$b_{n2} = \frac{C_{2n} - b_{21} b_{n1}}{b_{22}}$$

and

$$C_{33} = b_{31} b_{31} + b_{32} b_{32} + b_{33} b_{33} \quad \text{and} \quad b_{33} = \sqrt{C_{33} - b_{31}^2 - b_{32}^2}$$

$$C_{34} = b_{31} b_{41} + b_{32} b_{42} + b_{33} b_{43} \quad \text{and} \quad b_{43} = \frac{C_{34} - b_{31} b_{41} - b_{32} b_{42}}{b_{33}}$$

$$\vdots$$

$$b_{n3} = \frac{C_{3n} - b_{31} b_{n1} - b_{32} b_{n2}}{b_{33}}$$

Omitting Lag-One Cross-Correlation

If lag-one cross-correlation is not needed in the simulation, the following process will work. Matrix A can be taken as a diagonal matrix whose diagonal elements are the lag-one serial-correlation coefficients. Thus the (p,p)th element of A is $\hat{\rho}_x^{(p)}(1)$. The pth and qth element of X_{i+1} are

$$X_{i+1}^{(p)} = \rho_x^{(p)}(1) X_i^{(p)} + \sum_{s=1}^{m} b_{p,s} \epsilon_{i+1}^{(s)} \qquad (10\text{-}28)$$

$$X_{i+1}^{(q)} = \rho_x^{(q)}(1)X_i^{(q)} + \sum_{s=1}^{m} b_{q,s}\epsilon_{i+1}^{(s)} \tag{10-29}$$

with $b_{p,s}$ and $b_{q,s}$ the (p,s)th and (q,s)th elements of B.

Multiply both sides of Eq. 10-28 by $X_i^{(q)}$ and take expectations, giving

$$\bar{\rho}_x^{(p)(q)}(1) = \hat{\rho}_x^{(p)(q)}(0)\hat{\rho}_x^{(p)}(1) \tag{10-30}$$

where $\bar{\rho}_x^{(p)(q)}(1)$ is the lag-one cross-correlation of $X_{i+1}^{(p)}$ and $X_{i+1}^{(q)}$ in Eqs. 10-28 and 10-29. Then if $\hat{\rho}_x^{(p)(q)}(1)$ is replaced by $\bar{\rho}_x^{(p)(q)}(1)$ in matrix M_1, the synthetic sequence preserves the mean, standard deviation lag-one and lag-zero cross-correlations, but not the lag-one cross-correlation. Equation 10-30 can be employed to calculate the lag-one cross-correlation for insertion in matrix M_1.

10-6 Event Simulation

Sequential simulation described in Secs. 10-4 and 10-5 consists of step-by-step processes through the entire time period. Simulation of monthly or yearly streamflow or rainfall events is satisfactorily accomplished through this type of operation. However, for many hydrologic events which occur in shorter time intervals, the shear amount of computation prohibits sequential generation of these data for large-scale simulations. For example, calculation of simulated rainfall depths hour by hour requires 8,760 hourly values for each year of record. To achieve efficiency in computation, including only the events, such as storm depths, reduces the length of time to do the simulation significantly. Such a technique is called "event simulation." The process simply resorts to statistical properties of *the time between events*, *their duration*, and some relationship between magnitude and duration.

The simulation process described is thus the generation of a random number having the statistical properties of the time between events, and another random number possessing the same attributes for its duration. A derived relationship of the magnitude of the event dependent upon its duration then allows a simulation of a series of events over a specified period of time. Hence calculation of each storm rainfall depth is undertaken only when a storm event occurs.

Inverse Transformations

Generation of a random number T_n having known statistical properties can be achieved by an inverse transformation. A uniform random number uniformly distributed between 0 and 1 provides the desired distribution. The transformation is customarily stated as "given a particular realization R_n of random variable X having a uniform probability density function on the interval 0–1, compute a corresponding number T_n which appears to be a realization of the random variable Y that has a specified

probability density function $g(T_n)$." Thus if the inverse function exists, $T_n = G^{-1}(R_n)$.

Inverse Exponential Transformation

Adopting an exponential distribution is one common approximation for time between events. It can also be employed for representing the duration. The desired probability density function of an exponential distribution is

$$g(y) = \lambda e^{-\lambda y} \qquad \begin{matrix} y \geq 0 \\ \lambda \geq 0 \end{matrix} \tag{10-31}$$

Consequently, since $G(T_n) = R_n$

$$G(T_n) = \int_0^{T_n} \lambda e^{-\lambda y}\, dy = -e^{-\lambda T_n} + 1 \tag{10-32}$$

or

$$R_n = 1 - e^{-\lambda T_n} \tag{10-33}$$

and solving for the desired transformation,

$$T_n = -\frac{1}{\lambda} \ln (1 - R_n) \tag{10-34}$$

The value $1/\lambda$ is equal to the mean μ of the set of data. Hence, by selecting a uniform random number R_n, a random number T_n can be produced having an exponential distribution with a mean of μ.

Inverse Log-Normal Transformation

Log-normal distributions for describing durations of an event are fairly common in simulating records of rainfall. This transformation is readily obtained by recourse to a standard normal variate which is a random number having a mean of zero and a standard deviation of one. Then to generate a variable T_n having a log-normal distribution with a mean of μ and a variance σ^2, the variable $\ln (T_n)$ will have a mean and variance of

$$\mu_y = \text{mean of } \ln (T_n) = \ln \mu - \frac{\sigma_y^2}{2} \tag{10-35}$$

$$\sigma_y^2 = \text{variance of } \ln (T_n) = \ln\left(\frac{\sigma^2}{\mu^2} + 1\right) \tag{10-36}$$

Now, forming a standard normal variate,

$$N_s = \frac{\ln (T_n) - \mu_y}{\sigma_y} \tag{10-37}$$

and solving for T_n gives

$$T_n = \exp (N_s \cdot \sigma_y + \mu_y) \tag{10-38}$$

Utilizing a standard normal variate random number with a zero mean and unit variance, a log-normal distribution can be computed possessing

a mean μ and variance σ^2. A similar transformation was used in Eq. 10-19 to find the Pearson III distribution discussed in Sec. 10-4. Applying event simulation to simulate a record of rainfall is a rapid means to generate monthly, seasonal or annual values of rainfall. A sample of this procedure is given in Example 10-2.

10-7 Validating Models

Care must be taken at all stages of construction of a simulation model to insure that the statistics of the historical records are preserved and the distributions patterning the event are in fact mathematically sound. Numerous tests exist for checking these statistics. Complete structuring of a simulation model should always include a study of the validity of assumptions made in forming it.

Example 10-2 An existing record of rainfall has been analyzed for the statistical properties of the mean time from storm to storm t_B in days; mean duration of the storm t_E in days; standard deviation of duration σ_E; and the regression equation for storm depth versus duration of the storm $D = at_E^b$. These values have been catagorized and tabulated by month for years of record.

Month	1	2	3	4	5	6
t_B	4.17	5.60	4.67	6.13	4.38	6.63
t_E	1.92	3.10	1.83	2.50	3.06	5.37
σ_E	.95	4.11	.83	1.59	2.22	1.87
a	.0703	.1530	.0850	.1370	.0708	1.860
b	1.595	1.183	1.617	1.380	2.130	.0340
Month	7	8	9	10	11	12
t_B	12.57	7.17	6.75	7.38	7.55	13.87
t_E	11.00	3.67	4.75	3.00	5.36	4.38
σ_E	9.02	2.80	1.48	2.00	7.46	4.21
a	.3830	.0706	.0100	.1570	.2690	.0317
b	.897	1.710	3.438	1.354	1.121	2.019

A simulation model structured as listed can be accomplished readily in most computer simulation languages.

1. Select R_n and insert in Eq. 10-33 to determine time from storm to storm.
2. Choose R_n and use Eq. 10-37 to get duration of storm.
3. Compute $D = at_E^b$ to find depth of storm.
4. Repeat process keeping track of month, seasons, and years. Simulation for a 30-yr record results in an average annual rainfall of 70.8 in. at the station.

References

1. Chen, W. W., and R. D. Shubinski, "Computer Simulation of Urban Storm Water Runoff," *Proc. A.S.C.E., J. Hydro. Div.*, Vol. 97, No. HY2 (February 1971), pp. 289–301.

2. Sittner, W. T., C. E. Schauss, and J. C. Monro, "Continuous Hydrograph Synthesis with an API-Type Hydrologic Model," *Water Resources Research*, Vol. 5, No. 5 (1969), pp. 1007–1022.

3. Crawford, N. H., and R. K. Linsley, Jr., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Department of Civil Engineering, Technical Report No. 39, July 1966.

4. Thomas, H. A., and M. B. Fiering, "Mathematical Synthesis of Streamflow Sequences for the Analysis of River Basins by Simulation," in A. Maass, et al, *Design of Water Resources Systems*. Cambridge, Mass.: Harvard University Press, 1962.

5. Matalas, N. C., "Mathematical Assessment of Synthetic Hydrology," *Water Resources Research*, Vol. 3, No. 4 (1967), pp. 937–945.

6. Beard, L. R., "Simulation of Daily Streamflow, "International Hydrology Symposium, Fort Collins, Colo., September 1967.

7. Benson, M. A., and N. C. Matalas, "Synthetic Hydrology Based on Regional Statistical Parameters," *Water Resources Research*, Vol. 3, No. 4 (1967), pp. 931–936.

8. Young, G. K., G. T. Orlob, and L. A. Roesner, "Decision Criteria for Using Stochastic Hydrology," *Proc. A.S.C.E., J. Hydr. Div.*, Vol. 96, No. HY4 (April 1970), pp. 911–926.

Problems

10-1. Review a current article in hydrologic simulation and identify the type or combination of types of simulation employed. Discuss the advantages and disadvantages.

10-2. Make a flow chart for a simulation approach for overland flow and gutter routing (FWQA Model) using hydraulic routing equations for (a) explicit method, (b) characteristics method.

10-3. Develop a flow chart for simulating routing of an API mode. through a system of reservoirs (a) three reservoirs in a series, (b) two reservoirs in series parallel to one reservoir.

10-4. Select a gauged stream in your geographic location and prepare a quarterly streamflow simulation model using (a) normal distribution, (b) log-normal distribution, (c) Pearson III distribution.

10-5. Can you convert the simulation problem in Example 10-1 to a log-normal distribution simulation? What difficulties are encountered with the data given in the example?

10-6. Using NOAA raingage station data, perform a regional regression describing the average annual rainfall as a function of latitude, longitude, and elevation.

10-7. Select a month of thunderstorm activity in your region. From published NOAA hourly rainfall data, fit a distribution to the time between storms, and duration of storms, for 20 yr of recorded data covering the selected month. Flow diagram an event type simulation model and prepare a computer program to simulate the timing of storm and nonstorm events for this month.

11

SNOW HYDROLOGY

11-1 Introduction

In many areas snow is the dominant source of streamflow. Mountainous areas in the west are prime examples. Goodell has indicated that about 90 percent of the yearly water supply in the high elevations of the Colorado Rockies is derived from snowfall.[2] Equally high proportions are also likely in the Sierras of California and numerous regions in the Northwest. A significant but lesser share of the annual water yield in the Northeast and Lake States also originates in snow. It is important that the hydrologist understand the nature and distribution of snowfall and mechanisms involved in the snowmelt process if he is to make adequate estimates of streamflow derived from this source.

Snowmelt usually begins in the spring. The runoff derived is normally out of phase with periods of greatest water need, therefore various control schemes such as storage reservoirs have been developed to minimize this problem. An additional point of significance is that some of the greatest floods result from combined large-scale rainstorms and snowmelt. Streamflow forecasting is highly dependent upon adequate knowledge of the extent and characteristics of snow fields within the watershed. Water yield from snowfall can be increased by minimizing the vaporization of snow and melt water. Timing the yield can be managed within limits by controlling the rate of snowmelt. Early results can be obtained by speeding the melt process whereas the snowmelt period can be extended or delayed by retarding it. Annual snowfall distribution in the United States is shown in Fig. 11-1.

An adequate understanding of meteorological factors is as much a prerequisite in considering the snowmelt process as it is in dealing with evapotranspiration. The atmosphere supplies moisture for both snowfall and condensation of water vapor on the snowpack, regulates the exchange of energy within a watershed, and is a controlling factor in snowmelt rates.

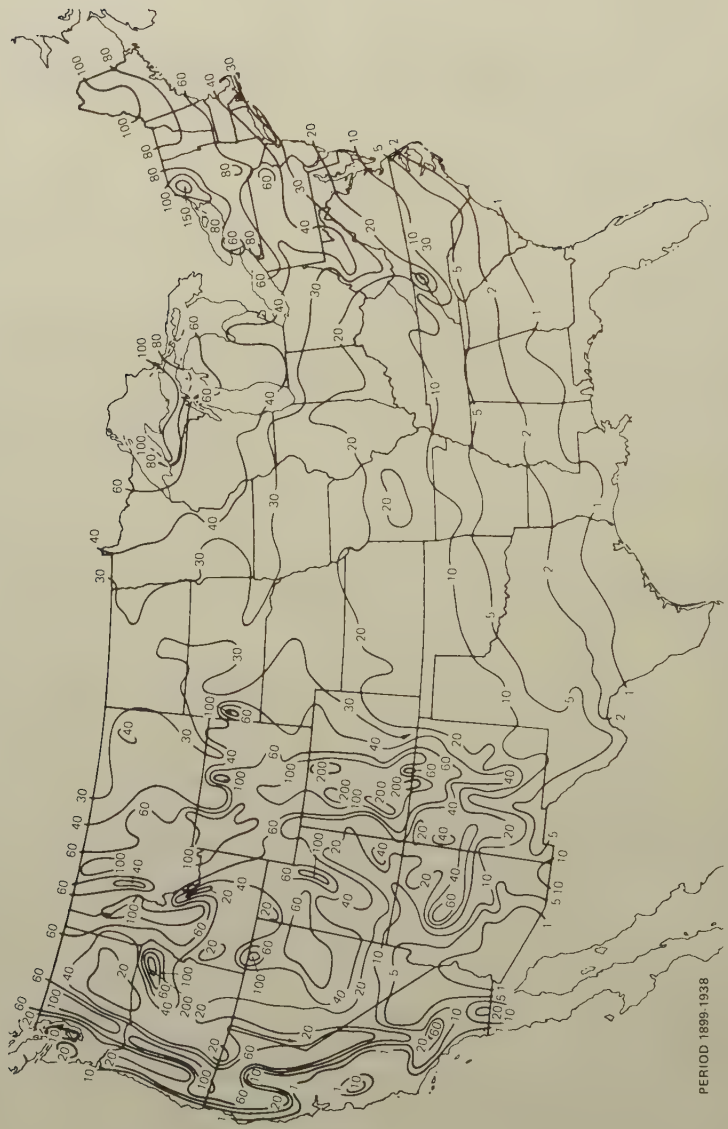


FIG. 11-1. Mean annual snowfall in the United States in inches, 1899–1938. (U. S. Weather Bureau.)

As in the rainfall-runoff process, geographic, geologic, topographic and vegetative factors also are operative in the snow accumulation-snowmelt runoff process.

For rainfall-runoff relationships, point rainfall measures are used in estimating areal and time distribution over the basin. A similar approach is taken in snow hydrology although the point-areal relationships are usually more complex. Soundly based mathematical equations can be used to determine the various components of snowmelt at a given location. Adequate measures of depth and other snowpack properties can also be obtained at specific locations. Use of these measurements in estimating amount and distribution in area and time of snow over large watershed areas is a much less rigorous procedure. Usually, average conditions related to particular areal subdivisions over time are used as the foundation for basin-wide hydrologic estimates. Such procedures are often in the category of index methods (Sec. 11-6).

11-2 Snow Accumulation and Runoff

Under usual conditions encountered in regions with heavy snowfall, runoff from the snowpack is the last occurrence in a series of events beginning when snowfall reaches the ground. The time interval from start to end of the process might vary from as little as a day or less to several months or more. Newly fallen snow has a density of about 0.10 (the percentage of snow volume its water equivalent would occupy) but as snow depth enlarges, settling and compaction increase the density.[8]

The temperature in a deep layer of accumulated snow is often well below freezing after prolonged cold periods. When milder weather sets in, melting occurs first at the snowpack surface. This initial meltwater moves only slightly below the surface and again freezes through contact with colder underlying snow. During the refreezing process, the heat of fusion released from melt water raises the snowpack temperature. Heat is also transferred to the snowpack from overlying air and the ground. During persistent warm periods, temperature of the entire snowpack continually rises and finally reaches 32°F. With continued melting, water begins flowing down through the pack. The initial melt component is retained on snow crystals in capillary films. Once the liquid water-holding capacity of the snow is reached, the snow is said to be *ripe*. Throughout the foregoing process, pack density increases due to refreezing of meltwater and build-up of capillary films. After the water-holding capacity is reached, the density remains relatively constant with continued melt. Melt water which exceeds the water-holding capacity will continue to move down through the snowpack until the ground is finally reached. At this point

runoff can occur. Three situations which may exist at the ground interface when melt water reaches it are described by Horton.[9,8]

First, consider the case where melt rate is less than the infiltration capacity of the soil. In addition, downward capillary pull of the soil coupled with gravity exceeds the same pull of the snow less gravity. Meltwater directly enters the soil and a slush layer is not formed.

The second case occurs when a soil's infiltration capacity is greater than the melt rate, but net capillary pull of the snowpack exceeds that of the soil aided by gravity. Capillary water builds up in the overlying snow until equilibrium is reached at which upward and downward forces balance. A slush layer forms and provides a supply of water which infiltrates the soil as rapidly as it enters the slush layer.

The final situation is one in which melt rate exceeds infiltration capacity. A slush layer forms and water infiltrates the soil at the infiltration capacity rate. Excess water acts in a manner analogous to surface runoff but at a much decreased overland flow rate.

As warm weather continues the melt process is maintained and accelerated until the snowcover is dissipated.

11-3 Snow Measurements and Surveys

Snow measurements are obtained through the use of standard and recording rain gauges, seasonal storage precipitation gauges, snow boards and snow stakes. Rain gauges are usually equipped with shields to reduce the effect of wind.[3] Snow boards are about 16 in. square, laid on the snow so that new snowfall which accumulates between observation periods will be found above them. Care must be taken to assure that adverse wind effects or other conditions do not produce an erroneous sample at the gauging location. Snow stakes are calibrated wooden posts driven into the ground for periodic observation of snow depth or inserted into the snowpack to determine its depth.

Direct measurements of snow depth at a single station are generally not very useful in making estimates of the distribution over large areas, since the measured depth may be highly unrepresentative due to drifting or blowing. To circumvent this problem, snow-surveying procedures have been developed. Such surveys provide information on snow depth, water equivalent, density, and quality at various points along a snow course. All these measures are of direct use to a hydrologist.

Water equivalent is the depth of water which would weigh the same amount as that of the sample. In this way snow can be described in terms of inches of water. Density is the percentage of snow volume which would be occupied by its water equivalent. Quality of the snow relates to the ice content of the snowpack and is expressed as a decimal

fraction. It is the ratio of weight of ice content to total weight. Snow quality is usually about 0.95 except during periods of rapid melt, when it may drop to 0.70 to 0.80 or less. The thermal quality of snow, Q_t, is the ratio of heat required to produce a particular amount of water from the snow, to the quantity of heat needed to produce the same amount of melt from pure ice at 32°F. Values of Q_t may exceed 100 percent at subfreezing temperatures. The density of dry snow is approximately 10 percent but there is considerable variability between samples. With aging, the density of snow increases to values on the order of 50 percent or greater.

A snow course includes a series of sampling locations, normally not less than 10 in number.[1] The various stations are spaced about 50–100 ft apart in a designed-in-advance geometric pattern. Points are permanently marked so the same locations will be surveyed each year—very important if snow course memoranda are to be correlated with areal snow cover and depth, expected runoff potential or other significant factors. Survey data are obtained directly by foresters and others, by aerial photographs and observations, and by automatic recording stations which telemeter information to a central processing location.

In the western United States the Soil Conservation Service coordinates many snow surveys. Various states, federal agencies, and private enterprises are also engaged in this type of activity. Sources of snow-survey data are summarized in Ref. 1.

11-4 Point and Areal Snow Characteristics

Estimation of areal snow depth and water equivalent from point measurement data is highly important in hydrologic forecasting.

Estimates of Areal Distribution of Snowfall

Normally, taking arithmetic averages or using Thiessen polygons does not provide reliable results for estimating areal snow distribution from point gaugings. This is because orographic and topographic effects are often pronounced, and gauging networks frequently are not dense enough to permit straightforward use of normal averaging techniques. Significantly however, regional orographic effects are relatively constant from year to year and storm to storm for tracts small when compared with the areal extent of general storms occurring in the region.[16] This factor permits many useful approaches in estimating areal snow distribution once the basic pattern has been found for a region.

One method used to estimate basin precipitation from point observations assumes that the ratio of station precipitation to basin precipitation is approximately constant for a storm or storms. This can be stated as[16]

$$\frac{P_b}{P_a} = \frac{N_b}{N_a} \tag{11-1}$$

or

$$P_b = P_a N_b / N_a \tag{11-2}$$

where P_b is the basin precipitation, P_a the observed precipitation at a point or group of stations, N_b the annual precipitation for the basin, and N_a the normal annual precipitation for the control station or stations. Normal annual precipitation is determined from a map (carefully prepared if it is to be representative) displaying the mean annual isohyets for the region. Precipitation is determined by planimetering areas between the isohyets. If the number of stations used and their distribution adequately depict the basin, Eq. 11-2 can provide a good approximation. For stations not uniformly distributed, weighting coefficients based on the percentage of the basin area portrayed by a gauge, are sometimes used in determining N_a for the group.

Another system used in estimating areal snowfall is the isopercental method. In this approach, storm or annual station precipitation is expressed as a percent of the normal annual total. Isopercental lines are drawn and can be superimposed on a normal annual precipitation map (NAP) to produce new isohyets representing the storm of interest. An NAP map indicates the general nature of the basin's topographic effects, while the isopercental map shows deviations from this pattern. The advantage of this method over preparing an isohyetal map directly is that relatively consistent storm-pattern features of the NAP can be taken into consideration as well as observed individual storm variations.

Estimates of Basin-Wide Water Equivalent

A hydrologist must be concerned not only with the amount and areal distribution of snowfall, but also with estimating the water equivalent of this snowpack over the basin, since in the final analysis it is this factor which determines runoff. Basin water equivalent may be given as an index or reported in a quantitative manner such as inches depth for the watershed.

The customary procedure for determining basin-water equivalent is to take observed data from snow course stations and to provide an index of basin conditions. Various procedures employ averages, weighted averages and other approaches to accomplish this.[16] The important point to remember is that usefulness of any index will be based on how well it represents overall basin conditions, not on how favorably it describes a particular point value. Indexes do not actually provide a quantitative evaluation of the property they cover. Instead they give relative changes in the factor. By introducing additional data however, an index can be

utilized in a prediction equation. For example, if basin-water equivalent can be estimated by subtracting the runoff and loss components from precipitation input, the index can be correlated with actual basin-water equivalent in a quantitative manner.

Areal Snow Cover

Estimates of areal distribution of snowfall are very helpful in making hydrologic forecasts. A knowledge of actual areal extent of snow cover on the ground at any given time is also applied in hydrograph synthesis and in making seasonal volumetric forecasts of runoff. Observations of snow cover are generally obtained by ground and air reconnaissance and photography. Between snow-cover surveys, approximations of the extent of snow cover are based on available hydrometeorological data. Snow-cover depletion patterns within a given basin are normally relatively uniform from year to year; thus snow-cover indexes can often be developed from data gathered at a few representative stations.

11-5 The Snowmelt Process

The snowmelt process converts ice content into water within the snowpack. Rates differ widely due to variations in causative factors to be discussed later. These divergencies are not as strikingly apparent when considering drainage from the snowpack, however, since the pack itself tends to filter out these nonuniformities so the drainage exhibits a more consistent rate.

Energy Sources for Snowmelt

Heat necessary to induce snowmelt is derived from short- and long-wave radiation, condensation of vapor, convection, air and ground conduction, and rainfall. The most important of these sources are convection, vapor condensation and radiation. Rainfall ranks about fourth in importance while conduction is usually a negligible source.

Energy Budget Considerations

If snowmelt is considered as a heat-transfer process, an energy-budget equation can be written to determine the heat equivalent of the snowmelt. Such an equation is of the form[16]

$$H_m = H_{rl} + H_{rs} + H_c + H_e + H_g + H_p + H_q \qquad (11\text{-}3)$$

where H_m = heat equivalent of snowmelt
H_{rl} = net long-wave radiation exchange between snowpack and surroundings
H_{rs} = absorbed solar radiation

H_c = heat transferred from air by convection
H_e = latent heat of vaporization derived from condensation
H_g = heat conduction from ground
H_p = rainfall heat content
H_q = internal energy change in snowpack

In this equation H_{rs}, H_g, and H_p are all positive; H_{rl} is usually negative in the open; H_e and H_q may take on positive or negative values; and H_c is normally positive. The actual amount of melt from a snowpack for a given total heat energy is a function of the snowpack's thermal quality. The heat energy required to produce a centimeter of water from pure ice at 32°F is 80 langleys (g-cal/cm^2). Therefore 203.2 langleys are needed to get an inch of runoff from a snowpack of 100 percent thermal quality. If the term H_m represents the combined total heat input in langleys, an equation for snowmelt M in inches is[16]

$$M = H_m/(203.2Q_t) \tag{11-4}$$

where Q_t is the thermal quality of the snowpack. Figure 11-2 gives a graphical solution to this equation for several values of Q_t.

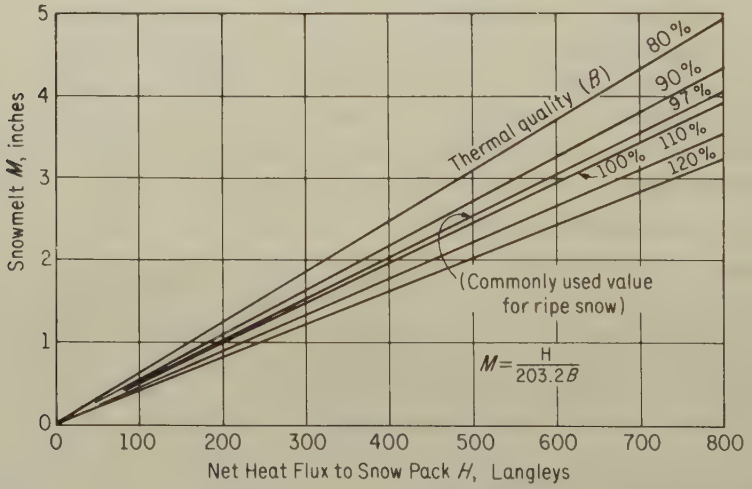


FIG. 11-2. Snowmelt resulting from thermal energy. (After U. S. Army Corps of Engineers, Ref. 16.)

Turbulent Exchange

The quantity of heat transferred to a snowpack by convection and condensation is commonly determined from turbulent-exchange equations. Such an approach has been widely used, since measurements of temperature and vapor pressure must be made in the turbulent zone where vertical water vapor, temperature, and wind-velocity gradients are con-

trolled by the action of eddies. In the following two subsections several practical equations for estimating condensation and convection melt will be given. Here a combined theoretical equation is presented to better acquaint the student with the theory of turbulent exchange.

The basic turbulent-exchange equation can be written as[16]

$$Q = A\, dq/dz \tag{11-5}$$

where Q is the time rate of flow of a specified property of the air such as water vapor through a unit horizontal area, dq/dz the vertical gradient of the property, q the property, z the elevation, and A an exchange coefficient. Property q must be unaffected by the vertical transport. Properties pertinent to this discussion are air temperature, water vapor, and wind velocity. Theoretically, potential temperature should be utilized but air temperatures measured at normal distances above the snowpack will not cause serious errors. The potential temperature of dry air is that which the air would take if brought adiabatically from its actual pressure to a standard pressure.

Gradients of the various properties of importance here follow a power-law distribution where conditions of atmospheric stability exist.[17] This qualification is characteristic of the atmosphere's state over snowfields.[16] Logarithmic profiles are more nearly representative of neutral or unstable atmospheric conditions. The power law provides that the ratio of values of a property determined at two levels above the snow is equivalent to the ratio of the levels raised to some power. Thus

$$q_2/q_1 = (z_2/z_1)^{1/n} \tag{11-6}$$

where q is the value of the property, z the elevation, with subscripts denoting the level, and n is the power-law exponent. If z_1 is made equal to 1, q_1 assumed to be the property value at this height, and the subscript dropped for the second level, Eq. 11-6 becomes

$$q = q_1 z^{1/n} \tag{11-7}$$

The magnitude of q is taken as the difference in values of q measured at level z and the snow surface. For example, if $T = 38°\text{F}$ at height z, and temperature is the property of interest, then $q = (38 - 32) = 6°\text{F}$. Gradient of the property dq/dz can be obtained by differentiating Eq. 11-7:

$$dq/dz = (q_1/n)\, z^{(1-n)/n} \tag{11-8}$$

If this expression is substituted in the basic turbulent-exchange equation (11-5), the following relationship obtains:

$$Q = A(q_1/n)\, z^{(1-n)/n} \tag{11-9}$$

Thus eddy exchange of the property at a specified elevation z is determined from observations of the property at unity level. The exchange

coefficient is also related to elevation z. For equilibrium conditions up to the usual levels of measurement of moisture and temperature, gradients of these variables are such that the eddy transfer of moisture and heat is constant with height. Then exchange coefficient A must be inversely related to the property gradient, or

$$A/A_1 = (dq/dz)_1/(dq/dz) \tag{11-10}$$

Substituting in Eq. 11-8 for dq/dz, there results

$$A = A_1 z^{(n-1)/n} \tag{11-11}$$

since $(dq/dz)_1 = (q_1/n)$ for $z = 1$. Now if the value of A from Eq. 11-11 is inserted in Eq. 11-9,

$$Q = A_1(q_1/n) \tag{11-12}$$

The exchange coefficient at an observation level has been shown to be directly proportional to the wind velocity measured at that elevation.[18] Therefore it may be written that

$$A_1 = kv_1 \tag{11-13}$$

where v_1 is the wind velocity at level one and k is a constant of proportionality. Substituting for A_1 in Eq. 11-12,

$$Q = (k/n)\, q_1 v_1 \tag{11-14}$$

Using the power-law equation (11-7) it is seen that

$$q_1 = qz^{-1/n} \tag{11-15}$$

and

$$v_1 = vz^{-1/n} \tag{11-16}$$

Making these substitutions in Eq. 11-14, denoting the observation level of v as z_b, and that of q as z_a (since these may be different), Eq. 11-14 becomes

$$Q = (k/n)(z_a z_b)^{-1/n} q_a v_b \tag{11-17}$$

This is a generalized turbulent-exchange equation. Consideration will now be given to developing specific theoretical equations for condensation and convection melt.

First, consider the case of condensation melt. The property to be transported in this case is water vapor, and since the exchange coefficient expresses transfer of an air mass, it is necessary to determine moisture content of the air mass. This can be accomplished by using the specific humidity which gives the weight of water vapor contained in a unit weight of moist air. Equation 11-18 can be used to calculate specific humidity:

$$q = 0.622e/p_a \tag{11-18}$$

where e is the vapor pressure and p_a the total pressure of the moist air. Inserting this expression in Eq. 11-17 for q_a yields[16]

$$Q_e = (k/n)(z_a z_b)^{-1/n}(0.622/p)\, e_a v_b \qquad (11\text{-}19)$$

where Q_e is the moisture transfer to the snow surface per unit time.

Vapor pressure property e_a is the difference in vapor pressures between level z_a and the snow surface. The value of e_a may thus be either positive (condensation) or negative (evaporation). In addition to condensate on the snow surface, release of the latent heat of vaporization (about 600 cal/g) from the condensate will melt 7.5 g of snow per gram of condensate if the thermal quality is 100 percent. Multiplying moisture transfer Q_e by 8.5 (melt plus condensate per gram of condensate), the time rate of condensation melt (M_e) becomes

$$M_e = 8.5(k/n)(z_a z_b)^{-1/n}(0.622/p)\, e_a v_b \qquad (11\text{-}20)$$

Proportionality constant k is a complex function related to air density and other factors. Since density of air is a function of elevation, k also varies with height. The constant may be made independent of density, and therefore elevation by introducing a factor to compensate directly for the density-elevation relationship. Atmospheric pressure serves to accomplish this and the equation can be adjusted by multiplying by the ratio p/p_0 where p is the pressure at the snowfield elevation and p_0 the sea-level pressure. Introducing this ratio in Eq. 11-20 and a new constant k_1 which is related to sea-level pressure gives[16]

$$M_e = 8.5(k_1/n)(z_a z_b)^{-1/n}(0.622/p_0)\, e_a v_b \qquad (11\text{-}21)$$

For convection melt, the property of importance in Eq. 11-17 is air temperature. To convert air temperature into thermal units the specific heat of air c_p must be introduced. Putting these values in Eq. 11-17, heat transfer by eddy exchange H_c converts to[16]

$$H_c = (k/n)(z_a z_b)^{-1/n} c_p T_a v_b \qquad (11\text{-}22)$$

Since the latent heat of fusion is 80 cal/gram, convective snowmelt M_c in grams in the cgs system is given by $H_c/80$, or

$$M_c = (1/80)(k/n)(z_a z_b)^{-1/n} c_p T_a v_b \qquad (11\text{-}23)$$

Introducing the elevation-density correction p/p_0,

$$M_c = (1/80)(k_1/n)(z_a z_b)^{-1/n}(p/p_0)\, c_p T_a v_b \qquad (11\text{-}24)$$

Equations 11-21 and 11-24 can be combined into a single convection-condensation melt M_{ce} equation of the form[16]

$$M_{ce} = \frac{k_1}{n}(z_a z_b)^{-1/n}\left(\frac{1}{80}\frac{p}{p_0} c_p T_a + 8.5\,\frac{0.622}{p_0} e_a\right) v_b \qquad (11\text{-}25)$$

This is a generalized theoretical equation for snowmelt which results from the turbulent transfer of water vapor and heat to the snowpack. It is assumed the exchange coefficients for heat and water vapor are equal. Their evaluation is accomplished by experimentation.[16]

A combined physical equation of the general nature of Eq. 11-25 has been developed by Light.[10] Widely used, its individual convection and condensation melt components will be discussed in following sections. The combined form of the Light equation is

$$D = \frac{\rho k_0^2}{80 \ln (a/z_0) \ln (b/z_0)} U\left[c_p T + (e - 6.11)\frac{423}{p}\right] \qquad (11\text{-}26)$$

where D = effective snowmelt (cm/sec)
ρ = air density
k_0 = von Kármán's coefficient = 0.38
z_0 = roughness parameter = 0.25
a, b = levels at which wind velocity, temperature, and vapor pressure are measured respectively
U = wind velocity
c_p = specific heat of air
T = air temperature
e = vapor pressure of the air
p = atmospheric pressure

Convection

Heat for snowmelt is transferred from atmosphere to snowpack by convection. The amount of snowmelt by this process is related to temperature and wind velocity. The following equation can be used to estimate the 6-hr depth of snowmelt in inches by convection:[11]

$$D = KV(T - 32) \qquad (11\text{-}27)$$

where V = mean wind velocity (mph)
T = the air temperature (°F)

On the basis of the theory of air turbulence and heat transfer (turbulent exchange) a theoretical value for the exchange coefficient K of $0.00184 \times 10^{-0.0000156h}$ has been given by Light.[10] In this relationship h, the elevation in feet, is used to reflect the change in baromeric pressure due to difference in altitude. The expression is said to represent conditions for an open, level snowfield where measurements of wind and temperature are made at heights of 50 and 10 ft respectively above the snow. Values of the expression $10^{-0.0000156h}$ vary from 10.0 at sea level to 0.70 at 10,000 ft of elevation. Actual values of K are normally less than the theoretical figure due to such factors as forest cover. Empirical 6-hr K values have been reported in the literature.[11]

Anderson and Crawford give an expression for hourly snowmelt due to convection as

$$M = \frac{cV(T_a - 32)}{Q_t} \qquad (11\text{-}28)$$

where M is the hourly melt in inches, V the wind velocity in miles per hour, T_a the surface air temperature (°F), Q_t the snow quality, and c is a turbulent exchange coefficient determined empirically.[8] Temperature measurements are at 4 ft, wind gauged at 15 ft. The corresponding value of c is reported as 0.0002.

Condensation

Heat given off by condensing water vapor in a snowpack is often times the most important heat source particularly when temperatures are in the higher ranges (50–60°F). To melt a pound of ice at 32°F, a thermal input of 144 Btu is required. A pound of moisture originating from the condensation process at 32°F, produces about 1073 Btu. On this basis, one inch of condensate will produce approximately 7.5 in. of water from the snow. A total yield of around 8.5 in. of snowmelt including the condensate will thus be derived.

A water-vapor supply at the snow surface is formed by the turbulent exchange process and consequently a mass-transfer equation similar to those presented for evaporation studies fits the melt process. An equation for hourly snowmelt from condensation takes the form[8]

$$M = \frac{bV}{Q_t}(e_a - 6.11) \qquad (11\text{-}29)$$

where b is an empirical constant, e_a is the vapor pressure of the air (mb), numerical value 6.11 is the saturation vapor pressure (mb) over ice at 32°F, and M, Q_t and V are as previously defined. The constant b has a value of 0.001 for temperature and wind measurements at 4 and 15 ft respectively.[8]

A similar expression but for 6-hr snowmelt (D) is given as

$$D = K_1 V(e_a - 6.11) \qquad (11\text{-}30)$$

where the theoretical value of K_1 is said by Light to equal 0.00578 if wind and temperature data are obtained at the 50- and 10-ft levels respectively and the snowfield is level and open.[10] Actual figures based on observation are generally lower than this due mainly to forest influences. A value of 0.0032 has been reported by Wilson for three study basins in Wyoming. For condensation melt to occur, the dewpoint temperature must exceed 32°F. When it drops below that level, evaporation occurs at the snow surface. An equation for snow evaporation takes the form

$$E = \frac{kV(e_a - e_s)}{Q_t} \qquad (11\text{-}31)$$

where E is the hourly evaporation in inches, e_s the saturation vapor pressure over the snow, k an empirical constant, and V, e_a, and Q_t are as defined before.[8] In the expression $k = 0.0001$, temperature and wind

measurements are taken as for Eq. 11-30, and temperature of the air is assumed equal to that of the snow surface for temperatures below 32°F.

Radiation Melt

The net amount of short- and long-wave radiation received by a snowpack can be a very important source of heat energy for snowmelt. Under clear skies, the most significant variables in radiation melt are insolation, reflectivity or albedo of the snow, and air temperature. Humidity effects, while existent, are usually not important. When cloud cover exists, striking changes in the amount of radiation from an open snow field are in evidence. The general nature of these effects is illustrated on Fig. 11-3.[16] Combined short- and long-wave radiation exchange as a function of cloud height and cover is represented. Radiation melt is shown to be more significant in the spring than in winter. It should also be noted that winter radiation melt tends to increase with cloud cover and decreasing cloud height as a result of the more dominant role played by long-wave radiation during that period.

Forest canopies also exhibit important characteristics in regulating radiative heat exchange. These effects differ somewhat from those exhibited by the cloud cover, especially where short-wave radiation is concerned. Clouds and trees both limit insolation, but clouds are very reflective while a large amount of the intercepted insolation is absorbed by the forest. Consequently, the forest is warmed and part of the incident energy directly transferred to snow in the form of long-wave radiation; an additional fraction is transferred indirectly by air also heated by the forest.

Figure 11-4 illustrates some effects of forest canopy on radiation snowmelt. The figure typifies average conditions for a coniferous cover in the middle latitudes.[16] In winter, maximum radiation melt is associated with complete forest cover, and in spring the greatest radiation melt occurs in the open. Generalizations should not be drawn from these curves which do indicate relative seasonal effects of forest cover on radiation melt for the conditions described. Another factor affecting radiation melt is land slope and its aspect. Radiation received by north-facing slopes is less than that for south exposure inclines in the northern hemisphere for example.

Solar energy provides an important source of heat for snowmelt. Above the earth's atmosphere, the thermal equivalent of solar radiation normal to the radiation path is 1.94 langleys/min (one langley is approximately 3.97×10^{-3} Btu/cm^2). Actual amount of radiation reaching the snow pack is modified by many factors such as degree of cloudiness, topography, and vegetal cover. The importance of vegetal cover in influencing snowmelt, long recognized, has prompted many forest management schemes to regulate snowmelt.[1,2,12]

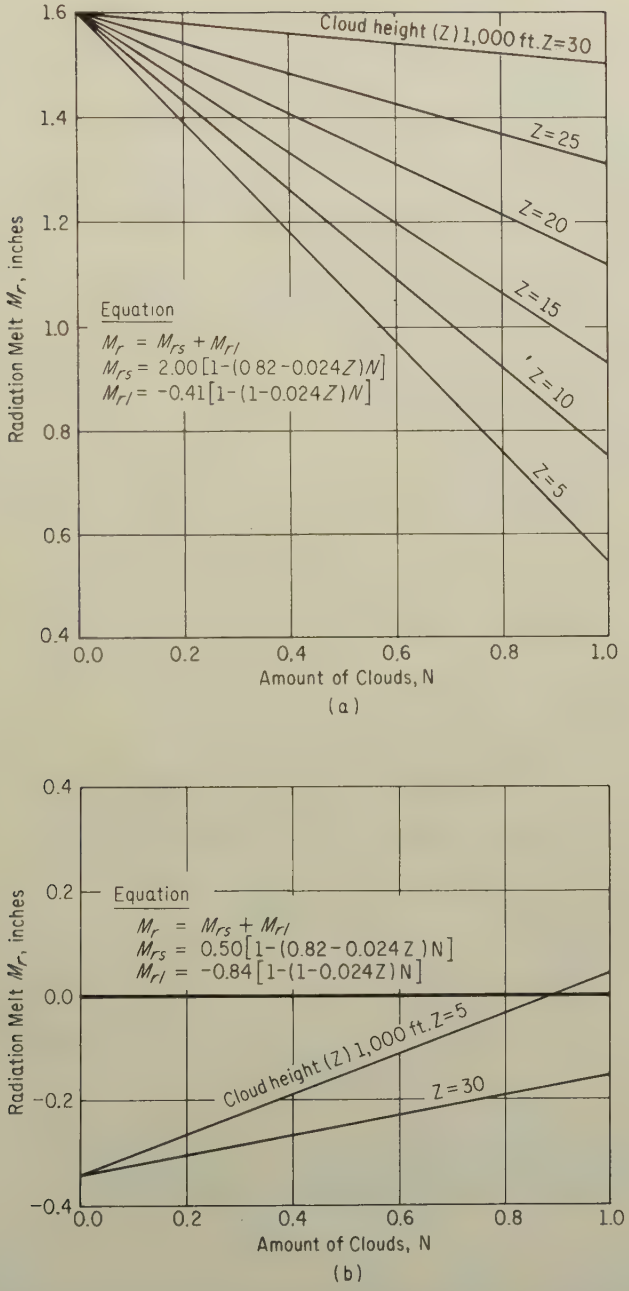


FIG. 11-3. Daily radiation melt in the open with cloudy skies: (a) during spring–20 May; and (b) during winter–15 February. (After U. S. Army Corps of Engineers, Ref. 16.)

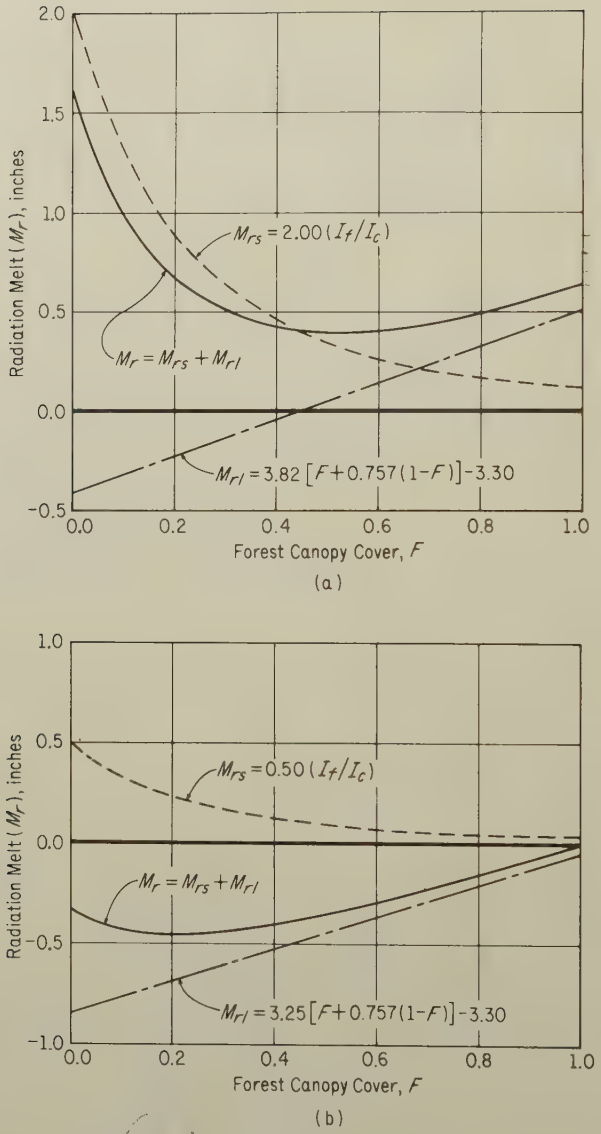


FIG. 11-4. Daily radiation melt in the forest with clear skies: (a) during spring–20 May; and (b) during winter–15 February. (After U. S. Army Corps of Engineers, Ref. 16.)

Two basic laws are applicable to radiation. Planck's law states that the temperature of a black body is related to spectral distribution of energy which it radiates. Integration of Planck's law for all wavelengths produces Stefan's law,

$$R_a = \sigma T^4 \tag{11-32}$$

where R_a is the total radiation, σ is Stefan's constant (0.826×10^{-10} Langley/min/(°K)$^{-4}$, and T is the temperature (°K). Because snow radiates as a black body, the amount of radiation is related to its temperature (Planck's law), and total energy radiated is according to Stefan's law. Long-wave radiation by a snow pack is determined in a complex fashion through the interactions of temperature, forest cover, and cloud conditions.

Direct solar short-wave radiation received at the snow surface is not all transferred to sensible heat. Part of the radiation is reflected and thus lost for melt purposes. Short-wave reflection is known as *albedo* and ranges from about 40 percent for melting snow late in the season to approximately 80 percent for newly fallen snow. Values as high as 90 percent have also been reported in several cases.[13] This property of the snow pack to reflect large fractions of the insolation explains why the covers persist and air temperatures remain low during clear, sunny, winter periods.

That portion of short-wave radiation not reflected is available for snowmelt, may become long-wave radiation, or be conducted within the snowpack. Some heat may also be absorbed by the ground with no resultant melt if the ground is frozen.

An expression for hourly short-wave radiation snowmelt is given as[16]

$$M = H_m/(203.2Q_l) \tag{11-33}$$

where H_m is the net absorbed radiation in langleys, 203.2 is a conversion factor for changing langleys to inches of water. When the snow quality is 1, long-wave radiation is exchanged between the snow cover and its surroundings. Snowmelt from net positive long-wave radiation follows Eq. 11-33. If net long-wave radiation is negative (back radiation) there is an equivalent heat loss from the snowpack.

An approximate method of estimating 12-hr snowmelt D_{12} (periods midnight to noon, noon to midnight) from direct solar radiation has been given by Wilson.[11] The relationship is of the form

$$D_{12} = D_0(1 - 7.5m) \tag{11-34}$$

D_0 being the snowmelt occurring in a half-day in clear weather, and m the degree of cloudiness (0 for clear weather, 1.0 for complete overcast). Suggested values for D_0 are 0.35 in. (March), 0.42 in. (April), 0.48 in. (May), 0.53 in. (June) within latitudes 40 to 48 degrees.[11]

Rainfall

Heat derived from rainfall is generally small since during those periods when rainfall occurs on a snowpack, temperature of the rain is probably

quite low. Nevertheless, at higher temperatures, rainfall may constitute a significant heat source, does effect the aging process of the snow and frequently is very important in this respect. An equation for hourly snowmelt from rainfall is[8]

$$M = \frac{P(T_w - 32)}{144 Q_t} \tag{11-35}$$

where P is the rainfall in inches and T_w the wet-bulb temperature assumed to be that of the rain. This equation is based upon the relationship between heat required to melt ice (144 Btu per pound of ice) and the amount of heat given up by a pound of water when its temperature is decreased by one degree.

Daily snowmelt by rainfall estimates is given by

$$M_d = 0.007 P_d(T_a - 32) \tag{11-36}$$

where M_d is the daily snowmelt (in.), P_d the daily rainfall (in.), and T_a the mean daily air temperature (°F) of saturated air taken at the 10 ft level.[14]

Conduction

Major sources of heat energy to the snowpack are radiation, convection, and condensation. Under usual conditions reliable determination of hourly or daily melt quantities can be founded on these heat sources plus rainfall if it occurs. An additional source of heat, negligible in daily melt computations but perhaps significant over an entire melt season, is ground conduction.

Ground-conduction melt is the result of upward transfer of heat from ground to snowpack due to thermal energy which was stored in the ground during the preceding summer and early fall. This heat source can produce meltwater during winter and early spring periods when snowmelt at the surface does not normally occur. Heat transfer by ground conduction can be expressed by the relationship[16]

$$H_q = K\, dT/dz \tag{11-37}$$

where K is the thermal conductivity of soil and dT/dz the temperature gradient perpendicular to soil surface.

Snowmelt from ground conduction is generally exceedingly small. Wilson notes that after about 30 days of continuous snow cover, heat transferred from the ground to the snow is insignificant.[11] The amount of snowmelt from ground conduction during a snowmelt season has been estimated at approximately 0.02 in./day.[14] Ground conduction does act to provide moisture to the soil however, and thus when other favorable conditions for snowmelt occur, a more rapid development of runoff can be expected.

This section has emphasized the physics of snowmelt. The manner in which heat can be provided to initiate the melt process was discussed. Equations 11-27 through 11-31 and 11-33 to 11-37 inclusive can be used to estimate the melt at a given point. The task of computing runoff from snowmelt in a basin can not be approached in such a simple fashion since there are many complex factors operative. The remainder of this chapter will be devoted to the general subject of runoff from snowmelt investigations. Figure 11-5 illustrates hourly variation in the principal heat fluxes to a snowpack for a cloudy day.

11-6 Snowmelt-Runoff Determinations

Various approaches to runoff determination from snowmelt have been followed. They range from relatively simple correlation analyses which completely ignore the physical snowmelt process to relatively sophisticated methods using physical equations. Most techniques can be considered as based on degree-day correlations, analyses of recession curves, correlation analyses, physical equations, or various indexes. Each will be discussed in the following paragraphs.

Purposes of Snowmelt-Runoff Estimates

Snowmelt-runoff estimates are extremely important for many regions of the United States and other countries in (1) forecasting seasonal water yields for a diversity of water supply purposes; (2) regulating rivers and storage works; (3) implementing flood-control programs; and (4) selecting design floods for particular watersheds. Maximum floods in many areas are often due to a combination of rainfall and snowmelt runoff. In effect, determination of snowmelt runoff has the same utility as the calculation of runoff from rainfall. In some areas it will in fact be the more important of the two.

Snowpack Condition

The manner in which runoff from either rainfall or snowmelt is affected by conditions prevalent within the snowpack is of primary interest to a hydrologist. Various views on storage characteristics of a snowpack have been advanced.[16] These range from the concept that a snowpack can retain large amounts of liquid water to the hypothesis that snowpack storage is negligible. There is no universally applicable relationship and it becomes important to base any runoff considerations on a knowledge of the character of a snowpack at the time of study. Winter runoff is related to a snowpack's condition whereas in the spring, once active melt begins, little or no delay in transport of melt of rainfall through the snowpack occurs.

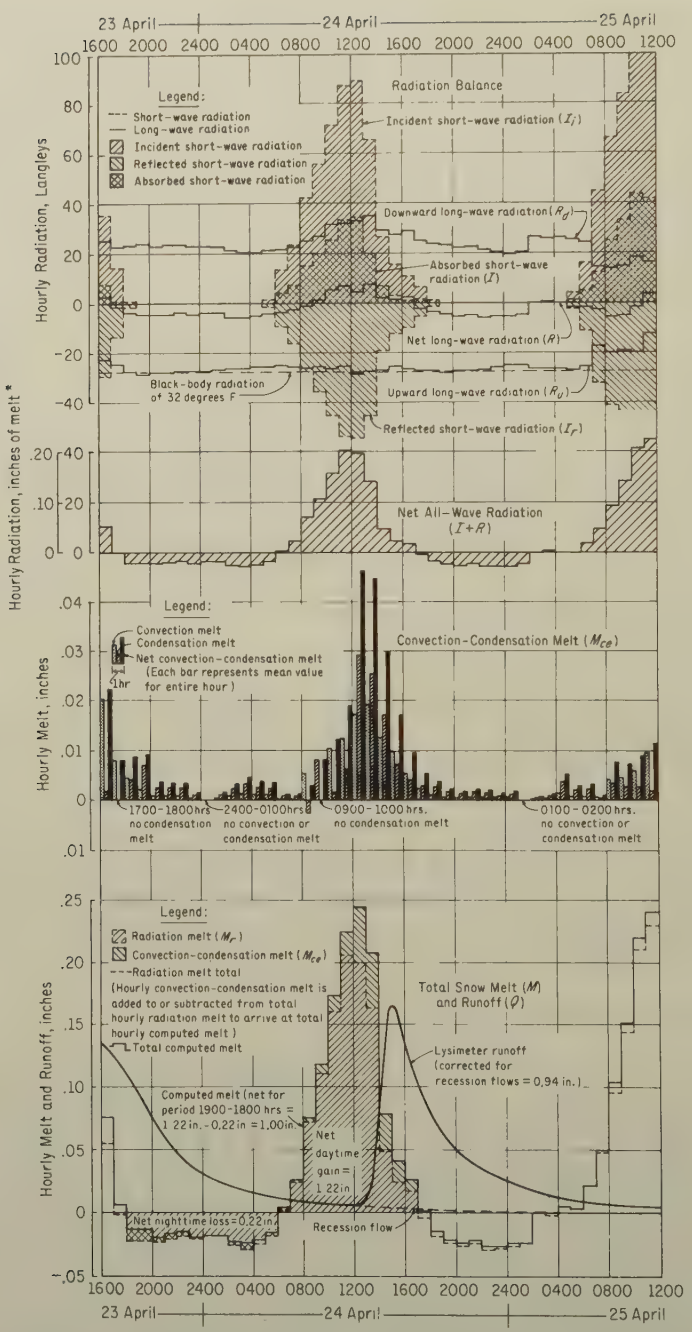

23 April
24 April
25 April
1600 2000 2400 0400 0800 1200 1600 2000 2400 0400 0800 1200
Hourly Radiation, Langleys
Legend:
Short-wave radiation
Long-wave radiation
Incident short-wave radiation
Reflected short-wave radiation
Absorbed short-wave radiation
Radiation Balance
Incident short-wave radiation (I_i)
Downward long-wave radiation (R_d)
Absorbed short-wave radiation (I')
Net long-wave radiation (R)
Black-body radiation of 32 degrees F
Upward long-wave radiation (R_u)
Reflected short-wave radiation (I_r)
Net All-Wave Radiation ($I+R$)
Hourly Radiation, inches of melt*
Hourly Melt, inches
Legend:
Convection melt
Condensation melt
Net convection-condensation melt
(Each bar represents mean value for entire hour)
1hr
Convection-Condensation Melt (M_{ce})
1700-1800hrs no condensation melt
2400-0100hrs no convection or condensation melt
0900-1000 hrs. no condensation melt
0100-0200 hrs. no convection or condensation melt
Hourly Melt and Runoff, inches
Legend:
Radiation melt (M_r)
Convection-condensation melt (M_{ce})
Radiation melt total
(Hourly convection-condensation melt is added to or subtracted from total hourly radiation melt to arrive at total hourly computed melt)
Total computed melt
Total Snow Melt (M) and Runoff (Q)
Lysimeter runoff (corrected for recession flows = 0.94 in.)
Computed melt (net for period 1900-1800 hrs = 1.22 in. - 0.22 in = 1.00 in.
Net daytime gain = 1.22 in
Net nighttime loss = 0.22 in.
Recession flow
1600 2000 2400 0400 0800 1200 1600 2000 2400 0400 0800 1200
23 April
24 April
25 April

FIG. 11-5. Hourly variation in principal heat fluxes to a snowpack for a cloudy day. (After U. S. Army Corps of Engineers, Ref. 16.)

For drainage basins in mountainous areas, snowpack storage effects may be approximated by subdividing the watershed into relatively uniform areas. Normally this will be accomplished by using elevation zones. Snowpack at the lowest levels may be conditioned to readily transmit rain or meltwater while in higher elevations, a liquid water deficit may prevail. At uppermost elevations, the snowpack may be very dry and cold and thus in a condition for optimum storage of water. Storage potential of the watershed zones must be based on representative measurements of snow depth, density, temperature, water equivalent, moisture content, and snowpack character. Snowpack character relates to physical structure of the pack. Unfortunately, adequate measures of all of these factors are not always available or easily obtained. Estimates of changes between sampling periods are usually indexed to readily observed meteorologic variables.

Formulation of snowpack storage and time delay characteristics can be fashioned by assuming a homogeneous pack. In this case storage is related directly to the liquid water deficit and cold content of the pack. Time delay is a function of inflow rate. It is considered that the snowpack storage potential must be entirely satisfied before runoff begins. In reality this is not the case but the assumption permits an analysis to be made. As melt proceeds, the storage potential of any snowpack diminishes.

Storage of a snowpack before runoff commences is considered to be the sum of the equivalent water requirement to raise the temperature of the snowpack to 0°C (cold content W_c), and the liquid water-holding capacity of the snowpack. If the cold content is given in inches of water needed to bring a snowpack temperature to 0°C, it may be represented by[16]

$$W_c = \frac{W_0 T_s}{160} \tag{11-38}$$

where T_s is the mean snowpack temperature below zero degrees, and W_o the initial water equivalent of the snowpack in inches. The time t_c in hours needed to raise the snowpack temperature to zero is thus given by

$$t_c = \frac{W_0 T_s}{160(i + m)} \tag{11-39}$$

where i is the rainfall intensity (in./hr) and m the rate of melt (in./hr). Storage required to meet the liquid water deficit of the snowpack is given by

$$S_f = \frac{f_p}{100}(W_0 + W_c) \tag{11-40}$$

where S_f is the amount of water stored (in inches) and f_p the percent deficiency in liquid water of the snowpack. The time in hours t_f needed

to fill the storage S_f is given by

$$t_f = \frac{f_p(W_0 + W_c)}{100(i + m)} \tag{11-41}$$

It has been specified that total storage potential S_p to be met prior to runoff is given as

$$S_p = W_c + S_f \tag{11-42}$$

This is also known as "permanent" storage, since it is not available to runoff until the snowpack has finally melted. An additional storage component transitory storage S_t is that water stored in the snowpack while moving through it to become runoff. Until initiation of runoff, the transitory storage in inches can be expressed as

$$S_t = \frac{D(i + m)}{V} \tag{11-43}$$

where D is the depth of the snowpack (in feet) and V the rate of transmission through the snowpack (ft/hr). The delay time of water in passing through the snowpack t_t is thus

$$t_t = \frac{D}{V} \tag{11-44}$$

for t_t in hours. Assuming W_c very small compared with W_0, depth of the snowpack is given by

$$D = \frac{W_0}{\rho_s} \tag{11-45}$$

with ρ_s the density of the snowpack. Then

$$t_t = \frac{W_0}{\rho_s V} \tag{11-46}$$

Before runoff commences, total water S stored in the snowpacks, in inches, is given by

$$S = W_c + S_f + S_t \tag{11-47}$$

which can also be written as

$$S = W_0\left\{\frac{T_s}{160} + \frac{f_p}{100} + \frac{(i + m)}{\rho_s V}\right\} \tag{11-48}$$

Total time in hours which passes before runoff is produced is thus[16]

$$t = t_c + t_f + t_t \tag{11-49}$$

or

$$t = W_0\left\{\frac{T_s}{160(i + m)} + \frac{f_p}{100(i + m)} + \frac{1}{\rho_s V}\right\} \tag{11-50}$$

After establishing active runoff from the snowpack, the only significant

term in Eq. 11-49 is t_l, and this is usually small compared with the overall basin lag and can be neglected. With increased snowmelt and runoff, additional increments of water previously withheld by snow blockage to drainage outlets and other factors are released. Adequate quantification of this cannot be accomplished at present.[16] A deep snowpack, say 15 ft, having a mean temperature of −5°C could store about 4 in. of liquid water before the onset of runoff. Figure 11-6 illustrates the nature of water balance in a snowpack during a rainstorm.

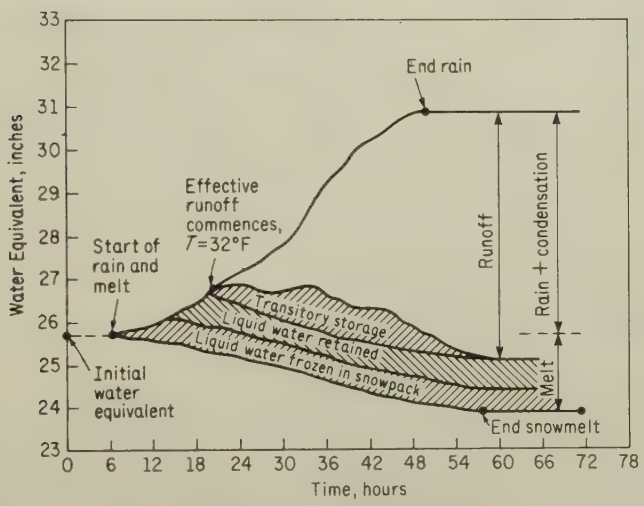


FIG. 11-6. Water balance in a snowpack during rainfall.

Indexes

Hydrologic indexes are made up of hydrologic or meterologic variables to describe their functioning. The index variable is more easily measured or handier than the element it represents. When mean fixed relationships are known to exist between point measurements and watershed values, indexes can be used to record both areal and temporal aspects of basin values. Indexes serve to permit (1) readily obtainable observations to depict hydrologic variables or processes which themselves cannot be easily measured; and (2) simplification of computational methods by allowing individual observations or groups of observations to replace watershed values in time and space. Adequacy of an index is based on (1) ability of the index to adequately describe the physical process it represents; (2) random variability of the observation; (3) the degree to which the point observation is typical of actual conditions; and (4) nature of variability between the point measurement and basin means.[16] Indexes may be equations or simple coefficients, and variable or constant.

The types of data required to make comprehensive thermal-budget studies are normally unavailable in whole or part for watersheds other than ones which themselves are experimental areas. As a result, a hydrologist must make the best use of information at hand. The most commonly available data are daily maximum and minimum temperature, humidity, and wind velocity. Less prevalent are continuous measurements of these and few stations record solar radiation or duration of sunshine. Hourly cloudiness data can sometimes be obtained from local airport weather stations.

A completely general index for reliably describing snowmelt-runoff relationships for all basins has not been established. Most indexes include coefficients valid only for specific topographic, meteorologic, hydrologic, and seasonal conditions and are therefore limited in applicability to other watersheds. Table 11-1 shows some types of indexes which have been successfully used in snowmelt investigations.

TABLE 11-1 Some Indexes Used to Describe Thermal Budget Variables

Thermal budget component	Index
Absorbed short-wave radiation	1. Duration of sunshine data. 2. Diurnal temperature range.
Long-wave radiation*	1. Air temperature for heavily forested areas. 2. For open areas long-wave radiation should be estimated.
Convective heat exchange	1. $(T_a - T_b)\,V$, where T_a is air temperature, T_b the snow surface temperature or base temperature, and V the wind speed.
Heat of condensation	1. $(e_a - e_s)\,V$, where e_a and e_s are vapor pressures of air and snow surface or a base value, and V is wind speed.

*Figure 11-7 illustrates an approximate linear relationship between melt and long-wave radiation used by the U.S. Army Corps of Engineers for index purposes.

The snowmelt runoff equation stated in terms of thermal budget indexes is

$$Y = a + \sum_{i=1}^{n} b_i X_i \tag{11-51}$$

where Y is the snowmelt runoff, a is a regression constant, the b_i's are the regression coefficients, and the X_i's are individual indexes. Various indexes usable to represent the terms of Eq. 11-51 are selected and a standard regression analysis performed to determine a and b_i. It should be noted that every term in the heat-budget equation is not always significant for a particular analysis, and thus the number of X_i's will vary for different basins and conditions. A final melt equation developed by the U.S. Army Corps of Engineers for the partly forested Boise River

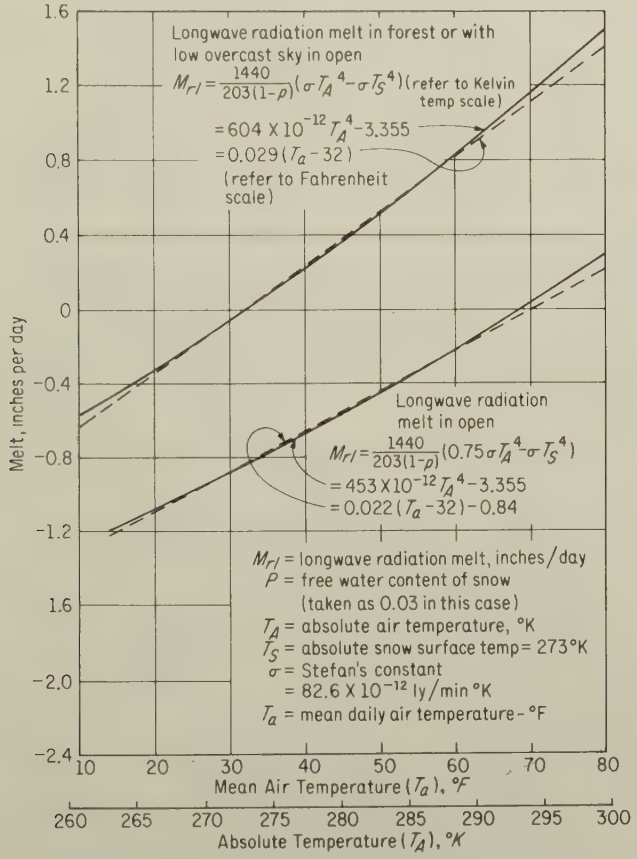

Longwave radiation melt in forest or with low overcast sky in open
$M_{rl} = \frac{1440}{203(1-p)}(\sigma T_A^4 - \sigma T_S^4)$ (refer to Kelvin temp scale)
$= 604 \times 10^{-12} T_A^4 - 3.355$
$= 0.029(T_a - 32)$
(refer to Fahrenheit scale)
Longwave radiation melt in open
$M_{rl} = \frac{1440}{203(1-p)}(0.75\sigma T_A^4 - \sigma T_S^4)$
$= 453 \times 10^{-12} T_A^4 - 3.355$
$= 0.022(T_a - 32) - 0.84$
M_{rl} = longwave radiation melt, inches/day
P = free water content of snow (taken as 0.03 in this case)
T_A = absolute air temperature, °K
T_S = absolute snow surface temp = 273°K
σ = Stefan's constant
$= 82.6 \times 10^{-12}$ ly/min °K
T_a = mean daily air temperature - °F
Melt, inches per day
1.6
1.2
0.8
0.4
0
-0.4
-0.8
-1.2
1.6
-2.0
-2.4
10 20 30 40 50 60 70 80
Mean Air Temperature (T_a), °F
260 265 270 275 280 285 290 295 300
Absolute Temperature (T_A), °K

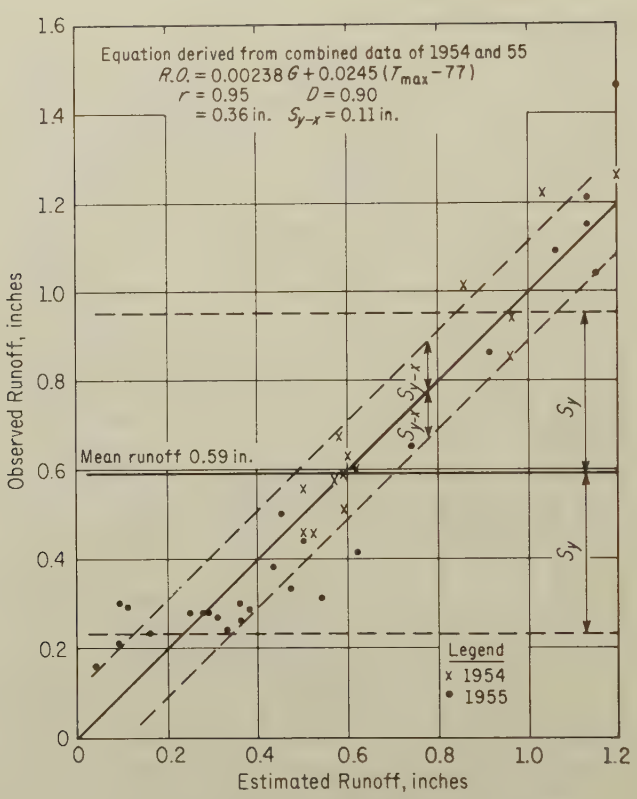
FIG. 11-7. Longwave radiation melt, with linear approximation. (After U. S. Army Corps of Engineers, Ref. 16.)

basin above Twin Springs, Idaho was

$$Q = 0.00238G + 0.0245(T_{max} - 77) \tag{11-52}$$

where Q is the daily snowmelt runoff in inches over the snow covered area, G is an estimated value of the daily allwave radiation exchange in the open (langleys), and T_{max} is the daily maximum temperature at Boise (°F).[16] The equation is said to predict daily snowmelt runoff values within 0.11 in. of observed values about 67 percent of the time. Figure 11-8 illustrates this relationship.

In attempting to develop suitable indexes for snowmelt, a hydrologist should seek the approach most closely resembling the thermal budget of the area, within the limitations of available data.



$R.O.$ = daily generated snowmelt runoff in inches depth over snow-covered area

G = daily net allwave radiation absorbed by snow in open, langleys

T_{max} = daily maximum temperature, Boise, °F

r = coefficient of correlation

D = coefficient of determination

s_y = standard deviation of observed runoff, inches

S_{yx} = standard error of estimated runoff, inches

FIG. 11-8. Observed versus estimated runoff. (After U.S. Army Corps of Engineers, Ref. 16.)

Temperature Indexes

Atmospheric temperature is an extremely useful parameter in snowmelt determination. It reflects the extent of radiation, vapor pressure of the air, and is also sensitive to air motion. Frequently it is the only adequate meteorological variable regularly on hand so widespread use has been made of degree-day relationships in snowmelt computations.

A *degree day* is defined as a deviation of one degree from a given datum temperature consistently over a 24-hr period. In snowmelt computations, the reference temperature is usually 32°F. If the mean daily temperature is 43°F for example, this is equivalent to 11 degree days above 32°F. If the temperature does not drop below freezing during the 24-hr period there will be 24 degree hours for each degree departure above 32°F. In this example there would be 264 degree hours for the day of observation.

Various ways of estimating the mean temperature have enabled investigators to take several approaches. One method is to simply average the maximum and minimum daily temperatures. Bases other than 32°F are also used. Regardless of the particular attack employed, a degree hour or degree day is an index to the amount of heat present for snowmelt or other purposes and has proven useful in point-snowmelt and runoff from snowmelt determinations.

Standard practice in developing snowmelt relationships on the basis of temperature is to correlate degree days or degree hours with snowmelt or basin runoff. In some cases, other factors are introduced to define forest cover effects and/or other influences. Another approach often used is to calculate a degree-day factor—the ratio of runoff or snowmelt to accumulated degree days which produced the runoff or melt. Unfortunately, the degree-day factor has been found to vary seasonally and between basins, therefore single representative values should be used with caution. Point-degree-day factors for snow-covered basins range from 0.015 to 0.20 in. per degree day when melting occurs. Gartska states that an average point value of 0.05 can be used to represent spring snowmelt provided caution is used.[1] Linsley and others state that basin mean degree-day factors are usually between 0.06 and 0.15 in./degree day under conditions of continuous snow cover and at melting temperatures. Figure 11-9 illustrates temperature index equations for springtime snowmelt on clear and forested areas.[16]

Generalized Basin Snowmelt Equations

Extensive studies by the U.S. Army Corps of Engineers at various laboratories in the West have produced several general equations for snowmelt during (1) rain-free periods, and (2) periods of rain.[16] When rain is falling, heat transfer by convection and condensation is of prime importance. Solar radiation is slight and long-wave radiation can be readily determined from theoretical considerations. When rain-free periods prevail, both solar and terrestrial radiation become material and may require direct evaluation. Convection and condensation are usually less critical during rainless intervals. The equations are summarized as follows:[16]

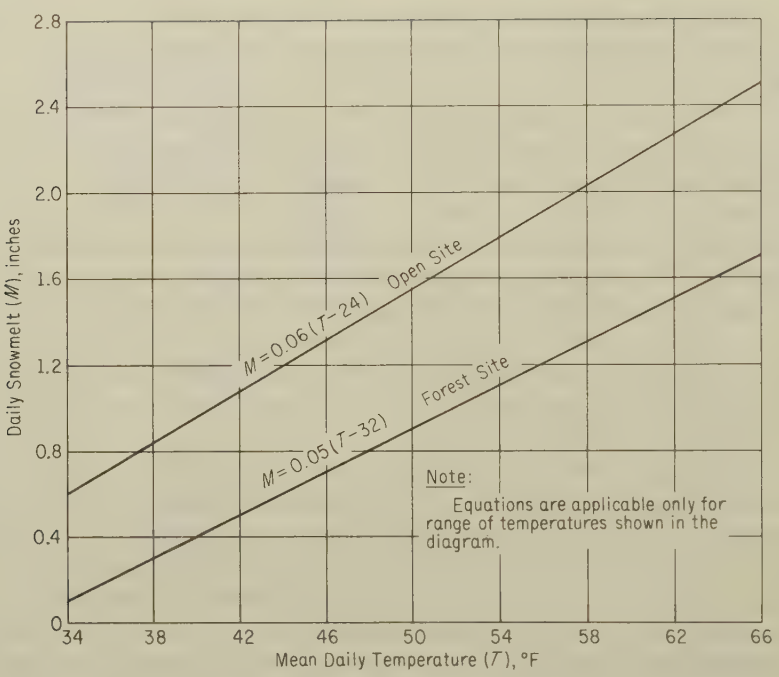


FIG. 11-9. Mean temperature index. (After U. S. Army Corps of Engineers, Ref. 16.)

A. Equations for periods with rainfall.

(1) For open (cover below 10 percent) or partly forested (cover from 10 to 60 percent) watersheds:

$$M = (0.029 + 0.0084kv + 0.007\,P_r)(T_a - 32) + 0.09 \qquad (11\text{-}53)$$

(2) For heavily forested areas (over 80 percent cover)

$$M = (0.074 + 0.007P_r)(T_a - 32) + 0.05 \qquad (11\text{-}54)$$

where M is the daily snowmelt (in./day), P_r the rainfall intensity (in./day), T_a the temperature of saturated air at 10-ft level (°F), v the average wind velocity at 50-ft level (mph) and k the basin constant which includes forest and topographic effects, and represents average exposure of the area to wind. Values decrease from 1.0 for clear plains areas to about 0.3 for dense forests.

B. Equations for rain-free periods.

(1) For heavily forested areas:

$$M = 0.074\,(0.53\,T'_a + 0.47\,T'_d) \qquad (11\text{-}55)$$

(2) For forested areas (cover of 60 to 80 percent)

$$M = k(0.0084v)(0.22T'_a + 0.78\,T'_d) + 0.029\,T_a \qquad (11\text{-}56)$$

(3) For partly forested areas

$$M = k'(1 - F)(0.0040I_i)(1 - a) + k(0.0084v)(0.22T'_a + 0.78T'_d) + F(0.029T'_a) \qquad (11\text{-}57)$$

(4) For open areas:

$$M = k'(0.00508I_i)(1 - a) + (1 - N)(0.0212\,T'_a - 0.84) + N(0.029\mathrm{T}'_c) + k(0.0084v)(0.22T'_a + 0.78T'_d) \qquad (11\text{-}58)$$

where M, v, and k are as previously described, T'_a is the difference between the 10-ft air snow surface (°F) temperatures; and T'_d is the difference between 10-ft dewpoint and snow-surface temperatures (°F), I_i is the observed or estimated insolation (langleys), a the observed or estimated mean snow surface albedo, k' the basin short-wave radiation melt factor (varies from 0.9 to 1.1) which is related to mean exposure of open areas compared to an unshielded horizontal surface, F the mean basin forest-canopy cover (decimal fraction), T'_c the difference between cloud-base and snow-surface temperatures (°F), and N the estimated cloud cover (decimal fraction).

Note that use of equations of the type given must be related to the areal extent of snow cover if realistic values are to be obtained. Present methods of determining this are not totally adequate.

The Water Budget

The water budget can be used to estimate snowmelt runoff from a watershed.[16] Such an approach has particular merit for areas where hydrometeorological records are short. Difficulty with the method is the usual lack of satisfactory data to properly quantify the various components. A hydrologic budget equation for the earth's surface (Eq. 1-2) can be written as

$$R = P - L - \Delta S \qquad (11\text{-}59)$$

where P is the gross precipitation, R the runoff, L the losses, and ΔS the change in storage. For snowmelt computations this equation is modified somewhat.

Gross precipitation for a given period P will now be defined as the sum of precipitations in the form of snow P_s and rain P_r, or

$$P = P_r + P_s \qquad (11\text{-}60)$$

This may also be written as

$$P = P_r + P_s = P_n + L_i \qquad (11\text{-}61)$$

where P_n = net precipitation
L_i = interception loss
A further refinement yields

$$P = P_{rn} + L_{ri} + P_{sn} + L_{si} \tag{11-62}$$

where P_{rn} and P_{sn} denote net rainfall and snowfall, L_{ri} and L_{si} the rain and snow interception respectively. Figure 11-10 indicates the nature of

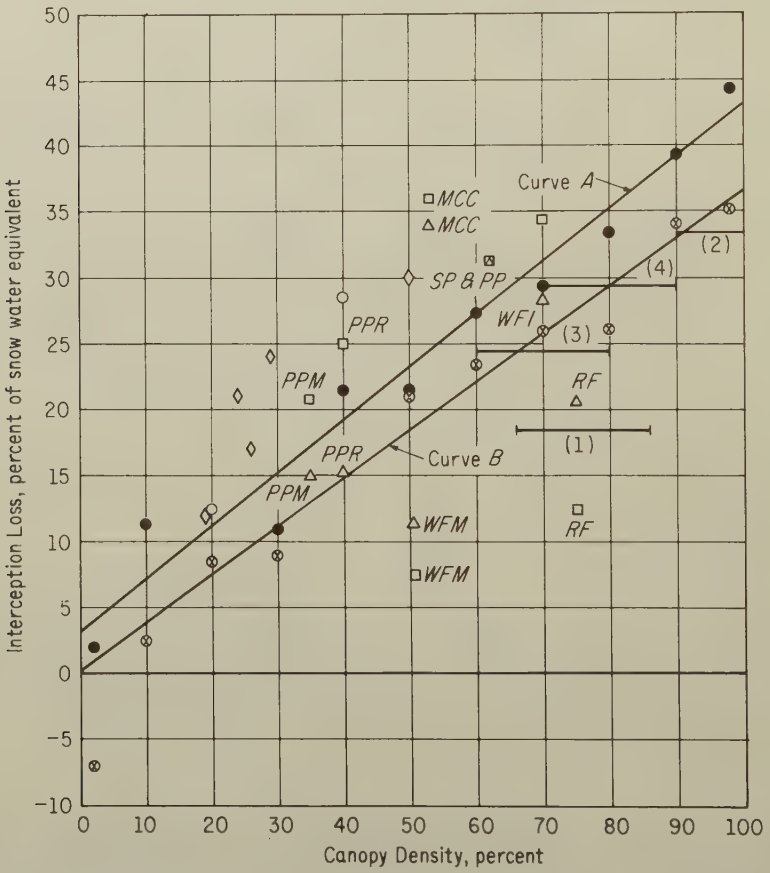


FIG. 11-10. Snowfall interception loss. (After U.S. Army Corps of Engineers, Ref. 16.)

snow interception by forested areas. Additional information on interception can be found in Chapter 2.

Total loss L will be

$$L = L_{si} + L_{ri} + L_e + Q_{sm} \tag{11-63}$$

where L_e is the evapotranspiration loss and Q_{sm} the change in available soil moisture. Storage term ΔS is then given as

$$\Delta S = (W_2 - W_1) - Q_g \tag{11-64}$$

where W_2 and W_1 are the final and initial water equivalents of the snowpack and Q_g represents ground and channel storage. Inserting values for P, L, and ΔS from Eqs. 11-62 through 11-64 in Eq. 11-59 gives

$$R = P_{rn} + L_{ri} + P_{sn} + L_{si} - L_{si} - L_{ri} - L_e - Q_{sm} - (W_2 - W_1) - Q_g \tag{11-65}$$

and canceling positive and negative values of L_{ri} and L_{si} produces

$$R = P_{rn} + P_{sn} - (W_2 - W_1) - Q_{sm} - Q_g - L_e \tag{11-66}$$

Expression $P_{sn} - (W_2 - W_1)$ represents snowmelt M and therefore

$$Q = P_{rn} + M - Q_{sm} - Q_g - L_e \tag{11-67}$$

If reliable estimates of the terms in Eq. 11-67 can be secured, basin discharge Q is computable.

Hydrograph Recessions

Recession curves have been discussed in Chapter 3 and take the general form

$$Q = Q_0 e^{-kt} \tag{11-68}$$

where Q is the discharge at time t, Q_0 the initial rate of flow, and k is a recession constant. Studies of daily streamflow by hydrographs permit evaluating the amount of runoff derived from snowmelt. The technique used is essentially one of separation of the daily hydrographs. Figure 11-11 (not to scale and oversimplified to illustrate the point) illustrates the

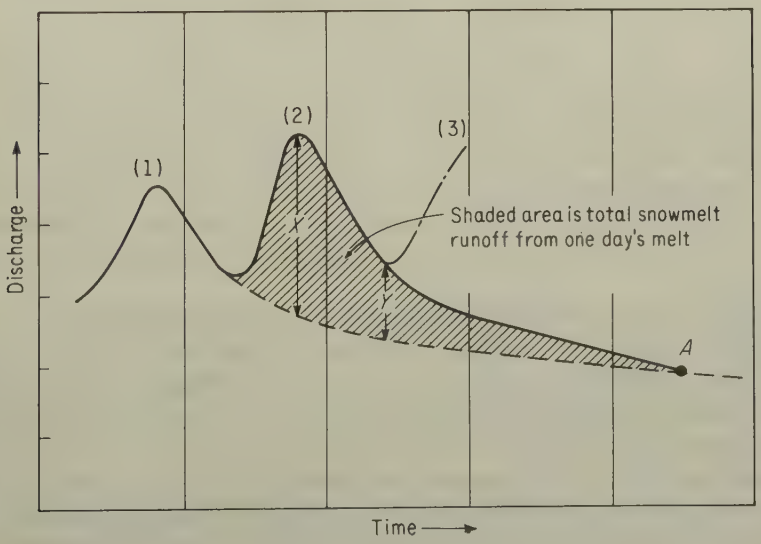


FIG. 11-11. Separation of a snowmelt hydrograph.

procedure. Assume the first, second, and succeeding peaks respectively fit snowmelt days. If the ultimate recession curve is extended backward in time, at a point A the recession curve from hydrograph 2 will intersect it. The area between recessions from hydrograph 1 and hydrograph 2 (shown cross-hatched) are the melt attributed to day 1. In like manner, a series of snowmelt hydrographs can be studied to determine their individual melt components. By observing such hydrograph features as the height to peak X, height to trough Y, and form of the recession, volumetric and rate forecasts of snowmelt runoff can be made. A more comprehensive treatment of this subject can be found in Ref. 20.

Hydrograph Synthesis

Syntheses of runoff hydrographs associated with snow hydrology are of two types. The first is a short-term forecast. The second kind is the development of flow distribution for a complete melt season or a particular rain-on-snow occurrence. Short-term forecasting is very helpful in preparing plans to operate reservoirs or other flow controls, while synthesis of particular storm-period hydrographs is basic to calculating design floods. To forecast a few days in advance, only the present state of a snowfield and streamflow need be known to make an estimate. For long-term forecasting, it is necessary to have reliable prediction of various meteorological parameters in addition to a knowledge of the initial conditions. Known historic parameters can be used for reconstructing historic flows while assumed or generated parameters satisfy design flood syntheses. Figure 11-12 displays some common hydrometeorological data.

In snowmelt hydrograph syntheses, several factors (not of great concern where only rainfall exists) must be carefully considered. First, a drainage basin with snow cover cannot be accepted as a homogeneous system, since areal extent of the blanket is highly important. Where only snowmelt flows are developed, the contributing area need not be the entire drainage—only that portion with snow cover. If rainfall occurs during the snow-cover period, contributions can come from bare areas while other expanses may produce combined runoff. The nature of losses in such cases may differ greatly for nonsnow overlayed and covered locations.

Altitude is an exceedingly pertinent factor in the hydrology of tracts subjected to snowfall. Rates of snowmelt decrease with elevation due to a general reduction in temperature with height. Orographic effects and the temperature-elevation relationships tend to raise the amount of precipitation with altitude. Greater snow-cover depth occurs due to increased precipitation and reduced melt rates. As a result, basinwide melt and cover-area increase with height as the snowline is approached, then diminish with elevation over the higher places normally completely snow

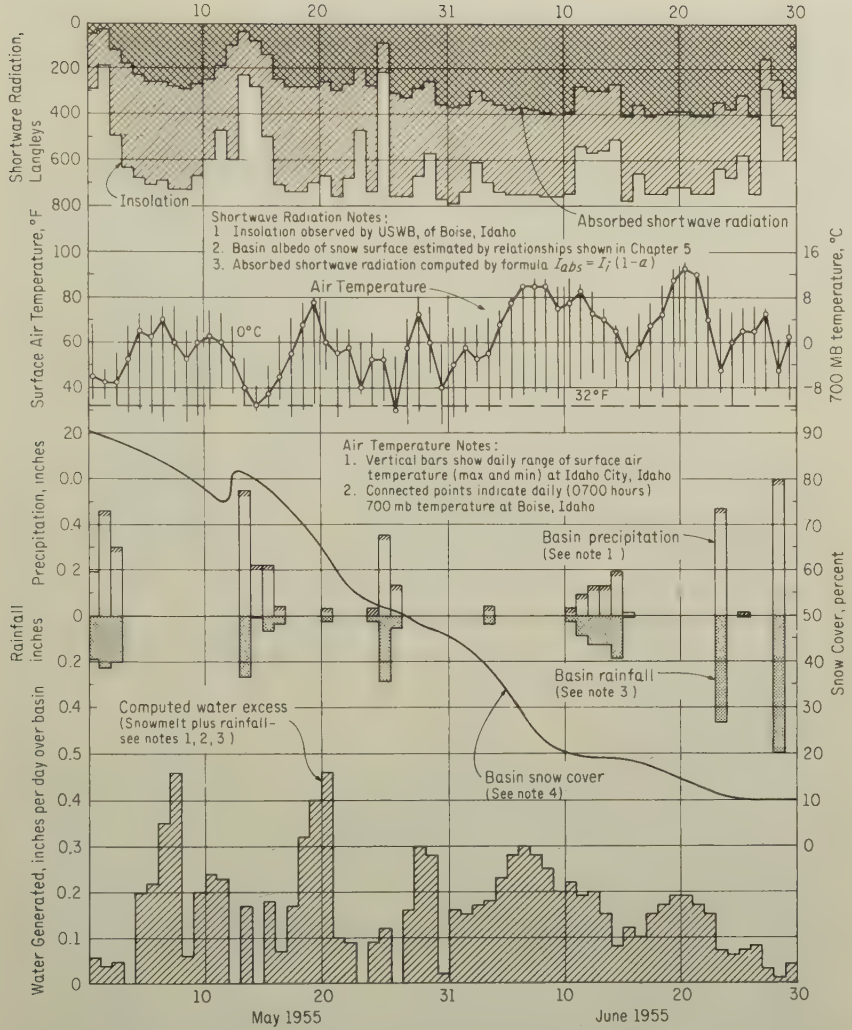

Shortwave Radiation, Langleys
Insolation
Absorbed shortwave radiation
Shortwave Radiation Notes:
1. Insolation observed by USWB, of Boise, Idaho
2. Basin albedo of snow surface estimated by relationships shown in Chapter 5
3. Absorbed shortwave radiation computed by formula $I_{abs} = I_i(1-\alpha)$
Surface Air Temperature, °F
Air Temperature
0°C
32°F
700 MB temperature, °C
Air Temperature Notes:
1. Vertical bars show daily range of surface air temperature (max and min) at Idaho City, Idaho
2. Connected points indicate daily (0700 hours) 700 mb temperature at Boise, Idaho
Rainfall inches
Precipitation, inches
Basin precipitation (See note 1)
Basin rainfall (See note 3)
Snow Cover, percent
Computed water excess (Snowmelt plus rainfall-see notes 1, 2, 3)
Basin snow cover (See note 4)
Water Generated, inches per day over basin
10 20 31 10 20 30
May 1955
June 1955

FIG. 11-12. Hydrometeorological data and computation of water generated. (After U. S. Army Corps of Engineers, Ref. 16.)

covered until late in the season. A snowpack exhibits another important trait in relation to rainstorms. In the spring, relatively little runoff occurs from snow-free regions compared with that from a snowfield for moderate rainfalls. During very cold weather, the situation during heavy rains is often reversed, since a dry snowpack can retain significant amounts of water.

Two basic approaches introduce elevation effects into procedures for hydrograph synthesis.[16] The first divides the basin into a series of eleva-

tion zones where snow depth, precipitation losses, and melt are assumed uniform. A second method considers the watershed as a unit, so adjustments are made to account for areal extent of snow cover, varying melt rates, precipitation, and other factors.

To synthesize a snowmelt hydrograph, information on precipitation losses, snowmelt, and the time distribution of runoff are needed. Snowmelt is generally estimated by index methods for forecasting but in design flood synthesis, the heat-budget approach is most used. Precipitation is determined from gaugings and historic or generated data. Losses are defined in two ways where snowmelt is involved. For rain-on-snow hydrographs all the water is considered a loss if delayed very long in reaching a stream. This is basically the concept of direct runoff employed in rainstorm hydrograph analyses. For hydrographs derived principally from snowmelt, only that part of the water which becomes evapotranspiration, or deep percolation, or permanently retained in the snowpack is considered to be lost. Assessing the time distribution of runoff from snow-covered areas is commonly done with unit hydrographs or storage-routing techniques. For rain-on-snow events, normal rainfall type unit hydrographs are applied while for distribution of strictly snowmelt excess, special "long-tailed" unit graphs are employed. Storage-routing techniques are widely exercised to synthesize spring snowmelt hydrographs, perhaps dividing them into several components and different representative storage times.

The time distribution of snowmelt runoff differs from that of rainstorms due mainly to large contrasts in rates of runoff generation. For flood flows associated with rainfall only, direct runoff is the prime concern, and time distribution of base flow is only approximated. Big errors in estimates of base flow are not generally of any practical significance where major rainstorm floods occur. In rainstorm flows, infiltrated water is treated as part of the base flow component and little effort is directed toward determining its time distribution when it appears as runoff. In using the unit-hydrograph approach to estimate snowmelt hydrographs it is customary to separate the surface and subsurface components and route them independently.

Storage routing has been used extensively for routing floods through reservoirs or river reaches. It is also applicable in preparing runoff hydrographs. In snowmelt-runoff estimates, rainfall and meltwater are treated as inputs to be routed through the basin, using storage times selected from the hydrologic characteristics of the watershed. Two basic hydrologic routing approaches are related to the assumption of (1) reservoir type storage, or (2) storage which is a function of inflow and outflow. These methods were treated in depth in Chapter 6.

Storage-routing techniques which separate runoff into surface and

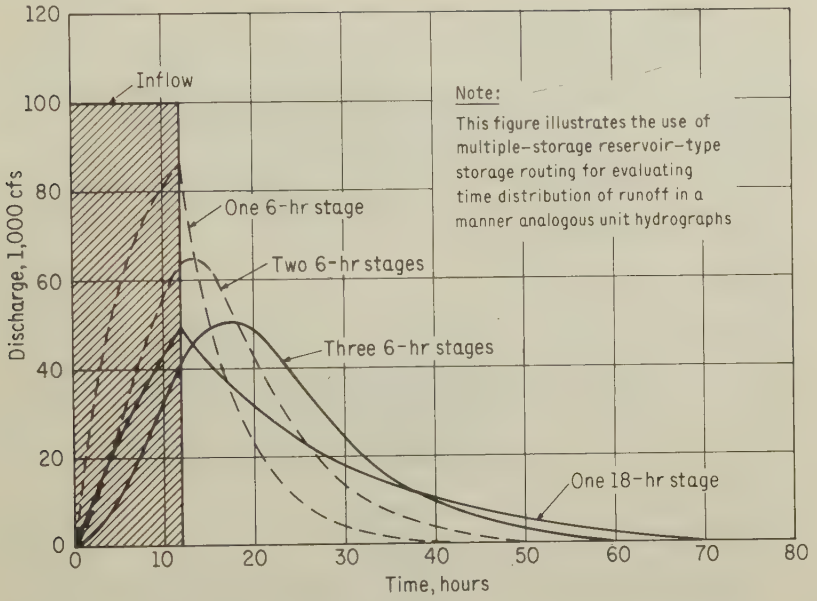


FIG. 11-13. Example of multiple-stage reservoir-type storage routing. (After U. S. Army Corps of Engineers, Ref. 16.)

groundwater components, assign different empirically derived storage times to each, and route them separately have been employed.[21] An additional system uses a multiple-storage reservoir-type storage scheduling.[16] In this method inflow is routed through two or more stages of storage successively. Figure 11-13 illustrates such an attack. Any desired travel time can be obtained by properly selecting storage time and number of stages. Retention times between steps may also be varied to reflect various basin hydrologic characteristics. Clark has suggested that use of single-stage routing after translating input in time permits computations to be simplified.[22]

The most practiced method for synthesizing snowmelt-runoff hydrographs has been the unit hydrograph. The character of snowmelt unit graphs differs primarily in time base length from that of rainstorm unit plots. As discussed in Chapter 3, rainstorm unit hydrographs often are derived from single isolated storm events. In snowmelt runoff, rates of water excess are small and approximately continuous. As a result, use of S-hydrographs is indicated.[16]

The S-hydrograph method has considerable utility since it allows: (1) adjustments to the derived unit hydrograph for nonuniform generation rates; (2) adjusting the observed time period to a desired interval; (3) ready adjustments of the area under the unit hydrograph to unit volume;

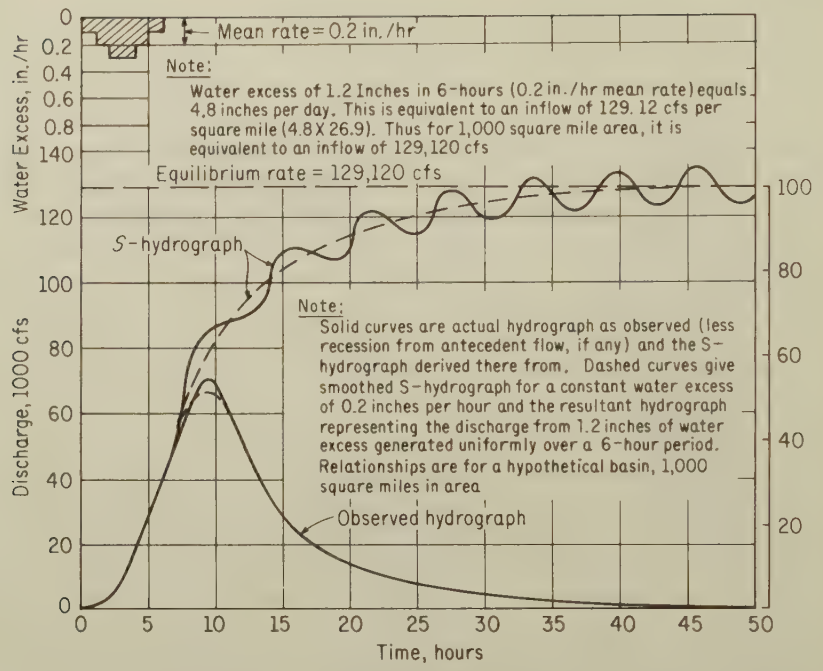

Mean rate = 0.2 in./hr
Note:
Water excess of 1.2 Inches in 6-hours (0.2 in./hr mean rate) equals 4.8 inches per day. This is equivalent to an inflow of 129.12 cfs per square mile (4.8 X 26.9). Thus for 1,000 square mile area, it is equivalent to an inflow of 129,120 cfs
Equilibrium rate = 129,120 cfs
S-hydrograph
Note:
Solid curves are actual hydrograph as observed (less recession from antecedent flow, if any) and the S-hydrograph derived there from. Dashed curves give smoothed S-hydrograph for a constant water excess of 0.2 inches per hour and the resultant hydrograph representing the discharge from 1.2 inches of water excess generated uniformly over a 6-hour period. Relationships are for a hypothetical basin, 1,000 square miles in area
Observed hydrograph
Water Excess, in./hr
Discharge, 1000 cfs
Time, hours

FIG. 11-14. S-hydrograph–unit-hydrograph relationships. (After U.S. Army Corps of Engineers, Ref. 16.)

(4) averaging several hydrographs to get a unit hydrograph; and (5) a particular unit hydrograph to be separated into two unit graphs of unequal generation periods (a particularly useful technique in snow hydrology). Figure 11-14 shows an example of the S-hydrograph method in adjusting for nonuniform generation rates of water excess.

Once a percentage S-hydrograph is derived, a unit hydrograph of any desired period can be obtained as indicated in Figure 11-15. Ordinates of the S-hydrograph are relative to the equilibrium rate associated with a specified water excess for the selected period. Choice of a "best period" for a snowmelt-unit hydrograph creates some problems. The rising limb is best defined by a relatively short period, whereas the long tail is correlated with a long interval. A method of choosing two points has been devised, one to represent the early periods of runoff and one to represent the later periods.[16] The procedure is illustrated on Fig. 11-16, although the recession time chosen is unrealistically shortened for purposes of explanation. Assume it is desirable to separate the S-hydrograph at point *D*. A horizontal line drawn through *D* cuts the original S-curve into two S-hydrographs, *CDB* and *ADE*. By choosing any desired time periods, unit hydrographs for each S-curve can be obtained by taking

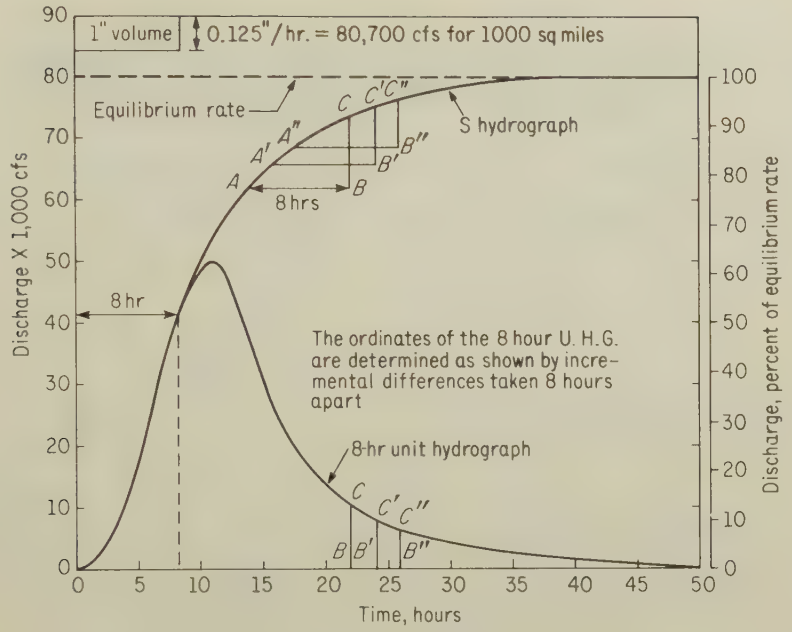

1" volume
0.125"/hr. = 80,700 cfs for 1000 sq miles
Equilibrium rate
S hydrograph
8 hrs
8 hr
The ordinates of the 8 hour U. H.G. are determined as shown by incremental differences taken 8 hours apart
8-hr unit hydrograph
Discharge X 1,000 cfs
Discharge, percent of equilibrium rate
Time, hours

FIG. 11-15. Derivation of unit hydrograph from S-hydrograph. (U. S. Army Corps of Engineers, 1956, Ref. 16.)

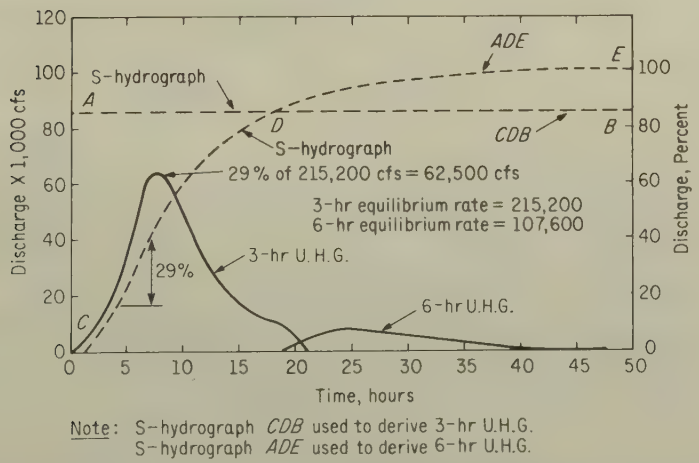


FIG. 11-16. Derivation of unit hydrographs having different periods from a divided S-hydrograph. (U. S. Army Corps of Engineers, 1956, Ref. 16.)

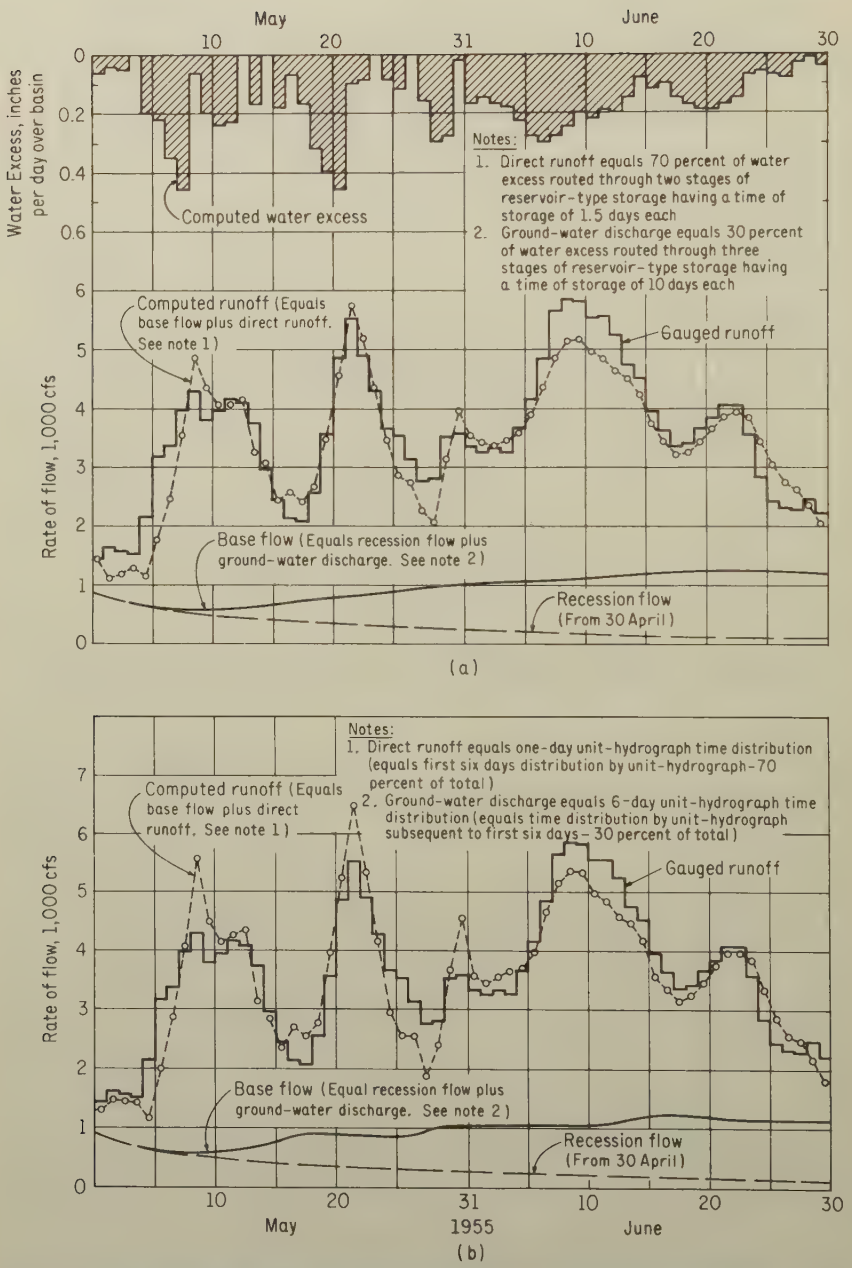


FIG. 11-17. Reconstitution of seasonal hydrograph of mean daily flows; (a) storage routing method; and (b) unit-hydrograph method. (After U.S. Army Corps of Engineers, Ref. 16.)

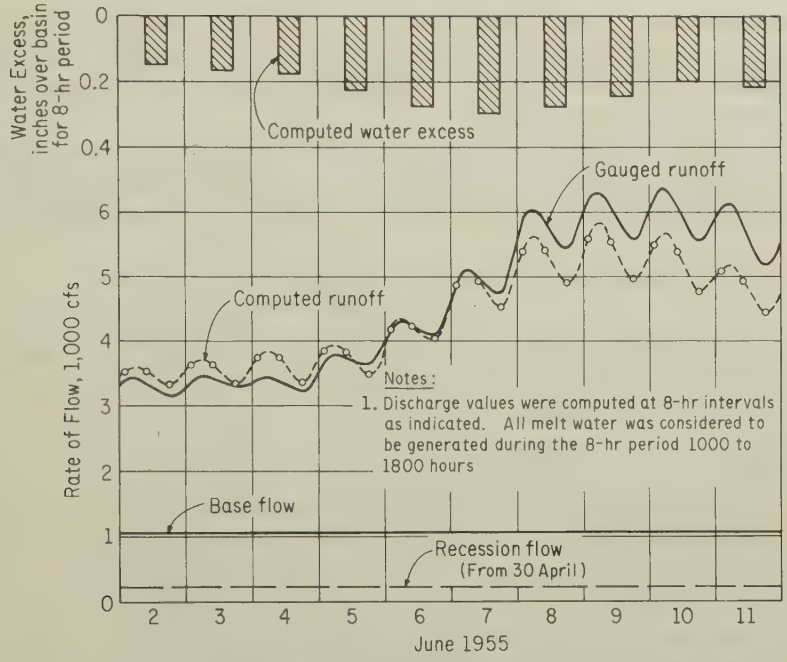


FIG. 11-18. Reconstitution of hydrograph showing diurnal flow variation. (After U. S. Army Corps of Engineers. Ref. 16.)

incremental differences and multiplying them by the applicable conversion factors. The figure describes the procedure covering a 3-hr period for curve *CDB* and a 6-hr period for curve *ADE*.

Figures 11-17 and 11-18 illustrate the reconstitution of several hydrographs using methods discussed in this section.

Runoff Forecasting

Dependable seasonal forecasts are essential to many aspects of water resources management and planning. In particular, they are indispensable to adequately planning multiple-purpose reservoirs where the various uses generally are in competition and sometimes not compatible. The increased emphasis being placed on water management for quality control also supports the need for better techniques in predicting seasonal flows.

Sparsity of data and inability to properly evaluate controlling factors in runoff processes are contributing causes to limitations on forecasting methods in many cases. Nevertheless, satisfactory forecasts can be made if proper attention is given to the significant variables. When factors of importance are reliably determined at the time of the prophecy, most errors result from conditions which set in after observations were made.

Subsequent events may have little effect on forecasts in some basins where approximate divergences can be fairly well quantified. Note also that an acceptable magnitude of error will vary with the circumstances for which the prediction is made.

Principal forecasting tools are the water budget, index methods, and combinations of the two. The water budget has already been discussed in detail, as have a number of snowmelt methods which usually involve

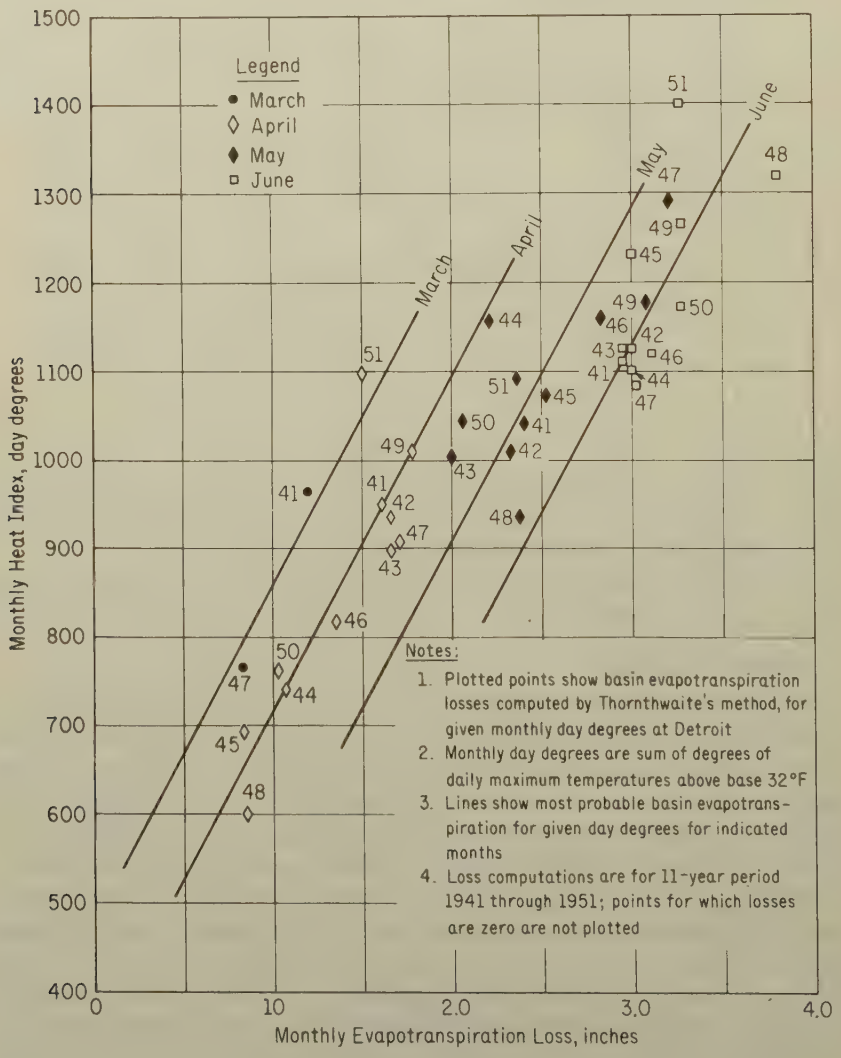


FIG. 11-19. Loss by evapotranspiration. (After U. S. Army Corps of Engineers, Ref. 16.)

correlation of specific variables with historic runoff records. Indexes represent factors such as precipitation, water equivalent, soil moisture, groundwater, and evapotranspiration. Figure 11-19 plots an evapotranspiration index. Reliability of the water-balance and index methods is fixed in large measure by the amount and adequacy of available hydrologic data.[19] Final proof of the method is its ability to accurately model the simulated system. Adequate forecasting by index methods normally requires records for approximately 25 yr.

Digital Simulation in Snow Hydrology

Since about 1960, Linsley, Crawford, and others have spent considerable time developing simulation models for watershed hydrology known as the Stanford Watershed Models.[7] Anderson, Crawford, and Linsley have evolved special subroutines for synthesizing snowmelt-runoff hydrographs for use in these models.[8] Their work will be briefly described to demonstrate the vital role played by the computer in furthering lengthy and detailed calculations of extensive snowmelt hydrographs.

Several models have been produced. One makes use of theoretical considerations in finding the various components of snowmelt. Condensation melt is calculated using Eq. 11-29; evaporation computed by Eq. 11-31; convection melt based on Eq. 11-28; and radiation melt tied to Eq. 11-33. Data requirements of the developed subroutine are highly

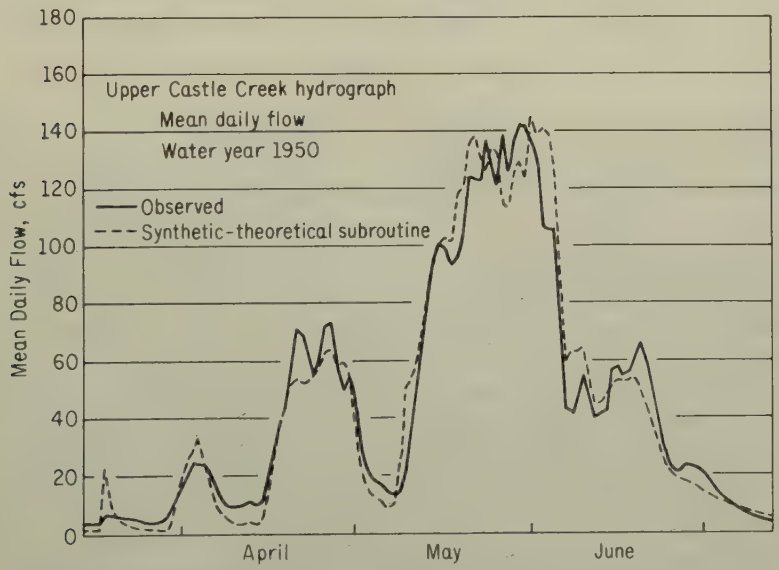


FIG. 11-20. Upper Castle Creek hydrograph. (After Anderson and Crawford, Ref. 8.)

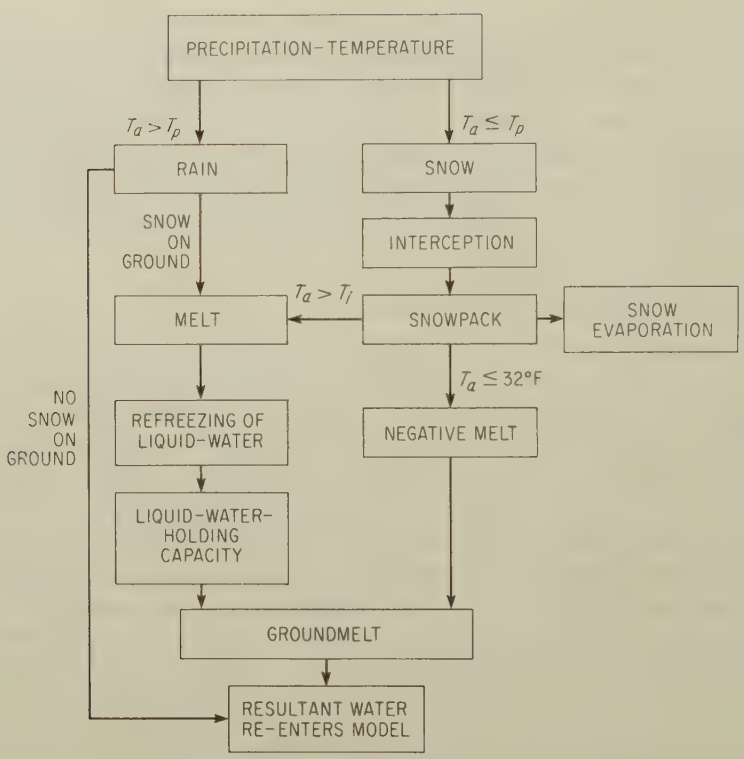


FIG. 11-21. Flow chart of snowmelt subroutine. (After Anderson and Crawford, Ref. 8.)

exacting and so have greatest utility on well-instrumented watersheds. Estimates of observed runoff compare reasonably well with observed flows for the years listed. Figure 11-20 shows the results of a computer run for 1950. A flow chart for the snowmelt routine is given in Fig. 11-21.

Many basins of hydrologic interest are not extensively or completely instrumented. Such conditions require adequate estimating techniques incorporating data to be prepared. With this in mind, Anderson et al. designed a subroutine to make use of temperature data only.[8] It was assumed that differentials measured between air and snow-surface temperature could be used. Reduced accuracy was noted in the comparison of outflow computations obtained by this method with those found through the more basic approach already discussed. Inability of the temperature differential to adequately reflect radiation melt was considered largely responsible.

A somewhat more sophisticated model was produced to include radi-

ation measurements. This snowmelt subroutine utilizes maximum and minimum daily temperatures, short-wave radiation (measured or estimated), and data on snow evaporation and precipitation. Calculations are on an hourly basis as precipitation adds to the snowpack or contributes to liquid water storage. Hourly net heat exchanges are calculated from temperature and radiation data. Heat exchange may be either positive or negative, the latter values indicating decreases in temperature of the snowpack which can be modeled by including a negative heat-storage term. As the snowpack warms, negative heat storage decreases until temperature of the snowpack reaches 32°F. Thereafter, positive heat initiates melt. When melt alone, or plus rainfall, exceeds the water-holding capacity of a snowpack, runoff can begin (see Sec. 11-2).

If the dewpoint temperature drops below freezing, evaporation from the snowpack can be reckoned from estimated potential values. Reflected short-wave radiation is continuously calculated to get variations in the amount of absorbed solar radiation. Areal extent of snow cover in various different zones is also constantly predicted. These zones are chosen on the basis of elevation and exposure.

Observed maximum and minimum temperatures are used to calculate hourly values, whereas changes with elevation are based on lapse-rate models. The amount of liquid water stored in a snowpack is limited by using an input parameter WC which restricts the quantity to a function of the snowpack supply. For the usual density range of snow, this factor is considered to be constant. Density of new snow at temperatures above 0°F is presumed to be related to temperature by

$$\text{DNS} = \text{INDS} + (T/100)^2 \tag{11-69}$$

where DNS = new snow density, INDS = snow density at or below datum temperature of 0°F, and T = temperature. Where snow density < 0.6, hourly decreases in depth are computed by multiplying the prevailing depth by a correction factor.

Where the form of precipitation is not shown by records, an assumption made is that for temperatures below freezing at an elevation 750 ft above a zone, precipitation is in the form of snow. Reliability of such a supposition is affected by the method of hourly temperature determination used. When temperatures are close to freezing, Linsley suggests that snow-depth observations be used to check the assumption. Solar radiation data are taken from daily observations and can be estimated. The gross amount of insolation is modified to include reflection losses, forest cover, slope, and exposure. A method of estimating net longwave radiation exchange based on studies of the U.S. Corps of Engineers is employed.[14] The equation for net long-wave radiation in Langleys per hour

is

$$\text{LW} = 27.5(1.0 - F)(0.76(T_r/273.0)^4 - 1.0) + 27.5(F)((T_r/273.0)^4 - 1.0) \quad (11\text{-}70)$$

where F denotes the fraction of the area in forest cover and T_r is absolute temperature (°R). Hourly long- and short-wave radiation are combined to produce total net radiation. The hourly radiation melt in inches is thus

$$M_r = \frac{H_m}{203.2} \quad (11\text{-}71)$$

Convective heat exchange is indexed to predict convective melt. The resulting expression is

$$M_c = \text{CONMELT}\ (T - 32) \quad (11\text{-}72)$$

in which M_c = convective melt (in.), T = temperature (°F), and CONMELT is a convection-condensation melt parameter. Since dewpoint data are not commonly available, condensation melt is not estimated separately. Parameter CONMELT includes condensation effects, however, when forecast from simulation runs. Rainfall snowmelt in inches is appraised from

$$M_p = \frac{(T - 32)PX}{144} \quad (11\text{-}73)$$

where PX is the rainfall in inches and T is the temperature (°F). Groundmelt is determined by a parameter DGM, which represents the water equivalent of daily melt in various zones whose elevations are reflected in the model as well as areal coverage of the snowpack. Where negative values result from Eqs. 11-71 or 11-72, the negative heat storage is increased.

Snowmelt simulation indicated in the preceding paragraphs is based on the availability of only limited data and its accuracy is affected accordingly. Where more complete facts are available, estimates of snowmelt can be improved and a better simulation model results. A complete discussion is given in Refs. 7 and 8.

References

1. Chow, Ven Te (ed.), *Handbook of Applied Hydrology*. New York: McGraw-Hill Book Company, 1964.

2. Goodell, B. C., "Snowpack Management for Optimum Water Benefits," Conference Preprint 379, ASCE Water Resources Engineering Conference, Denver, Colo., May 1966.

3. Butler, S. S., *Engineering Hydrology*. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957.

4. Yen, Y. C., "Heat Transfer by Vapor Transfer in Ventilated Snow," *J. Geophys. Res.*, Vol. 68 No. 4 (Feb. 15, 1963).

5. Hobbs, P. V., "The Effect of Time of the Physical Properties of Deposited Snow," *J. Geophys. Res.*, Vol. 70 No. 16 (Aug. 15, 1966).

6. Martinelli, M., Jr., "Some Hydrologic Aspects of Alpine Snowfields under Summer Conditions," *J. Geophys. Res.*, Vol. 64 No. 4 (April 1959).

7. Crawford, N. H., and Linsley, Jr., R. K., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Department of Civil Engineering, Stanford University, Technical Report No. 39, July 1966.

8. Anderson, E. A., and N. H. Crawford, "The Synthesis of Continuous Snowmelt Runoff Hydrographs on a Digital Computer," Department of Civil Engineering, Stanford University, Technical Report No. 36, June 1964.

9. Horton, R. E., "Phenomena of the Contact Zone between the Ground Surface and a Layer of Melting Snow," Transactions of Meetings of International Commission of Snow and Glaciers, Edinburgh, September 1936, International Association of Hydrology Bulletin 23.

10. Light, P., "Analysis of High Rates of Snow Melting," *Trans. Am. Geophys. Union*, Vol. 22, Part 1 (1941).

11. Wilson, W. T., "An Outline of the Thermodynamics of Snowmelt," *Trans. Am. Geophys. Union*, Vol. 22, Part 1 (1941).

12. Kittredge, J., *Forest Influences*. New York: McGraw-Hill Book Company, 1948.

13. Mantis, H. T., and others, "Review of the Properties of Snow and Ice," U.S. Army Corps of Engineers, Snow, Ice, and Permafrost Research Establishment, SIPRE Report 4, July 1951.

14. "Runoff From Snowmelt," U.S. Army Corps of Engineers, Engineering and Design Manuals, EM 1110-2-1406, January 1960.

15. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, *Hydrology for Engineers*. New York: McGraw-Hill Book Company, 1958.

16. "Snow Hydrology," North Pacific Division, Corps of Engineers, U..S. Army Corps of Engineers, Portland, Ore., June 30, 1956.

17. The Johns Hopkins University, Laboratory of Climatology, "Micrometeorology of the Surface Layer of the Atmosphere; the Flux of Momentum, Heat, and Water Vapor, Final Report," *Publications in Climatology*, Vol. 7 No. 2 (1954).

18. Sverdrup, H. U., "The Eddy Conductivity of the Air Over a Smooth Snow Field," *Geofys. Publik.*, Vol. 11 No. 7 (1936).

19. Quick, M. C., "River Flood Flows: Forecasts and Probabilities," Journal of the Hydraulics Division, *Proc. A.S.C.E.*, Vol. 91 No. HY3 (May 1965).

20. Gartska, W. U., L. D. Love, B. C. Goodell, and F. A. Bertle, "Factors Affecting Snowmelt and Streamflow," U.S. Bureau of Reclamation and U.S. Forest Service, 1958.

21. Zimmerman, A. L., "Reconstruction of the Snow-Melt Hydrograph in

the Payette River Basin," Proceedings of Western Snow Conference, Portland, Oreg., April 1955.

22. Clark, C. O., "Storage and the Unit Hydrograph," *Trans. A.S.C.E.*, Vol. 110 (1945).

Problems

11-1. Given a snowpack with thermal quality of 0.93, determine the snowmelt in inches if total heat input is 137 langleys.

11-2. Use Eq. 11-53 to estimate snowmelt at an elevation of 3,000 ft in a partly forested area if rainfall intensity = 0.3 in./day, wind velocity = 25 mph, and temperature of saturated air = 42°F.

11-3. Use the data of Prob. 11-2 and solve if the forest cover is dense.

APPENDIX A

Matrix Definition

A matrix is a set of quantities arranged in a rectangular array of m rows and n columns. A matrix can be expressed in the following manner:

$$P = [P_{ij}] = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mn} \end{bmatrix}$$

P stands for the entire array of quantities whereas the elements of the matrix are denoted by P_{ij}. A rectangular matrix of m rows and n columns is said to be of order $m \times n$, and if $m = n$, the matrix is square.

FUNDAMENTAL OPERATIONS

Transpose of a Matrix

The transpose P^T of a matrix P is found by simply interchanging rows for columns in the matrix P.

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \qquad P^T = \begin{bmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \\ P_{13} & P_{23} \end{bmatrix}$$

Determinant of a Matrix

The determinant of the elements of a square matrix P is called the determinant of the matrix P and is represented by $|P|$.

Cofactor

The cofactor of an element in the ith row and jth column of a square matrix P is the determinant formed by considering the matrix as a deter-

minant, suppressing the ith row and the jth column, and multiplying by $(-1)^{i+j}$.

The Adjoint Matrix

The adjoint matrix of a square matrix P is formed by replacing each element by its cofactor and transposing. It is denoted by adj P.

Equality of Matrices

Two matrices P and Z are equal if and only if they are the same order and all their corresponding elements are equal; that is, $P = Z$ provided that P and Z have the same order and $P_{ij} = Z_{ij}$.

Matrix Multiplication

Matrix multiplication has been defined in such a manner that operations involving linear transformations can be concisely expressed. The definition of multiplication states that two matrices can be multiplied together only when the number of columns of the first is equal to the number of rows of the second. Matrices satisfying this condition are termed conformable matrices. If two matrices P and U are conformable, their product is defined by the equation $PU = Q$, where

$$Q_{ij} = \sum_{k=1}^{P} P_{ik} U_{kj}$$

and the orders of P, U, and Q are $m \times p$, $p \times n$, and $m \times n$, respectively. As an example of the definition, consider the matrices

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \qquad U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix}$$

Here P is a matrix of order 3×3 and U of order 3×2, hence these matrices are comformable. The rule above gives the following product matrix, of order 3×2.

$$Q = PU = \begin{bmatrix} (p_{11}u_{11} + p_{12}u_{21} + p_{13}u_{31}) & (p_{11}u_{12} + p_{12}u_{22} + p_{13}u_{32}) \\ (p_{21}u_{11} + p_{22}u_{21} + p_{23}u_{31}) & (p_{21}u_{12} + p_{22}u_{22} + p_{23}u_{32}) \\ (p_{31}u_{11} + p_{32}u_{21} + p_{33}u_{31}) & (p_{31}u_{12} + p_{32}u_{22} + p_{33}u_{32}) \end{bmatrix}$$

In general, if a matrix of order $m \times p$ is multiplied by one of order $p \times n$, the product is a matrix of order $m \times n$.

The Inverse Matrix

If the determinant P of a square matrix P does not vanish, P is said to be *nonsingular*; it then possesses an inverse matrix P^{-1} such that $PP^{-1} = I = P^{-1}P$, where I is the unit matrix of the same order as P. The inverse matrix P^{-1} of the matrix P is given by

$$P^{-1} = \frac{\text{adj } P}{|P|}$$

Solution of a System of Linear Equations

The Gauss-Jordan elimination method is the best general procedure available for solving a system of linear equations $PU = Q$, where P is an arbitrary $n \times n$ matrix, U an arbitrary $n \times 1$ matrix, and Q the $n \times 1$ solution vector. The method consists of performing elementary transformations (adding or subtracting a multiple of a row to some other row, interchanging two rows, or multiplying a row by a constant) on the $n \times n + 1$ augmented matrix P, U until P has been reduced to a unit matrix, when U will become $P^{-1}U$. If matrix P is ill-conditioned (elements along the principal diagonal are less than the other matrix elements) it is highly desirable to choose the so-called *pivotal elements* to be the largest in magnitude columnwise in order to avoid large roundoff errors.

The technique is best explained by the following example.

$$\begin{aligned} u_1 - 2u_2 + 3u_3 &= 5 \\ 2u_1 + u_2 + u_3 &= 0 \\ 3u_1 + u_2 + u_3 &= 0 \end{aligned}$$

Steps involved in finding the solution are as follows.

$$\begin{bmatrix} 1 & -2 & 3 & 5 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{bmatrix}$$

Original Matrix.

First pivot chosen.

$$\begin{bmatrix} 3 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & -2 & 3 & 5 \end{bmatrix}$$

Rows 1 and 3 interchanged.

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 0 \\ 0 & -7/3 & 8/3 & 5 \end{bmatrix}$$

Row 1 divided by first pivot.

Rows 2 and 3 modified by row 1.

Second pivot chosen.

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 0 \\ 0 & -7/3 & 8/3 & 5 \\ 0 & 1/3 & 1/3 & 0 \end{bmatrix}$$

Rows 2 and 3 are interchanged.

$$\begin{bmatrix} 1 & 0 & 5/7 & 5/7 \\ 0 & 1 & -8/7 & -15/7 \\ 0 & 0 & 5/7 & 5/7 \end{bmatrix}$$

Row 2 divided by second pivot.

Rows 1 and 3 modified by row 2.

Third pivot chosen.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Row 3 divided by third pivot.

Rows 1 and 2 modified by row 3.

The solution vector after this final step is simply the last column on the right:

$$u_1 = 0, \quad u_2 = -1, \quad u_3 = 1$$

Error

An estimate of the error in computing P^{-1} can be found by computing the inverse of P^{-1} and expressing it as follows:

$$(P^{-1})^{-1} = P + 2E$$

where $E = e_{ij}$ is the error matrix for P^{-1}.

APPENDIX B

TABLE B-1 Water Properties, Constants, and Conversion Factors

Gas Constants (R)

$R = 0.0821$ (atm.)(liter)/(g-mole)(°K)
$R = 1.987$ g-cal/(g-mole)(°K)
$R = 1.987$ Btu/(lb-mole)(°R)

Acceleration of Gravity (Standard)

$g = 32.17$ ft/sec^2 $= 980.6$ cm/sec^2

Heat of Fusion of Water

79.7 cal/g = 144 Btu/lb

Heat of Vaporization of Water at 1.0 atm

540 cal/g = 970 Btu/lb

Specific Heat of Air

$C_P = 0.238$ cal/(g)(°C)

Density of Dry Air @ 0°C and 760 mm

0.001293 g/cu cm

Conversion Factors

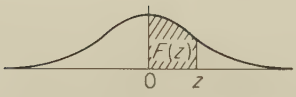
1 second-foot-day per square mile = 0.03719 inch
1 inch of runoff per square mile = 26.9 second-foot-days
= 53.3 acre-feet
= 2,323,200 cubic feet
1 cubic foot per second = 0.9917 acre-inch per hour
1 horsepower = 0.746 kilowatt
= 550 foot-pounds per second
$e = 2.71828$
$\log_{10} e = 0.43429$
$\log_e 10 = 2.30259$

Properties of Water

Temp., °F	Specific gravity	Unit weight, lb/ft^3	Heat of vaporization, Btu/lb	Kinematic viscosity, sq ft/sec	Vapor pressure		
					Mb	Psi	in. Hg
32	0.99987	62.416	1073	1.93×10^{-5}	6.11	0.09	0.18
40	0.99999	62.423	1066	1.67×10^{-5}	8.36	0.12	0.25
50	0.99975	62.408	1059	1.41×10^{-5}	12.19	0.18	0.36
60	0.99907	62.366	1054	1.21×10^{-5}	17.51	0.26	0.52
70	0.99802	62.300	1049	1.06×10^{-5}	24.79	0.36	0.74
80	0.99669	62.217	1044	0.929×10^{-5}	34.61	0.51	1.03
90	0.99510	62.118	1039	0.828×10^{-5}	47.68	0.70	1.42
100	0.99318	61.998	1033	0.741×10^{-5}	64.88	0.95	1.94

APPENDIX C

TABLE C-1



Areas under the Normal Curve

$$F(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0159	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4430	.4441
1.6	.4452	.4463	.4474	.4485	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4762	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4980	.4980	.4981
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998
⋮	⋮									
4.0	.499968									

After A. M. Neville and J. B. Kennedy, *Basic Statistical Methods for Engineers and Scientists* (Ref. 25).

TABLE C-2 K Values for Pearson Type III Distribution

Skew coeff., C_s	Recurrence interval in years: 1.0101	1.0526	1.1111	1.2500	2	5	10	25	50	100	200
	Percent chance: 99	95	90	80	50	20	10	4	2	1	0.5
					Positive Skew						
3.0	−0.667	−0.665	−0.660	−0.636	−0.396	0.420	1.180	2.278	3.152	4.051	4.970
2.9	−0.690	−0.688	−0.681	−0.651	−0.390	0.440	1.195	2.277	3.134	4.013	4.909
2.8	−0.714	−0.711	−0.702	−0.666	−0.384	0.460	1.210	2.275	3.114	3.973	4.847
2.7	−0.740	−0.736	−0.724	−0.681	−0.376	0.479	1.224	2.272	3.093	3.932	4.783
2.6	−0.769	−0.762	−0.747	−0.696	−0.368	0.499	1.238	2.267	3.071	3.889	4.718
2.5	−0.799	−0.790	−0.771	−0.711	−0.360	0.518	1.250	2.262	3.048	3.845	4.652
2.4	−0.832	−0.819	−0.795	−0.725	−0.351	0.537	1.262	2.256	3.023	3.800	4.584
2.3	−0.867	−0.850	−0.819	−0.739	−0.341	0.555	1.274	2.248	2.997	3.753	4.515
2.2	−0.905	−0.882	−0.844	−0.752	−0.330	0.574	1.284	2.240	2.970	3.705	4.444
2.1	−0.946	−0.914	−0.869	−0.765	−0.319	0.592	1.294	2.230	2.942	3.656	4.372
2.0	−0.990	−0.949	−0.895	−0.777	−0.307	0.609	1.302	2.219	2.912	3.605	4.298
1.9	−1.037	−0.984	−0.920	−0.788	−0.294	0.627	1.310	2.207	2.881	3.553	4.223
1.8	−1.087	−1.020	−0.945	−0.799	−0.282	0.643	1.318	2.193	2.848	3.499	4.147
1.7	−1.140	−1.056	−0.970	−0.808	−0.268	0.660	1.324	2.179	2.815	3.444	4.069
1.6	−1.197	−1.093	−0.994	−0.817	−0.254	0.675	1.329	2.163	2.780	3.388	3.990
1.5	−1.256	−1.131	−1.018	−0.825	−0.240	0.690	1.333	2.146	2.743	3.330	3.910
1.4	−1.318	−1.168	−1.041	−0.832	−0.225	0.705	1.337	2.128	2.706	3.271	3.828
1.3	−1.383	−1.206	−1.064	−0.838	−0.210	0.719	1.339	2.108	2.666	3.211	3.745
1.2	−1.449	−1.243	−1.086	−0.844	−0.195	0.732	1.340	2.087	2.626	3.149	3.661
1.1	−1.518	−1.280	−1.107	−0.848	−0.180	0.745	1.341	2.066	2.585	3.087	3.575
1.0	−1.588	−1.317	−1.128	−0.852	−0.164	0.758	1.340	2.043	2.542	3.022	3.489
.9	−1.660	−1.353	−1.147	−0.854	−0.148	0.769	1.339	2.018	2.498	2.957	3.401
.8	−1.733	−1.388	−1.166	−0.856	−0.132	0.780	1.336	1.993	2.453	2.891	3.312
.7	−1.806	−1.423	−1.183	−0.857	−0.116	0.790	1.333	1.967	2.407	2.824	3.223
.6	−1.880	−1.458	−1.200	−0.857	−0.099	0.800	1.328	1.939	2.359	2.755	3.132
.5	−1.955	−1.491	−1.216	−0.856	−0.083	0.808	1.323	1.910	2.311	2.686	3.041
.4	−2.029	−1.524	−1.231	−0.855	−0.066	0.816	1.317	1.880	2.261	2.615	2.949
.3	−2.104	−1.555	−1.245	−0.853	−0.050	0.824	1.309	1.849	2.211	2.544	2.856
.2	−2.178	−1.586	−1.258	−0.850	−0.033	0.830	1.301	1.818	2.159	2.472	2.763
.1	−2.252	−1.616	−1.270	−0.846	−0.017	0.836	1.292	1.785	2.107	2.400	2.670
.0	−2.326	−1.645	−1.282	−0.842	0	0.842	1.282	1.751	2.054	2.326	2.576
					Negative Skew						
− .1	−2.400	−1.673	−1.292	−0.836	0.017	0.846	1.270	1.716	2.000	2.252	2.482
− .2	−2.472	−1.700	−1.301	−0.830	0.033	0.850	1.258	1.680	1.945	2.178	2.388
− .3	−2.544	−1.726	−1.309	−0.824	0.050	0.853	1.245	1.643	1.890	2.104	2.294
− .4	−2.615	−1.750	−1.317	−0.816	0.066	0.855	1.231	1.606	1.834	2.029	2.201
− .5	−2.686	−1.774	−1.323	−0.808	0.083	0.856	1.216	1.567	1.777	1.955	2.108
− .6	−2.755	−1.797	−1.328	−0.800	0.099	0.857	1.200	1.528	1.720	1.880	2.016
− .7	−2.824	−1.819	−1.333	−0.790	0.116	0.857	1.183	1.488	1.663	1.806	1.926
− .8	−2.891	−1.839	−1.336	−0.780	0.132	0.856	1.166	1.448	1.606	1.733	1.837
− .9	−2.957	−1.858	−1.339	−0.769	0.148	0.854	1.147	1.407	1.549	1.660	1.749
−1.0	−3.022	−1.877	−1.340	−0.758	0.164	0.852	1.128	1.366	1.492	1.588	1.664
−1.1	−3.087	−1.894	−1.341	−0.745	0.180	0.848	1.107	1.324	1.435	1.518	1.581
−1.2	−3.149	−1.910	−1.340	−0.732	0.195	0.844	1.086	1.282	1.379	1.449	1.501
−1.3	−3.211	−1.925	−1.339	−0.719	0.210	0.838	1.064	1.240	1.324	1.383	1.424
−1.4	−3.271	−1.938	−1.337	−0.705	0.225	0.832	1.041	1.198	1.270	1.318	1.351
−1.5	−3.330	−1.951	−1.333	−0.690	0.240	0.825	1.018	1.157	1.217	1.256	1.282
−1.6	−3.388	−1.962	−1.329	−0.675	0.254	0.817	0.994	1.116	1.166	1.197	1.216

(*Continued on next page*)

TABLE C-2 (continued)

Skew coeff., C_s	Recurrence interval in years: 1.0101	1.0526	1.1111	1.2500	2	5	10	25	50	100	200
	Percent chance: 99	95	90	80	50	20	10	4	2	1	0.5
					Negative Skew						
−1.7	−3.444	−1.972	−1.324	−0.660	0.268	0.808	0.970	1.075	1.116	1.140	1.155
−1.8	−3.499	−1.981	−1.318	−0.643	0.282	0.799	0.945	1.035	1.069	1.087	1.097
−1.9	−3.553	−1.989	−1.310	−0.627	0.294	0.788	0.920	0.996	1.023	1.037	1.044
−2.0	−3.605	−1.996	−1.302	−0.609	0.307	0.777	0.895	0.959	0.980	0.990	0.995
−2.1	−3.656	−2.001	−1.294	−0.592	0.319	0.765	0.869	0.923	0.939	0.946	0.949
−2.2	−3.705	−2.006	−1.284	−0.574	0.330	0.752	0.844	0.888	0.900	0.905	0.907
−2.3	−3.753	−2.009	−1.274	−0.555	0.341	0.739	0.819	0.855	0.864	0.867	0.869
−2.4	−3.800	−2.011	−1.262	−0.537	0.351	0.725	0.795	0.823	0.830	0.832	0.833
−2.5	−3.845	−2.012	−1.250	−0.518	0.360	0.711	0.771	0.793	0.798	0.799	0.800
−2.6	−3.889	−2.013	−1.238	−0.499	0.368	0.696	0.747	0.764	0.768	0.769	0.769
−2.7	−3.932	−2.012	−1.224	−0.479	0.376	0.681	0.724	0.738	0.740	0.740	0.741
−2.8	−3.973	−2.010	−1.210	−0.460	0.384	0.666	0.702	0.712	0.714	0.714	0.714
−2.9	−4.013	−2.007	−1.195	−0.440	0.390	0.651	0.681	0.683	0.689	0.690	0.690
−3.0	−4.051	−2.003	−1.180	−0.420	0.396	0.636	0.660	0.666	0.666	0.667	0.667

After *Water Resources Council, Bulletin No. 15.*

INDEX

A

Absolute humidity, 14, 15
Abstractions, 61, 63
  Initial, 54
Adiabatic cooling, 19, 21
Air masses, continental, 19
Airport drainage, 297, 300
Albedo, 367
Analog
  electric, 262
  models, 230
Analysis
  depth-area-duration (DAD), 165
  regional, 147
Analytic solutions, 100
Anderson, E. A., 362, 391
Anisotropic media, 225
Anisotropy, 245
Annual series, 134
Antecedent condition, 294
Antecedent precipitation index (API), 60
API model, 334
Aquicludes, 226
Aquifers, 226
  artesian, 253
  confined, 227, 236, 252
  free, 261
  leaky, 261
  performance tests, 250
  permeability, 250
  unconfined, 236, 250
Aquifuge, 227
Aquitards, 226
Areal factor, 290
Artesian aquifer, 253
Atmospheric water
  absolute humidity, 14, 15
  dew-point temperature, 14, 15
  geographic variations, 18
  mixing ratio, 14, 15
  relative humidity, 14, 15
  specific humidity, 14
  temporal variations, 18
  vapor pressure, 14
Autocorrelation function, 158
Autocovariance function, 158

B

Base flow, 61, 62, 67, 69, 78, 86
  hydrograph, 65
  separation, 71, 72
Basin
  characteristics, 66
  isochrones, 187
  properties, 61
  small, 29
Bernoulli, 2
Biot–Fourier law, 257
Biswas, Asit K., 3
Bivariate normal distribution, 151
Blaney–Criddle method, 42
Boundary
  condition, 196, 239, 255
  constant head, 240
  impervious, 240
  reservoir, 240
Bowen's ratio, 36
Bureau of Reclamation, 40

C

Calibrated, 331
Capillary
  water, 354
  zone, 226
Cauchy–Riemann equation, 237
Central limit theorem, 130
Central tendency, 116, 117
Channel
  flow, 62
  precipitation, 61, 67
Characteristic(s)
  basin, 66
  curve, 195
  method, 188, 193
  storm, 66
Chézy equation, 2, 196
Clark, C. O., 73
Climatological data, 74, 76
Coalescence process, 20
Coastal regions, 262
Caoxial correlation, 59
Coefficient
  of permeability, 231, 232
  of skewness, 118
  of variation, 118
Cold content, 371
Complete series, 134
Complex potential, 239
Computers, digital, 262
Concentration
  curve, 61
  time of, 277, 305, 306
Condensation, 19, 358, 363
  hourly snowmelt, 363
  melt, 360
  nuclei, 19, 20
Conditional probability, 108
Conduction, 358, 368
  ground, 368
Cone of depression, 250, 254, 255, 257
Confidence limits, 131
Confined aquifers, 227, 236, 252
Constant head boundaries, 240
Constants, 401
Consumptive use, 41, 42
Continental air masses, 19
Continuity equation, 189, 234, 236
Continuous variables, 106, 157
Control zone, 53
Convection, 358, 362
  condensation melt equation, 361
  hourly snowmelt, 362
  melt, 361
Convective precipitation, 21
Conversion factors, 401
Correlation, 147
  coaxial, 59
  coefficient, 150
  cross, 344
  multiple, 154
  partial, 166
  serial, 157, 344
Courant condition, 198
Covariance, 149
Cover factors, 63, 64
Crawford, N. H., 100, 362, 391
Crest segment, 61
Critical storm method, 292
Cumulative distribution function (CDF), 111
Cumulous clouds, 22
Cyclonic precipitation, 21

D

Daily evaporation, 35
Dalton, 37
Dams, 316
  major, 317
  small, 270, 275
Danger reach, 318
Darcy's law, 2, 230, 233, 241, 242, 251
Data, 10
  hydrologic, 274, 316
  physiographic, 274
da Vinci, Leonardo, 2
De Glee, G. J., 261
Degree day, 377
Density, 354
Depression storage, 6, 28, 31, 46, 61, 310
  intensity, 47
Depth-area-duration (DAD), 322
  analysis, 165
  curves, 325
Derivatives, total, 194
Design, 269, 316
  floods, 318
  frequency, 270
  hydrograph, 275
  storm hydrograph, 85, 276, 316
Detention storage, 62

Deterministic simulation, 331
Deviation, standard, 118
Dew, 19
  point temperature, 14, 15
DeWiest, R. J. M., 242, 243, 261
Digital simulation, 100, 391
Dimensionless hydrograph, 91, 93
Direct runoff, 78, 79
  equation, 277, 288
  hydrograph, 62, 65, 78
  surface, 61
Discharge
  hydrograph, 61
  storage relation, 188
Discrete
  distribution, 114
  variables, 106
Distribution
  bivariate normal, 151
  characteristics, 116
  discrete, 114
  gamma, 93, 123
  gumbel's extremal, 123
  joint, 147
  log-normal, 122
  log Pearson III, 123
  marginal, 148
  moments of, 115
  multivariate, 151
  normal, 120
  Pearson Type III, 123
  plotting, 124
Downdraft, 23
Drainage, 298
  rate, 53
Drawdown, 254
  curve, 255
  total, 254
Drizzle, 19
Dupuit, Jules, 245
Dupuit's equation, 246, 248
Dupuit's parabola, 248
Dupuit's theory, 245
Dynamic cooling, 21

E

Effective
  duration, 78
  rainfall, 79
Emergency spillway, 270
  hydrograph, 277
Energy budget, 32
  method, 35
Entrainment, 22
Environmental problems, 10
Environmental quality, 11
Equation of state, 15
Equilibrium, 62
  equation, 250, 252
  tests, 250
Equipotential drops, 244
Equipotential lines, 236, 238, 241, 253
Eskinazi, Salamon, 239
Euler's equation, 233
Evaporation, 5, 6, 14, 31, 35
  daily, 35
  lake, 31, 39
  method for determining, 39
  pans, 38
  reduction, 39
  reservoir, 38
  surface, 39
Evapotranspiration, 6, 34, 43, 44
  loss, 390
Event simulation, 331, 347
Excess precipitation, 31, 48
Explicit method, 188, 191
Explicit routing, 219
Exponential transformation, 348

F

Field
  capacity, 40, 65, 67
  coefficient of permeability, 231
  permeameters, 250
Finite difference, 262
  approximation, 192
  network, 192, 196
Finite element, 262
First-order process, 155
Flood
  frequency-based, 318, 322
  frequency method, 296
  probable maximum, 318, 319
  standard project, 318, 321
  waves, 316
Flooding infiltrometers, 52
Flow
  chart, 332

Flow (*continued*)
mean daily, 388
nets, 241
refracted, 244
routine techniques, 102
steady confined radial, 252
steady unconfined radial, 250
Flowlines, 236, 237, 238, 253
Fluid compressibility, 235
Fog, 19
Forecasting, 102
runoff, 389
seasonal, 389
short-term, 382
tools, 390
Forest management, 364
Formation constants, 236
Franzini, J. B., 40
Free surfaces, 240
Freeboard, 270
design hydrograph, 290
Frequency, 107
analysis, 106, 124, 134, 159
based floods, 318, 322
factors, 126
relative, 107
studies, reliability of, 130
Frost, 19
FWQA Model, 331

G

Gamma distribution, 93, 123
Gas constant, 15
Generalized basin snowmelt equation, 377
Geology, 226
Graphical integration method, 178
Graphical solution, 253
Gray, D. M., 93
Gray method, 93, 96, 99
Gross precipitation, 29, 31
Gross rainfall, 48
Groundwater, 6, 28, 59, 223
basin, 226, 263
basin development, 262
depletion, 70
development, 249
flow, 5, 66, 223, 229, 262
hydrograph, 67
level, fluctuations in, 228
recession, 68
surfacewater relationships, 228
zone, 225
Gumbel distribution, 123, 129

H

Hagen-Poiseuille, 2
Hail, 19
Halley, 2
Hantush, M. S., 261
Harr, M. E., 233
Heterogeneous media, 225
Hexadecanol, 40
Hicks, W. I., 48
Histogram, 112
History, 1
Holtan, H. N., 52
Homer, 1
Homogeneous fluids, 225
Homogeneous media, 225
Horton, R. E., 51, 354
Huggins, L. F., 53
Humidity
absolute, 14, 15
relative, 14, 15
specific, 14
Hydraulic conductivity, 244, 245
Hydraulic routing, 173, 198
Hydrodynamic equation, 233
Hydrograph, 29, 60, 68, 70
analyses, 52
base flow, 61, 65
channel precipitation, 61
construction, 90
design, 275
design flood, 292
design storm, 85, 276, 316
dimensionless, 90
direct runoff, 62, 65, 78
direct surface runoff, 61, 87
discharge, 61
emergency spillway, 277
family, 290
freeboard design, 290
groundwater, 67
instantaneous unit, 73, 184
interflow, 61
outflow, 29
percentage *S*, 386
recessions, 381
seasonal, 388

Hydrograph (*continued*)
separation, 65
shape, 61, 65, 67
snowmelt, 381
stage, 61
synthesis, 382
time base, 72
unit, 73, 89, 279, 319, 385, 388
Hydrologic budget, 4, 5, 8
Hydrologic cycle, 1, 3
Hydrologic Data, 10, 274, 316
Hydrologic equation, 8
Hydrologic index, 373
Hydrologic model, 100
Hydrologic regional techniques, 269
Hydrologic routing, 173
Hydrologic simulation, 330
Hydrologic system, 4
Hydrology, 1
history of, 1
simulation, 100
snow, 351
Hydro-meteorological factors, 63
Hydrostatics, 228
Hyetograph, 187

I

Ice
content, 354
crystals, 22
crystal process, 20
Image wells, 255
Images, method of, 255
Impervious boundaries, 240
Independent, 107
Index, 54, 356, 374
flood method, 160
hydrologic, 373
infiltration, 54
initial abstractions, 54
methods, 390
ϕ, 54
precipitation, 60
temperature, 376
W, 54
Infiltration, 5, 6, 25, 28, 29, 31, 48, 52, 61, 63, 65, 67, 275, 305
capacity, 52, 53
capacity curves, 311
curves, 64
control zone, 53
drainage rate, 53
galleries, 249
hydrograph analyses, 52
index, 54
infiltrometer studies, 52
rate, 51
soil moisture content, 53
Infiltrometers
flooding, 52
sprinkling, 54
studies, 52
Initial condition, 196
Inlet time, 305
Instantaneous unit hydrograph, 73, 184
Intensity-duration-frequency curve, 137
Interception, 25, 28, 31, 44
losses, 46
storage, 44
total, 45
Interflow, 28, 61, 65, 66, 67
Intermediate zone, 226
Intrinsic permeability, 231, 232
Inverse
exponential transformation, 348
log-normal transformation, 348
transformations, 347
Irrotationality, 239
Islands, 262
Isochrones, 186
basin, 187
Isohyetal pattern, 30, 324
Isotropic medium, 225

J

Jacob, C. E., 236, 261
Joint distributions, 147
Joint probability, 108
Jones, P. B., 3

K

Kiefer, C. J., 48

L

Lag-one Markov chain, 338
Lag time, 88, 89, 91
Lagging, 81
methods, 81
procedures, 78

Lake evaporation, 31
Land-management practices, 29
Langleys, 364
Laplace equation, 234, 247
Leakage, 261
Light, P., 362
Linear equation, 399
Linear reservoirs, 184
Linsley, R. K., Jr., 46, 100, 391
Log-normal
  distribution, 122, 339
  transformation, 347
Log Pearson III distribution, 123, 126
Long-wave radiation, 357
Low-flow frequency, 102
Lull, 44

M

Major structure, 316
Manning's equation, 197, 303
Marginal distribution, 148
Mariotte, 2
Markov process, 155
  lag-one chain, 338
  second-order chain, 341
Mass-transfer techniques, 32, 36, 43
Matrix
  adjoint, 398
  algebra, 86
  cofactor, 397
  definition, 397
  determinant of, 397
  inverse, 399
  multiplication, 398
  notation, 85
  square, 86
  transpose of, 397
Mean, 116
  daily flow, 388
Median, 117
Meinzer, O. E., 3
Melt
  equation, 374
  water, 353
Meyer, A. F., 37
Mining, 228
  of water, 263
Minor structure, 269
Mixing ratio, 14, 15
Mockus, V., 92
Mode, 117
Moisture, atmospheric, 14
Moments, method of, 120
Momentum equation, 190
Monke, E. J., 53
Monomolecular films, 40
Multiple-correlation, 59
  coefficient, 154
Multiple-phase flow, 225
Multiple-regression techniques, 165
Multivariate generating, 344
Muskingum method, 174, 176, 184, 186
Mutually exclusive, 107

N

Nace, Raymond L., 3
Net precipitation, 29
Newton, D. W., 88
Newton's iteration technique, 199, 208
Newton's second law, 233
N. O. A. A., 76
Nonequilibrium
  equation, 257
  tests, 250
Nonhomogeneous aquifers, 245
Normal curve, areas under, 402
Normal distribution, 120, 339
Normal equation, 153
Numerical
  integration, 181
  methods, 262

O

Ohm's law, 230
Orographic precipitation, 21
Orthogonal flow net, 253
Outflow hydrograph, 29
Outlets, 316
Overland flow, 25, 62, 210, 310
  supply, 28

P

Palissy, Bernard, 2
Pan evaporation, 38, 59
  data, 32
Partial correlation coefficient, 166
Partial duration series, 135
Partial series, 134
Peak discharge, 89, 306, 318

Peak time, 89
Pearson Type III distribution, 123, 341, 403
Penman method, 42
Percentage *S*-hydrograph, 386
Permeability
  coefficient of, 231
  field coefficient of, 231
  intrinsic, 231
  specific, 231
Perrault, 2
ϕ-Index, 54
Philip, J. R., 52
Phreatophytes, 42
Physiography
  characteristics, 94
  data, 274
  factors, 29, 63
Pivotal elements, 399
Planck's law, 366, 367
Plants, 41
Plotting distributions, 124
Plotting position, 124
Population, 106
Pore pressure, 228
Porous medium, 223
Potential theory, 253
Precipitable water, 16
Precipitation, 5, 13,19, 20, 31, 44, 58
  artificial, 21
  coalescence process, 20
  convective, 21
  cyclonic, 21
  data, 23
  dew, 19
  distribution, 25
  drizzle, 19
  excess, 31, 48, 65
  fog, 19
  formation of, 20
  frost, 19
  gross input, 29, 31
  hail, 19
  ice crystal process, 20
  index, 60
  interception, 25
  losses, 31
  net input, 29
  orographic, 21
  pattern, 61
  probable maximum (PMP), 270, 318
  rain, 19
  regional variations, 25
  runoff equation, 102
  sleet, 19
  snow, 19
  temporal variations, 25
  variability, 23
Preliminary planning studies, 59
Probability, 107, 294
  conditional, 108
  density function (PDF), 113
  distribution, 111
  joint, 108
  paper, 124
  union, 108
Probable maximum flood, 318, 319
Probable maximum precipitation (PMP), 270, 318
Processes
  first-order, 155
  Markov, 155
  stationary, 106

Q

Quadrangles, 274

R

Radial flow, 250, 252
  equation, 255
Radiation
  daily melt, 365, 366
  long-wave, 357, 364, 367
  melt, 364, 365, 366
  short-wave, 364, 367
  solar, 357
Rain, 19
  gauge network, 29
  gauges, 354
  on-snow occurrence, 382
Rainfall, 367
  daily snowmelt by, 368
  duration, 301
  effective, 79
  excess, 85, 87, 288, 291
  gross, 48
  heat content, 358
  records, 76
  runoff relationships, 353
Rainfall intensity, 25
  time variations, 25
Rainmaking, 21

Random
  variable, 111
  walk, 338
Rating curve, 61
Rational method, 298, 305
Recession, 61
  constant, 70
  curves, 381
  hydrograph, 381
Recharge, 263
  intensity, 248
  wells, 243, 262
Recurrence intervals, 124
Recursion
  equation, 340
  formula, 338, 341
Refracted flow, 244
Regional analysis, 147
Regional simulation, 342
Regression, 152, 343
  line, 152
  theory, 152
Relative frequency, 107
Relative humidity, 14, 15
Reliability
  frequency studies, 130
  statistical studies, 130
Reservoir
  boundary, 240
  evaporation, 38
  linear, 184
  routing, 179, 207
Resistance–capacitance network, 262
Retention, 305
Return period, 109
Reynolds number, 230
Risk, 109
River routing, 188, 196
Road Research Laboratory Method, 186
Routing, 173, 322
  explicit, 205
  method, 305, 308
  Muskingum, 186
  period, 182
  reservoir, 179, 200
  river, 188, 196
  watershed, 183, 203
Runoff, 58
  determination, 101
  forecasting, 389
  Frequency, 101
  probability approach, 101
  snowpack, 372
  statistical approach, 101
  winter, 369

S

Salt-water intrusion, 262
Saturated zone, 226
Schomaker, C. E., 44
Schwartz, H. E., 73, 88
SCS method, 91, 275
S-curve, 387
Seasonal forecasts, 389
Seasonal hydrograph, 388
Second-order Markov chain, 341
Seepage
  lines of, 240
  surfaces of, 240
Separation
  base flow, 71, 72
  techniques, 71
Sequential simulation, 331, 338
Serial correlation, 157
S-hydrograph, 73, 78, 81, 82, 83, 385, 387
Sherman, L. K., 73
Simulation, 100, 330
  digital, 100, 391
  deterministic, 331
  event, 347
  hydrologic, 330
  regional, 342
  sequential, 331, 338
  stochastic, 331
Singular points, 242
Sinks, 243
Skewness, 116, 118
  coefficient, 118
Sleet, 19
Slush layer, 354
Snow, 19
  accumulation, 353
  accumulation-snowmelt runoff process, 353
  areal characteristics, 355
  boards, 354
  course, 355
  hydrology, 351, 391
  measurements, 354
  point characteristics, 355
  quality of, 354

Snow (*continued*)
runoff, 353
stakes, 354
surveys, 354
Snowcover, 354
areal, 357
Snowfall
areal, 356
areal distribution, 355
interception loss, 380
mean annual, 352
Snowmelt, 351, 358, 361, 367
daily by rainfall, 368
energy budget considerations, 357
energy sources, 357
generalized basin equations, 377
hourly condensation, 363
hourly convection, 362
hydrograph, 381
hydrograph syntheses, 382
process, 357
runoff, 385
runoff determinations, 369
runoff equation, 374
six-hour, 363
subroutine, 392
unit graph, 385
Snowpack, 89, 353, 361
condition, 369
runoff, 372
storage, 371
temperature, 371
water balance, 373
Snyder, F. F., 73, 88
Soil moisture, 6, 28, 59
content, 53
deficiency, 65, 67
index, 59
Soil water zone, 225
Solar energy, 364
Solar radiation, 357
Sources, 243
Southern California, 262
Spatially varied unsteady flow equation, 189
Specific humidity, 14
Specific permeability, 231
Specific yield, 226, 227
Spillways, 316
emergency, 270
Sprinkling infiltrometer, 54
Square matrix, 86
Stability, 198
Stage hydrograph, 61
Standard deviation, 118
Standard error, 131
of estimate, 154
Standard project flood, 318, 321
Standard project storm, 318, 321
Stanford Watershed Models, 101, 334, 391
Stationarity, 107
Stationary processes, 106
Statistical studies, reliability of, 130
Steady confined radial flow, 252
Steady flow, 233
Steady flow problem, 225
Steady incompressible flow, 241
Steady unconfined radial flow, 250
Stefan's law, 366, 367
Stochastic, 106
Markov, 155
simulation, 331
time series, 155
Stomatal transpiration, 40
Storage, 174
coefficient, 227, 235, 250
depression, 6, 28, 31, 46, 61, 310
detention, 62
permanent, 372
potential, 371
routing, 384, 388
transitory, 372
Storms, 29
characteristics of, 66
first-quartile, 293, 295
frequency-based, 318
generalized pattern, 325
second-quartile, 293, 294
standard project, 318, 321
Stream function, 237
Streamflow, 6, 13, 58
forecasting, 101
Streamlines, 237, 241
Structures, design
major, 316
minor, 269
Surface
detention, 25
flow phenomenon, 61
runoff, 5, 66, 67, 68
runoff hydrograph, 87
Sutton's equation, 37

Synthetic unit hydrograph, 73, 88, 89, 92, 94
  methods, 73, 88

T

Taylor, A. B., 73, 88
Taylor's series, 246
Temperature index, 376
Theis, C. V., 257
Theis equation, 258
Thermal energy, 358
Thiem, Adloph, 250
Thiessen polygons, 163, 355
Tholin, A. L., 48
Thornthwaite Holzman equation, 43
Thunderstorms
  cumulus stage, 22
  dissipating stage, 22
  downdraft, 23
  mature stage, 22
Time–area curve, 186, 187
  construction, 187
Time base, 72, 90
Time–series analyses, 101, 147, 155
  stochastic, 155
Todd, D. K., 243, 255
Total derivatives, 194
Total drawdown, 254
Tracers, 250
Transformation
  inverse, 347
  inverse exponential, 348
  inverse log-normal, 348
Transition, 155
Transmissivity, 236, 258
Transpiration, 5, 6, 40, 41
  consumptive use, 41
  field capacity, 40
  reduction, 42
  stomatal, 40
  wilting point, 40
Transposition limits, 322
Trial-and-error, 100
Triangular inflow, 185
Turbulent exchange, 358
  equation, 360
Turner, L. B., 48
Type curve, 257

U

Unconfined aquifer, 236, 250
Uniform-flow field, 253
Union probability, 108
Unit graph, 87
Unit hydrograph, 73, 76, 78, 95, 102, 275, 319, 385, 388
  derivation, 76
  duration, 88, 91
  matrix methods, 85
  synthetic methods, 88
Unsteady flow, 234, 257
  problem, 225
Urban storm drain, 302
U. S. Geological Survey, 76
U. S. Weather Bureau, 23

V

Vapor density, 15
Vapor pressure, 14
  gradient, 41
Vapor transfer, 31
Variability, 116, 117
  coefficient, 118
Variable hydraulic conductivity, 243
Variables
  continuous, 106, 157
  discrete, 106
  random, 111
Velocity potential, 232, 237, 238
Verified, 331
Viessman, W., Jr., 48
Vineyard, J. W., 88
Vitruvius, Marcus, 1

W

W Index, 54
Walton, Wm. C., 261
Water
  atmospheric, 18
  balance in snowpack, 373
  basin-wide equivalent, 356
  capillary, 354
  equivalent, 354
  mining, 263
  properties, 401
  vapor, 13
Water budget, 32, 379, 390
  equation, 263
  method, 43
Water resource development, 228
Watershed, 89
  data, 77
  models, 102

Watershed (*continued*)
routing, 183, 210
small, 29
Weighting factor, 174, 175
Well(s), 243, 249
fields, 254
function of, 257
image, 255
partially penetrating, 261
recharge, 243, 262
Wilting point, 40
Winter runoff, 369

504 28
$ 50